Massachusetts Institute of Technology

Manipulator Path Planning with Obstacles using Disjunctive Programming

ACC06 Short Paper

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Model-based Embedded & Robotic Systems

Motivation

- Would like robotic manipulators to be able to operate in cluttered environments

MIT Cooperative Construction Testbed

JPL LEMUR: In-space Inspection and Assembly

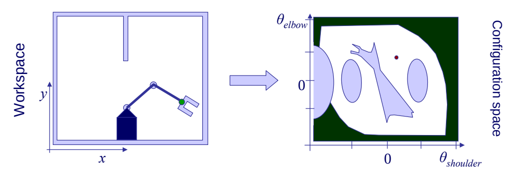
Problem Statement

- Given an initial configuration, find the **optimal** path that **avoids obstacles** and ensures that manipulator endpoint ends at goal position

LB1

Current Approaches

- Current manipulator path planning methods plan in **configuration space**
  1. Convert feasible region from **workspace** to **configuration space** offline



  2. Solve planning problem using existing methods
     - Potential field methods (Not optimal or complete)
     - Probabilistic Roadmaps (Optimal, complete in probabilistic sense)

New Approach: Key idea

- Recent methods have posed path planning problem for aircraft as **constrained optimization**
- Key ideas:
  1. Very simple models of plant are 'sufficient'
     – Double integrator with constraints on velocity and acceleration
     – Assume low-level controller can achieve anything within these constraints
     – So the state at any future time is a **linear function of control inputs**
  2. Obstacle avoidance can be posed as satisfaction of **disjunctive linear constraints**
  ➢ Solve efficiently using **disjunctive linear programming**
- Novel approach for **manipulators**:
  – Plan directly in **workspace** using constrained optimization approach
    • No pre-computation necessary
    • Optimal and complete

Assumptions

- Manipulator in 2D or 3D with **arbitrary number** of rotational joints
- Plan in discrete time
- Low velocity, accelerations
  – Dynamics can be ignored
- Key challenge: highly nonlinear kinematics

Slide 3

LB1 Note that this is not 'find joint angles to follow an endpoint trajectory'

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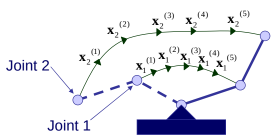
Workspace Planning Approach

- Instead of planning joint angles, **design finite trajectory for each joint in workspace**
  - **Joint 1:** $\mathbf{x}_1^{(1)}, \mathbf{x}_1^{(2)}, \ldots, \mathbf{x}_1^{(k)}$
  - **Joint 2 (endpoint):** $\mathbf{x}_2^{(1)}, \mathbf{x}_2^{(2)}, \ldots, \mathbf{x}_2^{(k)}$

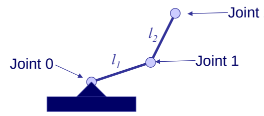


- Solving for joint angles given all joint locations straightforward
- Assume can achieve given joint angle using low level controller

LB3

Kinematic Constraints

- At any time step i, the desired joint positions must be **kinematically feasible**
  - Manipulator must be able to achieve the joint positions
- Joint 1 must be distance l_1 from joint 0
- Joint 2 must be distance l_2 from joint 1



- These are **quadratic constraints** on the desired joint positions

Kinematic Constraints

- Joint 1 must be distance l_1 from joint 0

$$\mathbf{x}_1^{(t)T} \mathbf{x}_1^{(t)} = l_1^2$$

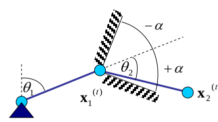
- Joint 2 must be distance l_2 from joint 1

$$\left(\mathbf{x}_2^{(t)} - \mathbf{x}_1^{(t)}\right)^T \left(\mathbf{x}_2^{(t)} - \mathbf{x}_1^{(t)}\right) = l_2^2$$

- These are **quadratic equality constraints** on the desired joint positions

Joint Angle Constraints

- Not all joint angles are feasible:

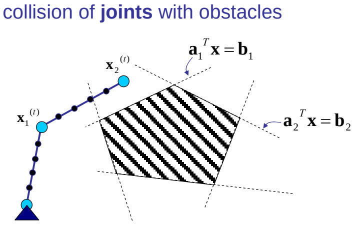


- Many joint angle constraints can be expressed as quadratic constraints also, for example:

$$\left(\mathbf{x}_2^{(t)} - \mathbf{x}_1^{(t)}\right)^T \left(\mathbf{x}_2^{(t)} - \mathbf{x}_1^{(t)}\right) \geq l_1 l_2 \cos\alpha$$

Obstacle Constraints

- Disjunctive linear constraints on **joint positions** prevents collision of **joints** with obstacles



$$\vee_{i=1\ldots N}\ \mathbf{a}_i^T \mathbf{x}_j^{(t)} > \mathbf{b}_i$$
$$\forall j, t$$

- Disjunctive linear constraints on **intermediate points** prevent collision of **links** with obstacles

$$\vee_{i=1\ldots N}\ \mathbf{a}_i^T \left(\lambda \mathbf{x}_j^{(t)} + (1-\lambda)\mathbf{x}_j^{(t)}\right) > \mathbf{b}_i$$

Dynamic Constraints

- Use linear constraints to encode highly simplified dynamics
  - Cartesian velocity constraints

$$\frac{\mathbf{x}_j^{(t)} - \mathbf{x}_j^{(t-1)}}{\Delta t} \leq \mathbf{v}_{\max}$$

- Encode goal constraint in terms of endpoint position

$$\mathbf{x}_2^{(k)} = goal$$

- Encode initial configuration constraint

Slide 7

LB3 Highlight example is 2 joint but general works

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Optimality

- Minimum control effort can be expressed piecewise linearly
- Minimum time can be expressed piecewise linearly
- Minimum energy can be expressed quadratically
- Minimum deviation of joint position from centre of workspace can be expressed quadratically

Summary

- Path planning for manipulators with obstacles expressed as **constrained optimization**:

Model Component	Constraint type
Limited dynamics	Linear, inequality
Obstacles	Disjunctive linear, inequality
Kinematics	Quadratic, equality
Joint limits	Quadratic, inequality

- Now show problem can be approximated as Disjunctive Linear Program
  – Globally optimal solution can be guaranteed

DLP Solution

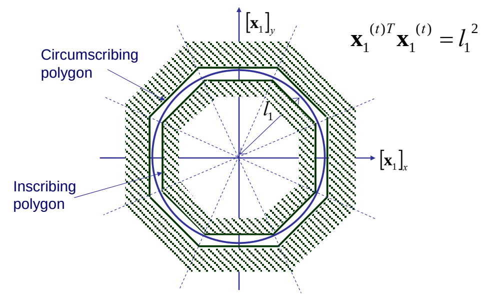
- Disjunctive Linear Programming encoding

Model Component	Constraint type	
Limited dynamics	Linear, inequality	✓
Obstacles	Disjunctive linear, inequality	✓
Kinematics	Quadratic, equality	?
Joint limits	Quadratic, inequality	?

- Main challenge: quadratic constraints
  – Can be approximated using disjunctive linear constraints

DLP Solution

- Approximating quadratic constraints

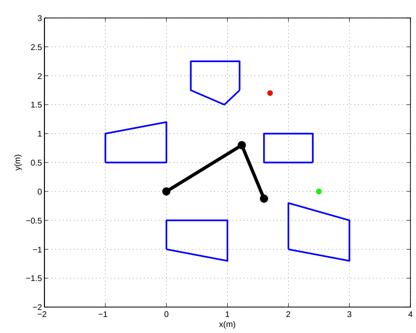


- Challenge: adding edges increases # disjunctions

Preliminary Results

- 3-D intractable
  – Approximate spherical constraints introduce very large number of disjunctive linear constraints
- 2-D solution to global optimum tractable for relatively small problems
  – Length of horizon and # joints drive complexity
- Will show typical results from 2-D

Preliminary Results



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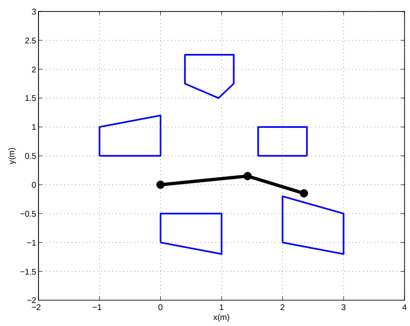

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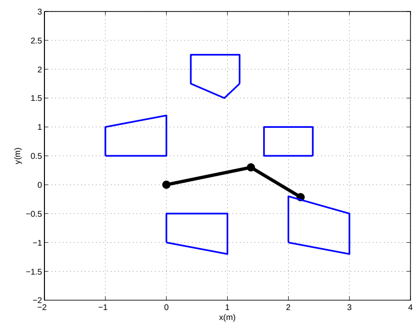

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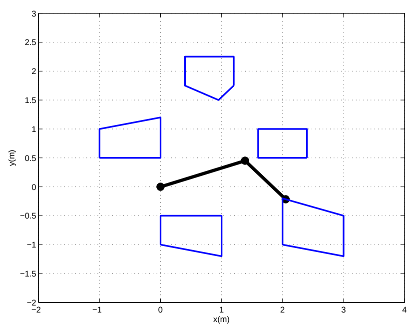

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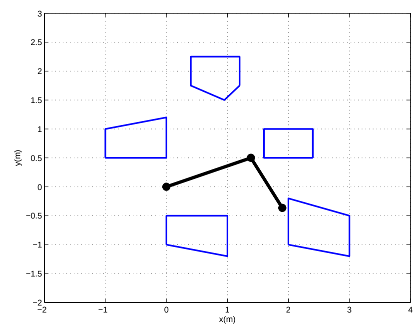

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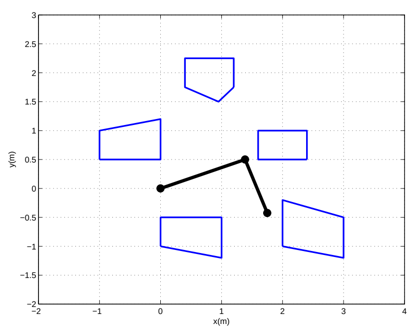

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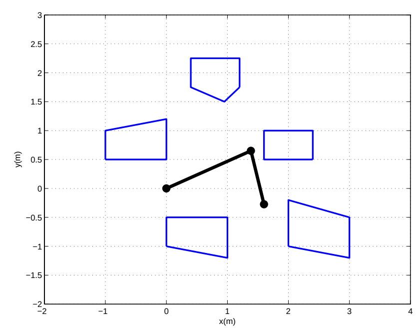

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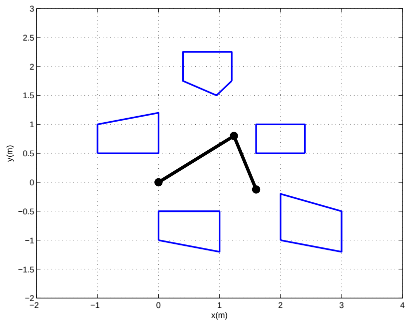

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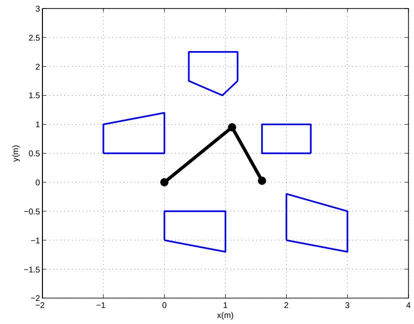

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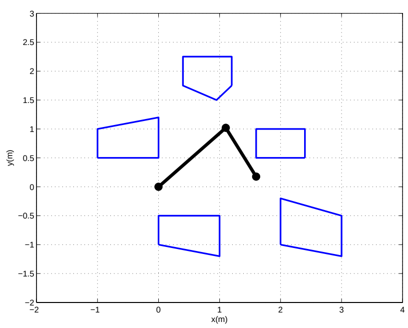

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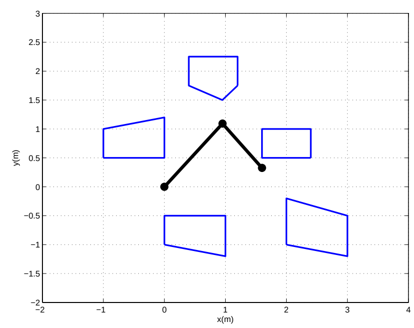

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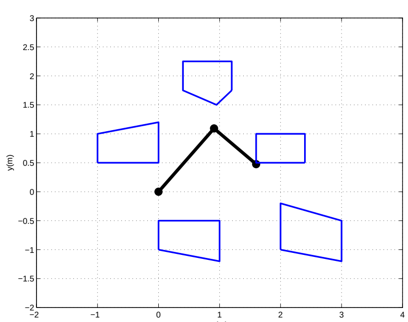

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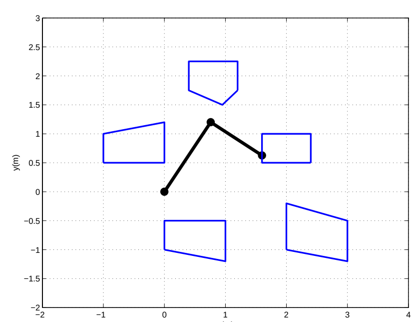

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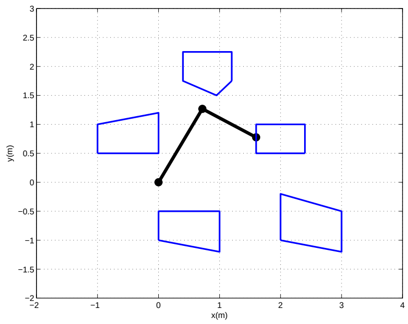

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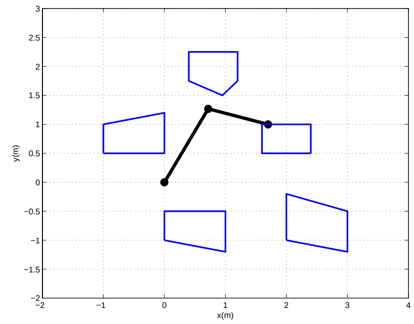

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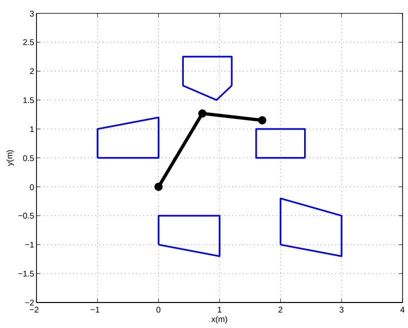

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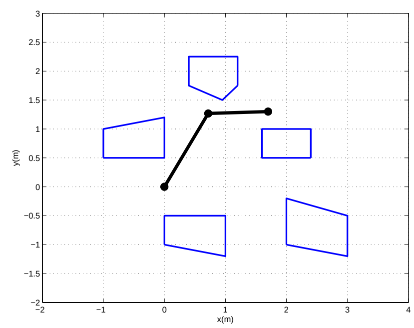

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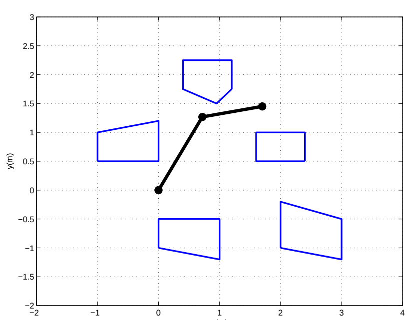

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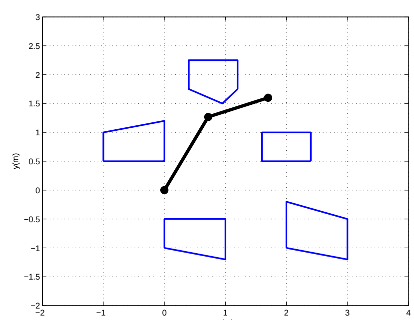

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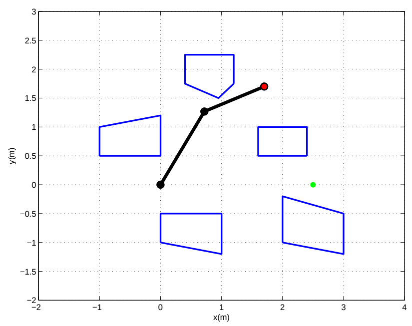

Conclusion

- Novel approach for manipulator path planning with obstacles
- Constrained optimization formulation appealing
  - Guarantees of optimality, completeness
- In practice, solution intractable with existing approaches for large problems
- Advances in quadratic disjunctive programming may make this approach practically effective
- Receding horizon formulation can reduce complexity

Questions?