Meteorites may follow a chaotic route to Earth

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It is widely believed that meteorites originate in the asteroid belt, but the precise dynamical mechanism whereby material is transported to Earth has eluded discovery. The observational data for the ordinary chondrites, the most common meteorites, impose severe constraints on any proposed mechanism. The ordinary chondrites are not strongly shocked, their cosmic ray exposure ages are typically <20 Myr, their radiants are concentrated near the antapex of Earth's motion and they show a pronounced 'afternoon excess' (for every meteorite which falls in the morning two fall in the afternoon). Wetherill[1] concluded that these data could only be explained by an "unobserved source" of material with perihelia near 1.0 AU and aphelia near Jupiter. His subsequent, more sophisticated investigations have not changed this basic conclusion. Recently I have shown[2,3] that there is a large chaotic zone in the phase space near the 3/1 mean motion commensurability with Jupiter and that the chaotic trajectories within this zone have particularly large variations in orbital eccentricity. Since asteroidal debris is quite easily injected into this chaotic zone, it could provide Wetherill's 'unobserved source' if chaotic trajectories which begin at asteroidal eccentricities ($e < 0.2$) reach such large eccentricities that Earth's orbit is crossed ($e > 0.57$)[4]. In this report I present a numerical integration which demonstrates that at least some of these chaotic trajectories do have the properties required to transport meteoritic material from the asteroid belt to Earth. Combined with the Monte Carlo calculations which show that the resulting meteorites are consistent with all the observational constraints, the case for this chaotic route to Earth is fairly strong.

Wetherill[5,6] has completed an extensive new Monte Carlo study to determine the meteoritic yield from this chaotic zone, with some reasonable assumptions concerning the actual dynamics. He assumes that on a timescale of 10^6 years, chaotic trajectories evolve to become Earth-crossing. During much of this time the trajectories are Mars-crossing. He estimates that there is a 50% chance that the material will become Earth-crossing before Mars removes it from the chaotic zone. (It has been suggested previously[7] that meteorites could be brought to Earth from the 3/1 resonance with the assistance of Mars. The timescale for this process is, however, longer than the time for collisional disruption of meteorite-sized bodies, and fragments from any larger bodies which have taken this route will not satisfy the orbital constraints for the ordinary chondrites. It is essential, therefore, that trajectories can become Earth-crossing without the assistance of Mars.) Once the fragment becomes Earth-crossing, however, perturbations from Earth remove it from the chaotic zone in 10^4 years, the approximate duration of the Earth-crossing spikes in eccentricity. With this assumed dynamical input, the subsequent evolution is followed with the Monte Carlo method[8,9]. Wetherill's model includes planetary close encounters, secular resonances, collisional destruction in space and atmospheric ablation. He concludes that major quantities of asteroidal fragments from bodies adjacent to the 3/1 gap can be transferred to Earth. His computed flux is comparable to the observed flux of ordinary chondrites. The afternoon excess of meteorite-sized fragments obtained directly from the chaotic zone is too large, but when the fall-times of these fragments are combined with the fall-times of fragments obtained from larger objects derived earlier from the 3/1 gap, the predicted fall-time ratio agrees with the ordinary chondrite value of 2/3. Similar agreement is found for the other observable parameters.

Greenberg and Chapman[10] have also considered the consequences of this transport mechanism; while they also compute a flux of chondrites consistent with the observed flux, their physical assumptions and methods are quite different from those of Wetherill. The purpose of this letter is not to enter that debate, but to make firm the dynamics on which it is based. If trajectories in the 3/1 chaotic zone do not have the required properties the discussion is moot.

In this article 'chaotic behaviour' refers to deterministic solutions of hamiltonian systems of equations that behave erratically and show sensitive dependence on initial conditions (positive Lyapunov characteristic exponent). On theoretical grounds[11], a chaotic zone is generally expected to be associated with each commensurability in a dynamical system, though these chaotic zones are often microscopically small. Using Schubart's numerical averaging procedure[12], Giffen[13] showed that the 2/1 commensurability in the planar-elliptic restricted three-body problem is accompanied by a sizeable chaotic zone. Scholl and Froeschle[14,15] extended Giffen's work to the other principal commensurabilities; while they found chaotic behaviour near the 3/1 commensurability, they concluded that the chaotic zone was small and unimportant. In fact, there is a large chaotic zone near the 3/1 commensurability, and trajectories within this zone have the peculiar property that their orbital eccentricity can remain low ($e < 0.1$) for as long as a million years and then suddenly assume very large values ($e > 0.3$)[2,3]. The long-term evolution of a typical chaotic trajectory has bursts of low eccentricity behaviour followed by bursts of high eccentricity behaviour, each of apparently random duration. Furthermore, when the effects of phase mixing are removed, the boundaries of the 3/1 Kirkwood gap correspond, within the errors of the asteroid orbital elements, to the boundary in the phase space of the chaotic zone. The 3/1 Kirkwood gap can be explained entirely by removal of the highly eccentric chaotic orbits and the also highly eccentric quasiperiodic librators by collisions with Mars and, as reported here, Earth.

These discoveries depended on the development of a new 'mapping' technique for studying the long-term evolution of trajectories near commensurabilities that is approximately 1,000 times faster than all previous methods. This method is approximate (as are the methods used in earlier studies), but the essential features of the picture presented above have been verified by conventional numerical integrations of the exact equations of motion for the planar-elliptic restricted three-body problem[3]. In particular, the width of the chaotic zone is in perfect agreement with that found using the mapping method. Even at eccentricities as high as 0.4, the trajectories using the mapping and the exact solutions are in good qualitative agreement, even though fourth-order terms in eccentricity are neglected in the disturbing function. Independent investigation has confirmed this qualitative agreement[16].

In the planar-elliptic problem, the eccentricity of chaotic trajectories which begin at asteroidal eccentricities ($e < 0.2$) seems to be limited to values <0.5; this is large enough for the trajectories to be Mars-crossing but not Earth-crossing. However, in the three-dimensional elliptic problem, chaotic trajectories computed with a mapping reach much larger eccentricities, large enough to be Earth-crossing ($e > 0.57$). Now the approximations used to derive the mapping are certainly invalid at such large eccentricities, but the success of the planar mapping at eccentricities as large as 0.4 encourages one to take these high eccentricities seriously. The actual eccentricity limit can be determined only by numerical integration of the exact equations of motion. Such integrations are time-consuming, but the answer is vital to our understanding of the origin of meteorites.

One approach to this problem is simply to integrate a chaotic trajectory until it reaches Earth-crossing eccentricities. Experience with the mapping, however, indicates that this may take several million years. Furthermore, not all chaotic trajectories are equivalent; there is an integral which divides the chaotic zone into a one-parameter family of chaotic zones. Some of these may have Earth-crossing trajectories and others not; we do not accurately know the extent of the eccentricity variations for any chaotic trajectory. It might then be necessary to integrate several chaotic trajectories for several million years to find the Earth-crossing route. This approach is not practical without a dedicated computer. Instead, I have sought, by trial and error, trajectories which begin near that point in the phase space where transitions from low eccentricity behaviour to high eccentricity behaviour occur, with as much guidance from the mapping as I have been able to glean. (While the mapping does give the correct qualitative behaviour, the correspondence is not precise enough to directly determine initial conditions.)

The numerical integrations include the Sun, the four major planets and a test body. Pluto is clearly not important because of its small mass. The inner planets are included with the Sun. I am attempting to show that the long-term evolution of a chaotic trajectory without close planetary encounters will bring material within the influence of Earth's gravitational field. It seems fair to ignore Earth until this is the case. The neglect of Mars perturbations is clearly more serious. I argue that Mars substantially affects the solution only during close encounters and that these close encounters may be taken into account after the fact in a statistical manner, much as Wetherill[6] has done. (His approach may not be entirely satisfactory. As mentioned previously, there is a family of chaotic zones, not just one. Minor encounters with Mars will be sufficient to cause a shift from one member of this family to another without removing the trajectory entirely from the chaotic zone. Several such shifts may occur before Earth is reached. Nevertheless, such trajectories would essentially be injected directly into Earth's control. Thus this synergism between Mars perturbations and chaotic dynamics is qualitatively different from previously proposed mechanisms[7] that rely on Mars perturbations to assist in the transfer of material from the Kirkwood gaps.)

The integrations were performed in equatorial rectangular coordinates referred to the mean equinox and equator 1950.0. The origin was the baricentre of the Sun and four inner planets. The unit of time was 40 mean solar days; the origin of time was taken to be 1982 August 19.0 (JD 2,445,200.5). The extrapolation algorithm of Bulirsch and Stoer[17] was used to perform the integrations. The relative accuracy per integration step was chosen to be 10^{-11}; the variable integration step was of the order of 1 yr. The twelfth-order Cowell method used by Cohen *et al.*[18] in their 1-Myr integration is less satisfactory when the eccentricities are large as in the integrations reported here. The initial conditions for the major planets were obtained from those used by Cohen *et al.* by numerical integration from their origin of time (JD 2,430,000.5) to mine. Their initial conditions for the major planets were in turn taken from the work of Eckert *et al.*[19], where the equations of motion can be found. No effort has been made to correct the initial conditions to properly account for my removal of Pluto. Because of the large Pluto mass used by Eckert *et al.* this introduces a relative discrepancy of one part in a million in the Cartesian coordinates just in the integration from their origin of time to mine. (I have verified, however, that when Pluto is included the program satisfactorily reproduces the tables of Eckert *et al.*[19].) The accuracy of these numerical integrations was tested directly by integrating the planets forward with one test meteoroid 170,000 years, and then integrating them back to the time origin with a different meteroid on an independent trajectory. Since the time step chosen by the integrator is mainly determined by the eccentricity of the meteoroid, these two integrations are completely independent. The initial conditions of the major planets were reproduced to one part in 10^4. The skeleton of the integration is thus adequately represented.

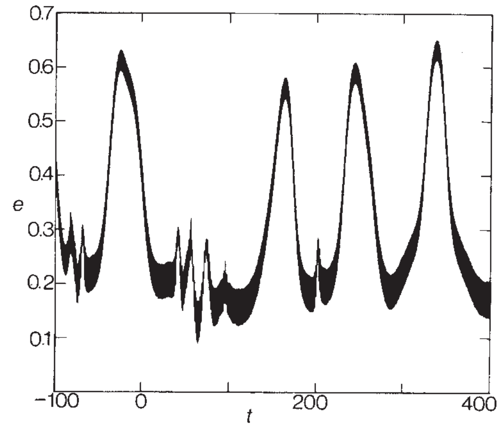

Fig. 1 The eccentricity (e) as a function of time (t, in millennia) plotted for the chaotic trajectory, which at times is deep in the asteroid belt (when $e < 0.15$) and at other times is strongly Earth-crossing ($e > 0.6$). The time origin is 1982 August 19.0.

It is well known that chaotic trajectories are difficult to follow precisely. In a chaotic zone neighbouring trajectories separate exponentially with time, and consequently so do round-off errors. Fortunately, the rate of this exponential separation is rather small for the 3/1 chaotic zone[3]; nevertheless, even a small exponential growth of errors is unbeatable. Now it has been argued[20] that even though any particular trajectory cannot be followed precisely, the qualitative behaviour of the trajectory will be correct if reasonable care is taken and furthermore that a true trajectory can be found which shadows the computed trajectory. This is not as wild a claim as it may seem, for it has been proven for the special class of Anosov dynamical systems. It is probably not strictly true for Hamiltonian systems in general, but most people who study chaotic behaviour in dynamical systems believe that it is practically true. The actual rate of separation of nearby trajectories in the 3/1 chaotic zone allows strict confidence to be placed in segments of 80,000 years[3]. Over longer times we may only hope that some true trajectory shadows

Table 1 Equatorial frame coordinates and velocities at JD 62043200.5

	x (AU)	y (AU)	z (AU)
Meteoroid	$0.262137541577 \times 10^{1}$	$-0.949476934922 \times 10^{0}$	$-0.179096284774 \times 10^{0}$
Jupiter	$-0.351023142831 \times 10^{1}$	$0.339348310152 \times 10^{1}$	$0.151726945011 \times 10^{1}$
Saturn	$0.777797463489 \times 10^{0}$	$-0.824734443595 \times 10^{1}$	$-0.369212909944 \times 10^{1}$
Uranus	$-0.748876946495 \times 10^{1}$	$-0.165985490544 \times 10^{2}$	$-0.644552105739 \times 10^{1}$
Neptune	$0.287412144601 \times 10^{2}$	$0.842938531215 \times 10^{1}$	$0.260621846977 \times 10^{1}$
	$\dot{x}$ (AU/40 days)	$\dot{y}$ (AU/40 days)	$\dot{z}$ (AU/40 days)
Meteoroid	$0.311286716224 \times 10^{0}$	$0.216759866523 \times 10^{0}$	$0.772871903870 \times 10^{-1}$
Jupiter	$-0.212645617591 \times 10^{0}$	$-0.206291875136 \times 10^{0}$	$-0.807809459066 \times 10^{-1}$
Saturn	$0.232711603323 \times 10^{0}$	$0.234902479708 \times 10^{-1}$	$0.286504328835 \times 10^{-2}$
Uranus	$0.145063206654 \times 10^{0}$	$-0.519858201645 \times 10^{-1}$	$-0.249322339022 \times 10^{-1}$
Neptune	$-0.382851184016 \times 10^{-1}$	$0.109983656044 \times 10^{0}$	$0.465724813196 \times 10^{-1}$

the computed trajectory. In any case 80,000 years is long enough to cover the most interesting behaviour.

I have made five numerical integrations, each spanning ~500,000 years. The initial conditions were chosen in the chaotic zone as determined by the three-dimensional mapping. The initial eccentricity for the first four was 0.15, the mean eccentricity of the asteroids; for three of these the resulting peak in eccentricity was between 0.5 and 0.54, insufficient for Earth-crossing. Earth's maximum aphelion occurs at 1.07 AU (ref. 21); for semimajor axes near the 3/1 commensurability ($a \approx 2.5$ AU) an eccentricity of 0.57 is required to just graze this distance. The results of the first three integrations were encouraging but not sufficient. The fourth integration, however, reached an eccentricity of 0.564 after 170,000 years. Its initial conditions (JD 2,445,200.5) were: semimajor axis $a = 0.4816 a_J$, eccentricity $e = 0.15$, inclination $= 5°$, mean anomaly $M = 180° + 3M_J$, longitude of perihelion $\varpi = \varpi_J$ and longitude of ascending node $\Omega = \varpi_J$. These elements are heliocentric osculating elements relative to the equinox and ecliptic 1950.0, and the subscript J refers to Jupiter. These initial conditions are on the representative plane[3]. This trajectory came so close to Earth-crossing (0.01 AU) that it appeared possible to change the coordinates enough to make it Earth-crossing and still remain in the chaotic zone. The first attempt to do this was, however, unsuccessful. The resulting trajectory was clearly quasiperiodic in nature. With a somewhat smaller increase in eccentricity, however, the route to Earth was found.

The initial conditions for this chaotic trajectory are given in Table 1. The trajectory was integrated both forward and backward in time from that point. The eccentricity as a function of time (in millennia) is shown in Fig. 1; the peak in eccentricity occurs near $t = 340{,}000$ yr with the value $e_{max} = 0.652$; the minimum perihelion distance is $q_{min} = 0.861$ AU, which is strongly Earth-crossing. The peak near $t = -20{,}000$ yr has $e_{max} = 0.632$ and $q_{min} = 0.910$ AU. The minimum in eccentricity occurs near $t = 60{,}000$ yr during a short burst of low eccentricity behaviour with the value $e_{min} = 0.089$; this value is lower than the mean eccentricity of the asteroids. The semimajor axis remains between 2.47 AU and 2.53 AU as expected. The inclination to the ecliptic varies between 1° and 8° in a way that is not obviously correlated with the eccentricity. The behaviour of the resonant argument $\sigma = l + 2\varpi - 3l_J$, where l is the mean longitude, is very complicated. During the eccentricity spikes it librates about a slowly varying centre which makes one revolution during the spike; during the low eccentricity bursts it circulates. The behaviour of the semimajor axis is correlated with the resonant argument: during the eccentricity spikes the semimajor axis oscillates across its full range of variation, but during the low eccentricity bursts it alternately oscillates on one side or the other of the centre of that range. This behaviour is familiar from the mapping studies, and consistent with the qualitative picture of these bursts presented previously[3]. The maximum Lyapunov exponent for this chaotic trajectory is $\lambda \approx 10^{-3.8}$ yr^{-1}.

The peak in eccentricity that occurs near 170,000 years, near our starting point, has $e_{max} = 0.583$ and $q_{min} = 1.034$ AU. Since q_{min} is less than Earth's maximum aphelion of 1.07 AU I was initially satisfied that this trajectory was indeed Earth-crossing; on closer examination this turned out not to be the case. I first checked that the argument of perihelion (the angle between the longitude of the ascending node and the longitude of perihelion was satisfactory. It turns out to be correlated with the peaks in eccentricity, and to be ideally placed for Earth-crossing (near zero). If it had been near 90° when the eccentricity was large, then Earth's orbit would not intersect the meteoroid's orbit—the meteoroid's orbit would have crossed the ecliptic near Mars. However, a more subtle avoidance mechanism was operative. During the peaks in eccentricity the longitude of perihelion aligns with Jupiter's longitude of perihelion. Although it is not commonly known, Earth's perihelion is also correlated with Jupiter's perihelion. Earth's maximum aphelion distance always occurs when the perihelia of Earth and Jupiter are aligned. Thus, if the meteoroid and Earth are both near their maximum eccentricities, their orbits are aligned so that no crossing occurs. It is a simple matter to calculate from Brouwer and van Woerkom's solution[22] of the secular motions of the planets that when Earth's perihelion is anti-aligned with Jupiter's perihelion, Earth's maximum aphelion is only 1.031 AU. During this peak the meteoroid came quite close to Earth, but "An inch is good as a mile". Nevertheless, the other peaks in eccentricity are strongly Earth-crossing and no avoidance mechanism is operative.

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1. Wetherill, G. W. *Science* **159**, 79-82 (1968).
2. Wisdom, J. *Astr. J.* **87**, 577-593 (1982).
3. Wisdom, J. *Icarus* **56**, 51-74 (1983).
4. Wisdom, J. *Meteoritics* **18**, 422-423 (1983).
5. Wetherill, G. W. *Meteoritics* **19**, 335 (1984).
6. Wetherill, G. W. *Meteoritics* (in the press).
7. Scholl, H. & Froeschlé, C. in *Comets, Asteroids, Meteorites; Interrelations, Evolution and Origins* (ed. Delsemme, A. H.) 293-295 (University of Toledo Press, 1977).
8. Arnold, J. R. *Astrophys. J.* **141**, 1536-1556 (1965).
9. Wetherill, G. W. in *Origin and Distribution of the Elements* (ed. Ahrens, L. H.) 423-443 (Pergamon, Oxford, 1968); *Icarus* **37**, 96-112 (1979).
10. Greenberg, R. & Chapman, C. R. *Icarus* **55**, 455-481 (1983).
11. Chirikov, B. V. *Phys. Rep.* **52**, 263-379 (1979).
12. Schubart, J. *Smithsonian Astrophys. Obs. Spec. Rep.* No. 149 (1964).
13. Giffen, R. *Astr. Astrophys.* **23**, 387-403 (1973).
14. Scholl, H. & Froeschlé, C. *Astr. Astrophys.* **33**, 455-458 (1974).
15. Froeschlé, C. & Scholl, H. *Astr. Astrophys.* **42**, 457-463 (1975).
16. Murray, C. D. & Fox, K. *Icarus* **59**, 221-233 (1984).
17. Bulirsch, R. & Stoer, J. *Numerische Math.* **8**, 1-13 (1966).
18. Cohen, C. J., Hubbard, E. C. & Oesterwinter, C. *Astr. Pap. of the American Ephemeris* **22**, 1 (1973).
19. Eckert, W. J., Brouwer, D. & Clemence, G. M. *Astr. Pap. of the American Ephemeris* **12** (1951).
20. Benettin, G., Casartelli, M., Galgani, L., Giorgilli, A. & Strelcyn, J.-M. *Nuovo Cim.* **44**, 183-195 (1978).
21. Berger, A. L. *Astr. Astrophys.* **51**, 127-135 (1976).
22. Brouwer, D. & van Woerkom, A. J. J. *Astr. Pap. of the American Ephemeris* **12**, 2 (1950).

Poorly graphitized carbon as a new cosmothermometer for primitive extraterrestrial materials

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The presence of carbon in primitive extraterrestrial materials has long been considered a useful indicator of prevailing geochemical conditions early in the formation of the Solar System. A recent addition to the suite of primitive materials available for study by cosmochemists includes particles collected from the stratosphere called chondritic porous (CP) aggregates[1]. Carbon-rich CP aggregates are less abundant in stratospheric collections and contain many low-temperature phases (such as layer silicates) as minor components[2,3]. We describe here the nature of the most abundant carbon phase in a carbon-rich CP aggregate (sample no. W7029*A) collected from the stratosphere as part of the Johnson Space Center (JSC) Cosmic Dust Program[4]. By comparison with experimental and terrestrial studies of poorly graphitized carbon (PGC), we show that the graphitization temperature, or the degree of ordering in the PGC, may provide a useful cosmothermometer for primitive extraterrestrial materials.

A detailed analytical electron microscope (AEM) investigation of CP aggregate W7029*A has established that parts of this aggregate experienced nominal flash-heating no higher than

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