Massachusetts Institute of Technology

Factored Symbolic Approach to Reactive Planning

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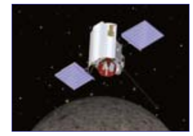
Model-based Embedded & Robotic Systems

CSAIL

Motivation for Reactive Planning

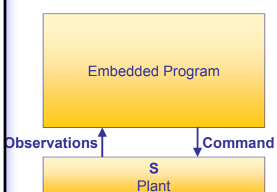


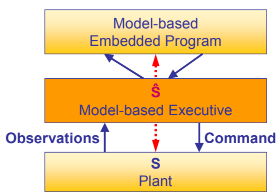


- Reason for Planning
  - Anomalies
    - Environmental
    - System
  - May require repair and/or reconfiguration capabilities.
- Reason for onboard reactive planning
  - Time-critical situations
  - Communication time delays
  - Situation in which no communication available

Model-base Programs Interact Directly with States

- Embedded programs interact with plant sensors and actuators:
  - Read sensors
  - Set actuators
- Model-based programs interact with plant state:
  - Read state
  - Write state



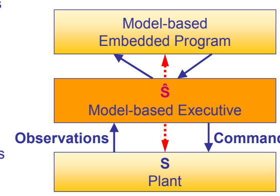


Complexity: Programmer must map between state and sensors/actuators.

Simplification: Model-based executive maps between state and sensors/actuators.

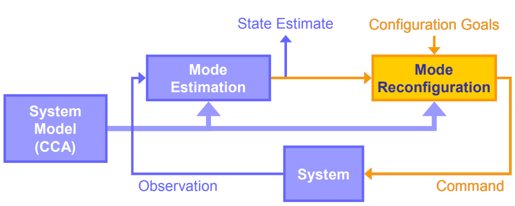
Increase Robustness through Model-based Programming

- Raised Level of Abstraction
  - Code in terms of desired state evolution
  - Fewer lines of code
  - Less chance of introducing bugs
- Executable Specification
  - Increase robustness by synthesizing executable code from the verified specification
  - Models are the specification of the system
  - Model-based Executive operates on the model, i.e. the specification
  - The model-based embedded program is guaranteed to meet the specification



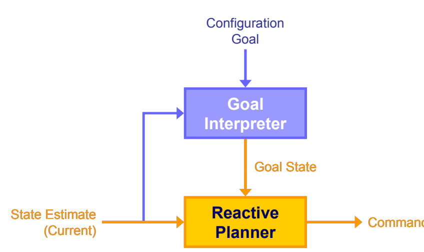
Model-based Executive: Deductive Controller

control nondeterministic system in a nondeterministic environment



Model-based Programming of Intelligent Embedded Systems and Robotic Explorers
[Williams et al., IEEE'03]

Mode Reconfiguration



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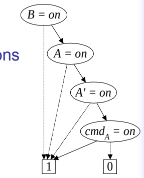
Past Approaches to Planning

- General-purpose Planner
  - Generates a sequence of commands that achieves the goal.
  - A sequence of commands lacks robustness within nondeterministic system and environment.
  - Replanning is expansive.
- Universal Planner
  - Maps all possible initial states to the appropriate actions.
  - State explosion problem
    - Assume:
      - x components
      - in average n number of states per component
    - Number of system states: $O(n^x)$
  - Must replan if the goal state changes.

Recent Advances in Reactive Planning: BDD-based Universal Planning

- Ordered Binary Decision Diagrams (BDD)
  - Compact representation of Boolean functions
  - Efficient algorithms for operating on Boolean functions
- Symbolic Model Checking
  - Use of BDDs for model checking
  - Reduce the state explosion problem
  - Has been very successful [Burch et al., IEEE'90]



- Recognized the similarity of Symbolic Model Checking and Planning [Cimatti et al., ECP'97]
  - Reduce the state explosion problem through the use of BDDs.
- BDD-based Universal planners have been developed:
  - Strong Plan, Strong Cyclic Plan, Optimistic Planning, Etc.

Recent Advances in Reactive Planning: Burton [Williams & Nayak, IJCAI97]

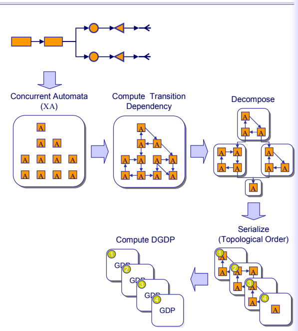
- Goal-directed plan :
  ⟨Current State, Goal State⟩ → Action
- Introduced a decomposition technique that enables subgoal serialization (i.e. in essence, applies a divide-and-conquer approach to reactive planning).
  - Mitigate the state space explosion problem.
  - Enable a compact encoding of a goal-directed plan.
- Only applicable to a limited subset of a planning problems (i.e. cannot generate a plan for a system with interdependent components).

Factored Symbolic Approach to Reactive Planning

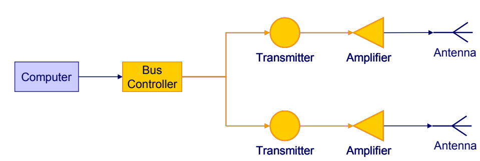
- Unify the two complementary approaches:
  - Address the state space explosion problem at the global level through decomposition : divide-and-conquer
  - Address the state space explosion problem at the subproblem level though BDD-based planning
- Extend the decomposition technique of [Williams & Nayak, IJCAI97] to problems with interdependent components.
- Extend the BDD-based Universal planning technique to generate a goal-directed plan.

Outline

- Spacecraft telecommunication system
- Model: Concurrent automata
- Decomposing the Problem: Transition dependency graph
- Reactive Plan for a Subproblem: Goal-directed plan
- Reactive Plan for the Problem: Decomposed goal-directed plan
- Executing the Plan

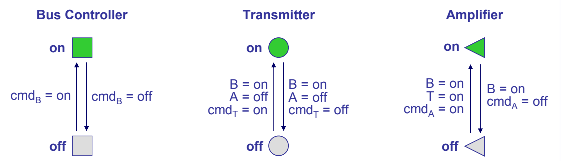


Telecommunication Subsystem Example



- Computer
  - Controls the devices and sends data to the devices.
- Bus Controller
  - Routes the commands and the data to the appropriate devices.
- Transmitter
  - Generates a signal that corresponds to the data to be transmitted.
- Amplifier
  - Amplifies the signal and transmits it to an antenna.

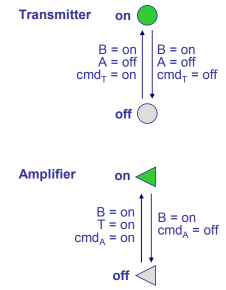
Concurrent Automata (CA)



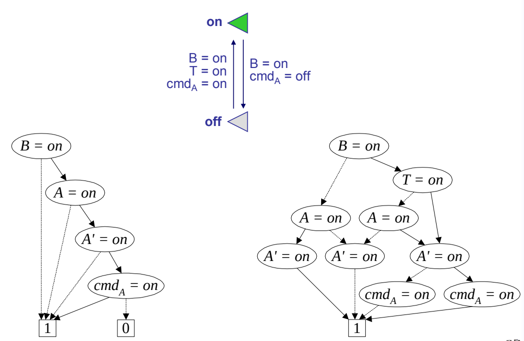
- Synchronous
  - Assume that each automaton performs a single state transition at each time step.
- Interleaved execution within a time step
  - A single main processor executes synchronous activities by interleaving.
  - Devices are not synchronized.

Interdependent Components

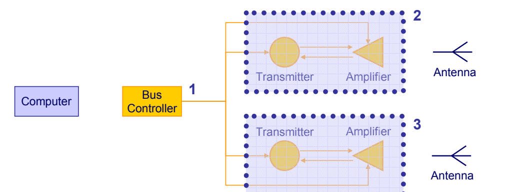
- Turning the transmitter on or off can generate a noise (i.e. transient signal).
- The transient signal may damage the amplifier.
- The amplified transient signal may damage other devices down stream of the amplifier.
- Constraint on the system:
  - The amplifier must be turned off before the transmitter can be turned on or off.
  - The transmitter must be turned on before the amplifier can be turned on.



BDD Encoding of a Concurrent Automaton

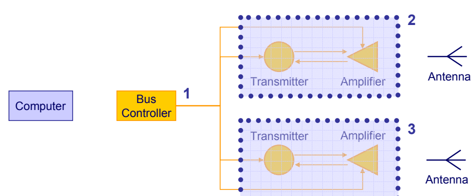


Transition Dependency Graph



- Transition Dependency Graph (TDG)
  - Vertex: for each automaton
  - Edge (v, u): if a transition of the automaton v is conditioned on the state of automaton u.
- Use Strongly Connected Components (SCC) algorithm to find the cyclic components.
- Compose SCC concurrent automata
  - New TDG is acyclic.
  - Serialize the subgoals in the inverse topological ordering.

Subgoal Serialization

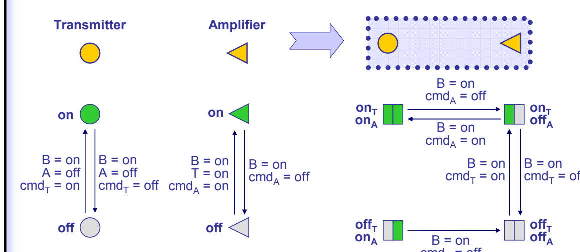


- Goal:
  - Bus Controller = on
  - Transmitter/Amplifier #1 = (on, on)
  - Transmitter/Amplifier #2 = (off, off)
- Solve each subgoal sequentially in the inverse topological order

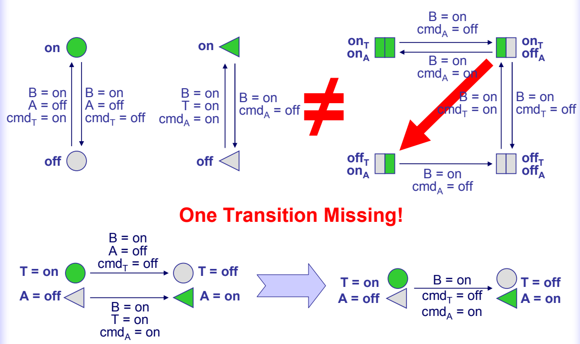
Composing Strongly Connected CA

- Compose all automata into a single automaton

$$R_{SCC} = \bigwedge R_i$$



Interdependent Concurrent Transitions



Simultaneous Commanding

- Both the transmitter and the amplifier depend on one another for the transition.
- The transmitter must be commanded "off" and the amplifier must be commanded "on" precisely at the same time.
- Due to concurrency via interleaving, simultaneous commanding cannot be guaranteed.
- If the amplifier were commanded on first, and then the transmitter is commanded off, the amplifier can be damaged.

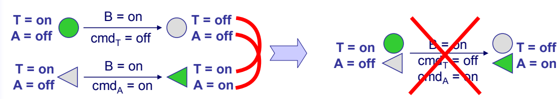
Assuring Proper Execution of Interdependent Transitions

- Enforce concurrency as interleaving:
  - For a given transition, the interdependent state constraints become the pre- and post-conditions.
  - No change to all other automata that are not independent.

Transmitter

Amplifier

Assuring Proper Execution of Interdependent Transitions

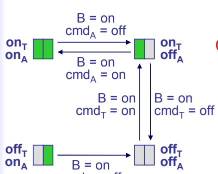


- Inconsistencies are automatically detected when conjoining the transition relations in BDDs.

$$R_{SCC} = \bigwedge R_i$$

- Similar to the Graphplan mutual exclusion rule.
  - Interference:
    - One transition deletes the precondition and/or effect of another.
  - Competing Needs:
    - Inconsistent preconditions

Goal-directed Plan

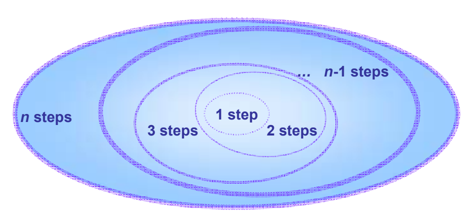


Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
On_T, Off_A	B = on cmd_A = on	idle	B = on cmd_T = off	fail
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

- Goal-directed Plan $\langle s,a,s'\rangle$: $\langle s, s'\rangle \rightarrow a$
  - s : current state
  - s' : goal state
  - a : first action/intermediate subgoals in a trajectory that eventually leads to s'
- Executing a goal-directed plan guarantees:
  - Progress toward the goal.
  - Finite number of actions to achieve the goal.
  - Optimal (shortest) trajectory under nominal conditions.

Computing Goal-Directed Plan: COMPUTEGDP(*T*)

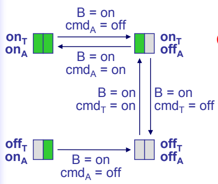
- Iteratively search backward breath-first for the goal-directed rules.
  - Find $\langle s,a,s'\rangle$ that can reach s' within 1 step
  - Find $\langle s,a,s'\rangle$ that can reach s' within 2 steps
  - …
  - Find $\langle s,a,s'\rangle$ that can reach s' within n steps



Generating Goal-Directed Plan

- ⟨*s*,*a*,*s'*⟩ that can reach *s'* within 1 step

Transition Relation



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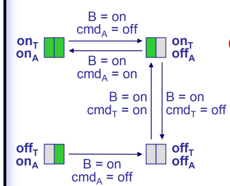
On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
On_T, Off_A	B = on cmd_A = on	idle	B = on cmd_T = off	fail
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Generating Goal-Directed Plan

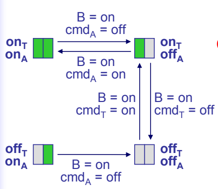
- ⟨*s*,*a*,*s'*⟩ that can reach *s'* within 2 steps
  - To the previous GDP add ⟨*s*,*a*,*s'*⟩ that can reach *s'* in 2 steps:
    - *s*: current state
    - *s'*: goal state that can be reached in 2 steps
    - *a*: first control action that must be commanded to eventually reach *s'*



Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
On_T, Off_A	B = on cmd_A = on	idle	B = on cmd_T = off	fail
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Generating Goal-Directed Plan

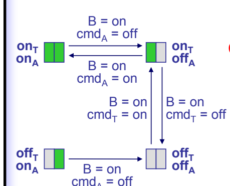
- ⟨*s*,*a*,*s'*⟩ that can reach *s'* within 3 steps
  - To the previous GDP add ⟨*s*,*a*,*s'*⟩ that can reach *s'* in 3 steps:
    - *s*: current state
    - *s'*: goal state that can be reached in 3 steps
    - *a*: first control action that must be commanded to eventually reach *s'*



Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
On_T, Off_A	B = on cmd_A = on	idle	B = on cmd_T = off	fail
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Generating Goal-Directed Plan

- ⟨*s*,*a*,*s'*⟩ that can reach *s'* within 4 steps
  - No new ⟨*s*,*a*,*s'*⟩ exists that can reach *s'* in 4 steps.
  - When the fixed-point is reached, generating the plan is complete.



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On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
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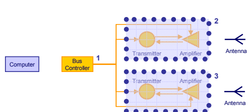
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Computing Decomposed Goal-directed Plan

- For each automaton compute a GDP.



Bus Controller

Current \ Goal	On	Off
On	idle	cmd_B = off
Off	cmd_B = on	idle

Transmitter & Amplifier

Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
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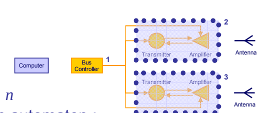
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Size of DGDP

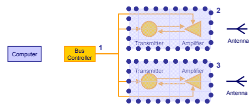


- Given
  - Number of concurrent automata : n
  - Average number of states in each automaton : m
  - Number of strongly connected components : l
  - Average number automata in a strongly connected component : w
  - Number of states for one composed automaton : $O(m^w)$
  - Size of a GDP : $O(m^{2w})$
  - Size of DGDP : $O(l \cdot m^{2w})$
- Approximately linear in the number of components
  - Assume m and w are constant.
  - $O(l \cdot m^{2w})$ is linear in $l < n$.
- Use of BDD makes each GDP even more compact.

Execution

- Achieve subgoals incrementally in the inverse topological order → subgoal serialization ordering

  1. Transmitter/Amplifier #2
  2. Transmitter/Amplifier #1
  3. Bus Controller



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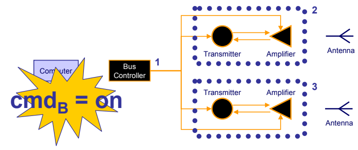
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Execution Example

- Current State:
  - B = off
  - T/A #1 = (off, off)
  - T/A #2 = (off, off)
- Goal State:
  - B = off
  - T/A #1 = (on,on)
  - T/A #2 = (off, off)



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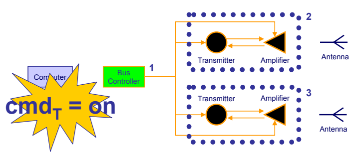

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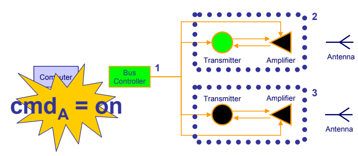

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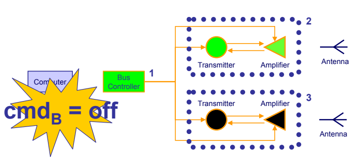

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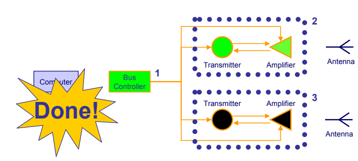
Off	cmd_B = on	idle

Transmitter & Amplifier

Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
On_T, Off_A	B = on cmd_A = on	idle	B = on cmd_T = off	fail
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

Execution Example

- Current State:
  - B = off
  - T/A #1 = (on, on)
  - T/A #2 = (off, off)
- Goal State:
  - B = off
  - T/A #1 = (on,on)
  - T/A #2 = (off, off)



Bus Controller

Current \ Goal	On	Off
On	idle	cmd_B = off
Off	cmd_B = on	idle

Transmitter & Amplifier

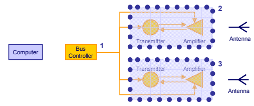
Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
On_T, On_A	idle	B = on cmd_A = off	B = on cmd_A = off	fail
On_T, Off_A	B = on cmd_A = on	idle	B = on cmd_T = off	fail
Off_T, Off_A	B = on cmd_T = on	B = on cmd_T = on	idle	fail
Off_T, On_A	B = on cmd_A = off	B = on cmd_A = off	B = on cmd_A = off	idle

DGDP Execution Capability

- Time complexity of one execution cycle.
  - GDP rule lookup is polynomial execution.
  - DGDP execution is $O(l)$, where l is the number of GDPs.
- DGDP is capable of real-time repair and reconfiguration.
  - Repair capability is necessary when anomalies occur during execution time (e.g. T/A #1 fails into a reparable state).
  - Reconfiguration capability is necessary when goal-states change quickly (e.g. turn on T/A #2 instead).

Transmitter & Amplifier

Current \ Goal	On_T, On_A	On_T, Off_A	Off_T, Off_A	Off_T, On_A
On_T, On_A	idle	B = on, cmd_A = off	B = on, cmd_A = off	fail
On_T, Off_A	B = on, cmd_A = on	idle	B = on, cmd_T = off	fail
Off_T, Off_A	B = on, cmd_T = on	B = on, cmd_T = on	idle	fail
Off_T, On_A	B = on, cmd_A = off	B = on, cmd_A = off	B = on, cmd_A = off	idle



Conclusion

- Factored Symbolic approach to Reactive Planning enables:
  - Compact decomposed goal-directed plan compilation through:
    - Decomposition
    - BDD encoding
  - Real-time execution capabilities:
    - Reactive repair
    - Reactive reconfiguration
    - Approximately linear in the number of components
- Possible Extension
  - Add probability and utility using ADD