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Note

Tidal dissipation at arbitrary eccentricity and obliquity

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Abstract

Expressions for tidal dissipation in a body in synchronous rotation at arbitrary orbital eccentricity and obliquity are derived. The rate of tidal dissipation for a synchronously rotating body is compared to that in a body in asymptotic nonsynchronous rotation.

Keywords: Extrasolar planets; Satellites, dynamics; Tides, solid body

1. Introduction

Time-dependent tidal distortion of a body leads to internal heating. For a synchronously rotating body in an eccentric orbit the rate of energy dissipation is reported to be (Peale and Cassen, 1978; Peale et al., 1979; Wisdom, 2004)

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{21}{2}\frac{k_2}{Q}\frac{GM^2nR^5e^2}{a^6}, \tag{1}$$

where k_2 is the satellite (secondary body) potential Love number, Q is the satellite effective tidal dissipation parameter, G is the gravitational constant, a is the orbit semimajor axis, M the mass of the host planet (primary body), n the orbital mean motion (and rotation rate), which is approximately $\sqrt{GM/a^3}$, R is the satellite radius, and e is the orbital eccentricity. The derivation of this formula assumes that the body is incompressible, the rotation is uniform and synchronous, and that the body is small enough that the displacement Love number h_2 is $5k_2/3$. The eccentricity has also been assumed to be small, and only the lowest order factor in eccentricity has been kept. Wisdom (2004) generalized this expression to include the lowest order terms in obliquity, forced synchronous libration, and spin–orbit secondary libration.

Though this expression has been adequate for most discussions of tidal dissipation in the Solar System where the orbital eccentricities are relatively small, there are also now situations for which it is inadequate. For instance, Garrick-Bethell et al. (2006) argued that the shape of the Moon was best explained if the moment differences of the Moon froze in during a period in which the Moon had large eccentricity. For synchronous rotation, the orbit that satisfies the shape constraint has an eccentricity of 0.49. We will see that in this case the familiar formula for tidal dissipation, Eq. (1), underestimates the rate of tidal dissipation by a factor of about 30. Another possible situation of interest is tidal dissipation in extrasolar planets. Extrasolar planets have been found to have a wide range of orbital eccentricities. For those extrasolar planets that are gas giants, it may be unlikely that they are in synchronous rotation, but rather may be expected to be in an asymptotic nonsynchronous state as was once proposed for Mercury (Peale and Gold, 1965; Levrard et al., 2007). Nevertheless, it may be expected that more rocky extrasolar planets will be discovered and that some of these may also have large orbital eccentricities and be in synchronous rotation. Extrasolar planets may also have large obliquity: Winn and Holman (2005) proposed that HD209458b might be in a high obliquity Cassini state, and that the enhanced tidal dissipation at large obliquity would inflate its radius (see also the discussion of HD209458b by Levrard et al., 2007).[1] It will be appropriate then to have a valid expression for tidal dissipation at arbitrary eccentricity and obliquity.

Here we generalize the familiar result for tidal dissipation in a synchronously rotating satellite, Eq. (1), and derive a concise formula applicable at arbitrary eccentricity and obliquity. We compare the rate of tidal heating in synchronous rotation to that in asymptotic nonsynchronous rotation (Levrard et al., 2007).

2. Derivation for synchronous rotation

The derivation of the generalizations of the tidal heating formula that is presented in Wisdom (2004) follows one of the derivations presented in Peale and Cassen (1978). Here we follow the same derivation, but, as appropriate, generalize the expressions so that they are applicable at arbitrary eccentricity and obliquity. In this section we assume the obliquity is zero.

Following Wisdom (2004),[2] let U_T be the tide-raising potential. The satellite may be thought of as consisting of a myriad of small constituent mass

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[1] In Wisdom (2004), it is clear that the expressions are only valid to second order in eccentricity, but I forgot to state that the results were also truncated at second order in the obliquity. Unfortunately, Winn and Holman (2005) used the expression for tidal dissipation in synchronous rotation presented in Wisdom (2004) at large obliquity. And Levrard et al. (2007) used the expressions in Wisdom (2004) for the synchronous dissipation rate at both high eccentricity and high obliquity.

[2] For ease of reading, some of that derivation is repeated here (and corrected).

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elements. The force on each mass element is the negative gradient of the potential energy, where the potential energy is the tidal potential multiplied by the mass of the constituent. The rate at which work is done on each constituent is the dot product of this force with the velocity of the constituent. Integrating over the volume of the satellite gives the rate at which work is done on the satellite.

The rate of energy dissipation in the satellite is

$$\frac{dE}{dt} = -\int_{\text{Body}} \rho \vec{v} \cdot \nabla U_T \, dV, \tag{2}$$

where ρ is the density and dV is the volume element. To a good approximation a satellite may be assumed to be incompressible $\nabla \cdot \vec{v} = 0$ (Peale and Cassen, 1978). The product rule gives

$$\nabla \cdot (U_T \vec{v}) = \vec{v} \cdot \nabla U_T + U_T \nabla \cdot \vec{v}, \tag{3}$$

so, with the assumption of incompressibility, the rate of energy dissipation is

$$\frac{dE}{dt} = -\int_{\text{Body}} \rho \nabla \cdot (U_T \vec{v}) \, dV. \tag{4}$$

If we ignore any variation of density in the body, Gauss's theorem allows us to write the rate of energy dissipation as a surface integral

$$\frac{dE}{dt} = -\rho \int_{\text{Surface}} U_T \vec{v} \cdot \vec{n} \, dS, \tag{5}$$

where $\vec{n}$ is the normal to the surface and dS is the surface area element. Now $\vec{v} \cdot \vec{n}$ is the rate at which the height of the surface changes. The height of the tide at any point on the surface is approximately

$$\Delta r = -\frac{h_2 U_T'}{g}, \tag{6}$$

where h_2 is the displacement Love number for the satellite, g is the local acceleration of gravity, and the prime on U_T indicates that the tidal potential is given a phase delay because the dissipative tidal response lags the forcing. So

$$\frac{dE}{dt} = \frac{\rho h_2}{g} \int_{\text{Surface}} U_T \frac{d}{dt}(U_T') \, dS. \tag{7}$$

The tide-raising gravity-gradient potential is

$$U_T = -\frac{GMR^2}{r^3} P_2(\cos\alpha), \tag{8}$$

where P_2 is the second Legendre polynomial, α is the angle at the center of the satellite between the planet to satellite line to the point in the satellite where the potential is being evaluated, R is the distance from the satellite center to the evaluation point, and r is the planet to satellite distance.

Consider motion in a fixed elliptical orbit, with eccentricity e. Choosing rectangular coordinates with the x-axis aligned with the orbit pericenter and the orbit in the (x, y)-plane, the orbital position is

$$\mathbf{o} = (r\cos f, r\sin f, 0) \tag{9}$$

with true anomaly f. In terms of planetocentric longitude λ and colatitude θ, the rectangular components of a surface element in the unrotated satellite (or at the initial time) are

$$\mathbf{s}_0 = (R\sin\theta\cos\lambda, R\sin\theta\sin\lambda, R\cos\theta). \tag{10}$$

Assuming uniform synchronous rotation about the z-axis (perpendicular to the orbit plane), the rectangular components of this element at time t are

$$\mathbf{s} = \mathbf{R}_z(nt)\mathbf{s}_0 = \big(R\sin\theta\cos(\lambda + nt), R\sin\theta\sin(\lambda + nt), R\cos\theta\big), \tag{11}$$

where $\mathbf{R}_z(nt)$ is an active right-handed rotation about the z-axis by the angle nt. The dot product of the surface element with the orbital position gives $\mathbf{o} \cdot \mathbf{s} = rR\cos\alpha$. This completes the expression for the tidal potential as a function of time and location on the surface. The delayed tidal potential U_T' is found by replacing nt by $nt + \Delta$ in the expression for U_T. The tidal model used here is the one where $1/Q$ is proportional to frequency. This is sometimes known as the Mignard model (Mignard, 1980).

The average rate of energy dissipation is found by carrying out the surface integral, Eq. (7), and averaging over an orbital period.

The surface integral may be carried out by expanding the integrand as a Poisson series in the angular variables. Any term containing λ then integrates to zero, the rest are multiplied by 2π. The θ integrals are simple. The details are unilluminating and will not be shown. The calculations were carried out with computer algebra, and checked by performing the integrals numerically. The result is

$$\frac{dE}{dt} = \frac{\rho h_2}{g} \frac{G^2 M^2 R^6 n}{a^6} \gamma, \tag{12}$$

where

$$\begin{aligned}
\frac{\gamma}{2\pi} = {} & \frac{3}{5}\left(\frac{a}{r}\right)^3 \left(\frac{a}{r'}\right)^5 \beta \sin(2\Delta + 2f - 2f') \\
& - \frac{3}{10}\left(\frac{a}{r}\right)^3 \left(\frac{a}{r'}\right)^4 \frac{e}{\beta} \sin(f') \\
& - \frac{9}{20}\left(\frac{a}{r}\right)^3 \left(\frac{a}{r'}\right)^4 \frac{e}{\beta} \sin(2\Delta + 2f - f') \\
& + \frac{9}{20}\left(\frac{a}{r}\right)^3 \left(\frac{a}{r'}\right)^4 \frac{e}{\beta} \sin(2\Delta + 2f - 3f') \\
& - \frac{3}{5}\left(\frac{a}{r}\right)^3 \left(\frac{a}{r'}\right)^3 \sin(2\Delta + 2f - 2f')
\end{aligned} \tag{13}$$

and where $\beta = (1 - e^2)^{1/2}$. The values of r' and f' are those of the radius r and true anomaly f for a mean anomaly of $nt + \Delta$.

We assume the dissipation is small and can therefore approximate

$$\gamma = \gamma_0 + \frac{d\gamma}{d\Delta}\Delta + \cdots \approx \gamma_0 + \frac{d\gamma}{d\Delta}\sin\Delta, \tag{14}$$

where γ_0 is the value of γ for $\Delta = 0$, and the derivative $d\gamma/d\Delta$ is evaluated at $\Delta = 0$. With this approximation the time average of the energy dissipation expression can be completed analytically.

Using

$$\frac{df}{d\mathcal{M}} = \beta\left(\frac{a}{r}\right)^2 \tag{15}$$

and

$$\frac{d(a/r)}{d\mathcal{M}} = -\frac{e\sin f}{\beta^2}\frac{df}{d\mathcal{M}}, \tag{16}$$

where $\mathcal{M}$ is the mean anomaly, we find

$$\begin{aligned}
\frac{1}{2\pi}\frac{d\gamma}{d\Delta} = {} & -\frac{6}{5}\left(\frac{a}{r}\right)^{10}\beta^2 - \frac{6}{5}\left(\frac{a}{r}\right)^9 e\cos f - \frac{6}{5}\left(\frac{a}{r}\right)^6 \\
& + \frac{12}{5}\left(\frac{a}{r}\right)^8 \left[\beta - \frac{e^2}{\beta^2}\big(1 - \cos(2f)\big)\right].
\end{aligned} \tag{17}$$

The time average of the energy dissipation can be found by integrating the energy dissipation over an orbital period and dividing by the orbital period. Note that γ_0 is proportional to $\sin f/r^7$, and its time average is zero. Thus we just need to calculate the average of $d\gamma/d\Delta$.

The integrals involving the radius and true anomaly can be expressed exactly in terms of Hansen functions X_k^{ij} (Plummer, 1960; Mignard, 1980). The time average of the energy dissipation is

$$\frac{dE}{dt} = \frac{\rho h_2}{g} \frac{G^2 M^2 R^6 n}{a^6} \left\langle \frac{d\gamma}{d\Delta} \right\rangle \sin\Delta, \tag{18}$$

where $\langle d\gamma/d\Delta \rangle$ is the average of $d\gamma/d\Delta$ over an orbit period:

$$\begin{aligned}
\frac{1}{2\pi}\left\langle \frac{d\gamma}{d\Delta} \right\rangle = {} & -\frac{6}{5} X_0^{-10,0}(e)\beta^2 - \frac{6}{5} X_0^{-9,1}(e)e - \frac{6}{5} X_0^{-6,0}(e) \\
& + \frac{12}{5}\left[X_0^{-8,0}(e)\left(\beta - \frac{e^2}{\beta^2}\right) + \frac{e^2}{\beta^2} X_0^{-8,2}(e)\right].
\end{aligned} \tag{19}$$

The Hansen functions satisfy the following identities (Mignard, 1980)[3]:

$$eX_0^{-(n-1),1}(e) = \beta^2 X_0^{-n,0}(e) - X_0^{-(n-1),0}(e),$$
$$X_0^{-n,2}(e) = -\frac{2\beta^2}{e(n-1)} X_0^{-(n+1),1}(e) + X_0^{-n,0}(e). \tag{20}$$

Using these identities, the expression $g = 4\pi G R\rho/3$, which is valid for a satellite of uniform density, the relation $k_2 = 3h_2/5$, which is valid for small homogeneous satellites, and the relation $\sin\Delta = 1/Q$, which is valid for small $1/Q$, we find that the average rate of energy dissipation is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{21}{2}\frac{k_2}{Q}\frac{GM^2R^5n}{a^6}\zeta(e), \tag{21}$$

where $\zeta(e)$ is an eccentricity-dependent factor

$$\zeta(e) = \frac{20}{49}\beta^2 X_0^{-10,0}(e) - \frac{6}{49}X_0^{-9,0}(e) - \frac{4}{7}\beta X_0^{-8,0}(e) + \frac{2}{7}X_0^{-6,0}(e). \tag{22}$$

Note that this is the same form as the familiar expression for tidal dissipation, but with the factor e^2 replaced by the factor $\zeta(e)$.

Mignard (1980) notes that these particular Hansen functions have closed form expressions in terms of the eccentricity. Thus the $\zeta(e)$ factor may be written explicitly:

$$\zeta(e) = \frac{2}{7}\frac{f_0(e)}{\beta^{15}} - \frac{4}{7}\frac{f_1(e)}{\beta^{12}} + \frac{2}{7}\frac{f_2(e)}{\beta^9}, \tag{23}$$

where

$$f_0(e) = 1 + \frac{31}{2}e^2 + \frac{255}{8}e^4 + \frac{185}{16}e^6 + \frac{25}{64}e^8,$$
$$f_1(e) = 1 + \frac{15}{2}e^2 + \frac{45}{8}e^4 + \frac{5}{16}e^6,$$
$$f_2(e) = 1 + 3e^2 + \frac{3}{8}e^4. \tag{24}$$

Recall that $\beta = (1-e^2)^{1/2}$.

The factor $\zeta(e)$ depends only on e^2 and has no constant term, so we may write it

$$\zeta(e) = e^2\eta(e), \tag{25}$$

where $\eta(e)$ is an enhancement factor. It is the factor by which the familiar formula for tidal dissipation is multiplied to get the correct formula for tidal dissipation at arbitrary eccentricity (with obliquity zero).

The enhancement factor $\eta(e)$ should be evaluated by the exact expressions above, but it is interesting to display it as a power series:

$$\eta(e) = 1 + 18e^2 + \frac{3329}{28}e^4 + \frac{55551}{112}e^6 + \frac{201669}{128}e^8 + \cdots. \tag{26}$$

This polynomial agrees with the polynomial representation of the energy dissipation rate given in Peale and Cassen (1978). The enhancement factor and this polynomial approximation are shown in Fig. 1. For moderate to large eccentricity even a high degree polynomial gives a poor approximation to the actual dissipation rate.

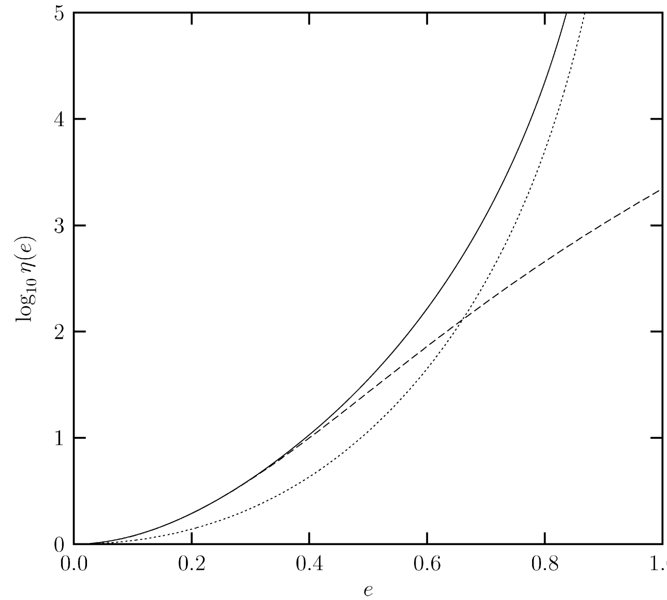

Fig. 1. The dissipation enhancement factor $\eta(e)$ for synchronous rotation is plotted as a function of eccentricity e. The enhancement factor is the factor by which the familiar formula for tidal dissipation is multiplied to get the actual rate of tidal dissipation at arbitrary eccentricity (at zero obliquity). The solid line shows the enhancement factor $\eta(e)$; the dashed line shows the 8th degree polynomial approximation. The dotted line shows the enhancement factor $\eta_L(e)$, valid for asymptotic nonsynchronous rotation.

3. Obliquity

In this section, we generalize the results of the previous section to include arbitrary obliquity of the satellite with respect to the orbit. Only key points will be presented, as the calculation is straightforward though tedious. Computer algebra was used.

Obliquity is introduced by modifying Eq. (11):

$$\mathbf{s} = \mathbf{R}_z(\Lambda)\mathbf{R}_x(I)\mathbf{R}_z(nt)\mathbf{s}_0, \tag{27}$$

where I is the obliquity, and Λ is a measure of the longitude of the node of the equator on the orbit plane with respect to the pericenter of the orbit, and $\mathbf{R}_i$ is an active right-handed rotation about the indicated axis by the indicated angle.

The generalization of Eq. (17) is found to be

$$\frac{1}{2\pi}\frac{\mathrm{d}\gamma}{\mathrm{d}\Delta} = -\frac{6}{5}\left(\frac{a}{r}\right)^{10}\beta^2 - \frac{6}{5}\left(\frac{a}{r}\right)^9 e\cos f - \frac{3}{5}\left(1+(\cos I)^2\right)\left(\frac{a}{r}\right)^6$$
$$+ \frac{12}{5}\left(\frac{a}{r}\right)^8\left[\beta\cos I - \frac{e^2}{\beta^2}\left(1-\cos(2f)\right)\right]$$
$$- \frac{3}{5}\left(\frac{a}{r}\right)^6(\sin I)^2\cos(2\Lambda - 2f). \tag{28}$$

The generalization of Eq. (22) is

$$\zeta(e,I) = \frac{20}{49}\beta^2 X_0^{-10,0}(e) - \frac{6}{49}X_0^{-9,0}(e)$$
$$- \frac{4}{7}\beta X_0^{-8,0}(e)\cos I + \frac{1}{7}X_0^{-6,0}(e)\left(1+(\cos I)^2\right)$$
$$+ \frac{1}{7}\cos(2\Lambda)(\sin I)^2 X_0^{-6,2}(e). \tag{29}$$

Note the dependence on Λ.

Finally, the generalization of Eq. (23) is

$$\zeta(e,I) = \frac{2}{7}\frac{f_0(e)}{\beta^{15}} - \frac{4}{7}\frac{f_1(e)}{\beta^{12}}\cos I + \frac{1}{7}\frac{f_2(e)}{\beta^9}\left(1+(\cos I)^2\right)$$
$$+ \frac{3}{14}\frac{e^2 f_3(e)}{\beta^{13}}(\sin I)^2\cos(2\Lambda), \tag{30}$$

where

$$f_3(e) = 1 - \frac{11}{6}e^2 + \frac{2}{3}e^4 + \frac{1}{6}e^6. \tag{31}$$

The rate of tidal dissipation is given by Eq. (21), with $\zeta(e)$ replaced by its generalization $\zeta(e,I)$.[4]

It is interesting to note that a Λ-dependent term survives; with both obliquity and eccentricity the rate of dissipation depends on the longitude of the

[3] The second identity has a typographical error in Mignard (1980).

[4] To second order in eccentricity and obliquity, this expression agrees with corresponding terms given in Wisdom (2004).

equator relative to the pericenter. However, if the relative precession of the equator and the pericenter is rapid compared to timescales for changes in the eccentricity and obliquity, then the Λ-dependent term will average to zero.

4. Tidal dissipation for asymptotic nonsynchronous rotation

Levrard et al. (2007) derived expressions for the asymptotic rate of rotation and rate of tidal dissipation for a satellite (or extrasolar planet) that is not locked in a spin–orbit resonance. The assumed tidal model is the same as the one assumed here. The derived equilibrium rotation rate is

$$\omega_{\mathrm{eq}} = \frac{N(e)}{\Omega(e)} \frac{2x}{1+x^2} n, \tag{32}$$

where

$$N(e) = \frac{1}{\beta^{12}}\left(1 + \frac{15}{2}e^2 + \frac{45}{8}e^4 + \frac{5}{16}e^6\right) = \frac{f_1(e)}{\beta^{12}}, \tag{33}$$

$$\Omega(e) = \frac{1}{\beta^9}\left(1 + 3e^2 + \frac{3}{8}e^4\right) = \frac{f_2(e)}{\beta^9}, \tag{34}$$

and $x = \cos I$ for obliquity I. The derived rate of tidal dissipation is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{21}{2}\frac{k_2}{Q}\frac{GM^2R^5n}{a^6}\zeta_L(e,x), \tag{35}$$

where $\zeta_L(e,x)$ is

$$\zeta_L(e,x) = \frac{2}{7}\left[N_a(e) - \frac{(N(e))^2}{\Omega(e)}\frac{2x^2}{1+x^2}\right], \tag{36}$$

and

$$N_a(e) = \frac{1}{\beta^{15}}\left(1 + \frac{31}{2}e^2 + \frac{255}{8}e^4 + \frac{185}{16}e^6 + \frac{25}{64}e^8\right) = \frac{f_0(e)}{\beta^{15}}. \tag{37}$$

For zero obliquity ($x = 1$), we may write

$$\zeta_L(e,1) = e^2\eta_L(e), \tag{38}$$

defining the nonsynchronous tidal dissipation enhancement factor $\eta_L(e)$. This is the factor by which the familiar formula for tidal dissipation is multiplied to get the tidal dissipation in an asymptotic nonsynchronous rotation state, with zero obliquity. It is interesting to display a power series representation of the enhancement factor $\eta_L(e)$:

$$\eta_L(e) = 1 + \frac{54}{7}e^2 + \frac{1133}{28}e^4 + \frac{31845}{224}e^6 + \cdots. \tag{39}$$

Levrard et al. (2007) compared the derived rate of dissipation for the nonsynchronous case (valid at arbitrary eccentricity and obliquity) to Eq. (45) of Wisdom (2004) (which is valid only at small eccentricity and/or small obliquity; only second-order terms were kept). They found that dissipation in the nonsynchronous case was much larger than in the synchronous case. Now that we have an expression for the rate of tidal dissipation that is valid at arbitrary eccentricity it is appropriate to make a new comparison of the derived rates, using expressions that are both valid at arbitrary eccentricity. To this end, the enhancement factor $\eta_L(e)$ is also plotted in Fig. 1. Contrary to the conclusion of Levrard et al. (2007), and somewhat surprisingly, we see that the rate of tidal dissipation for synchronous rotation is always greater than for asymptotic nonsynchronous rotation, at zero obliquity. As e tends to 1, both of these enhancement factors are proportional to β^{-15} and the ratio $\eta(e)/\eta_L(e)$ tends to $195/41$ or about 4.756. Numerically, we find that the dissipation rate in synchronous rotation is greater than or equal to that in asymptotic rotation for any obliquity and eccentricity. This is illustrated in Fig. 2.

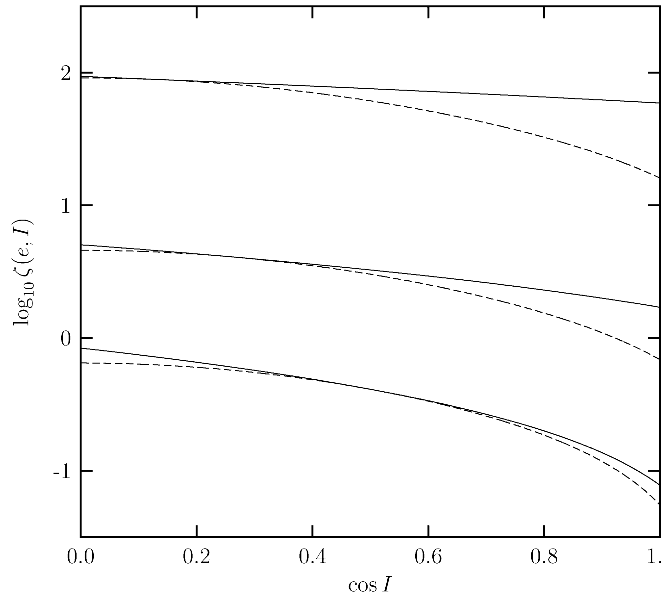

Fig. 2. The tidal dissipation factor $\zeta(e,I)$ is plotted (solid line) versus the cosine of the obliquity I for eccentricities 0.2, 0.4, and 0.6. Dissipation increases with increasing eccentricity. Also plotted (dashed line) is the corresponding dissipation factor $\zeta_L(e,I)$ for asymptotic nonsynchronous rotation. Tidal dissipation in synchronous rotation is greater than or equal to the dissipation in asymptotic nonsynchronous rotation.

5. Summary

We have derived a concise, closed form expression for the rate of tidal dissipation in a synchronously rotating body for arbitrary eccentricity and obliquity.

The derivation presented here is strictly valid only for homogeneous bodies (constant density), which are small enough that the Love numbers satisfy $k_2 = (3/5)h_2$. The familiar expression for tidal dissipation, Eq. (1), has the same domain of applicability.

The rate of dissipation is enhanced over the familiar formula, Eq. (1), by several orders of magnitude at high eccentricity (Fig. 1), and diverges as the eccentricity approaches unity.

The rate of tidal dissipation in a synchronously rotating satellite is larger than that in an asymptotic nonsynchronous rotation state, contrary to the conclusions of Levrard et al. (2007).

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References

Garrick-Bethell, I., Wisdom, J., Zuber, M.T., 2006. Evidence for a past high eccentricity lunar orbit. Science 313, 652–655.

Levrard, B., Correia, A.C.M., Chabrier, G., Baraffe, I., Selsis, F., Laskar, J., 2007. Tidal dissipation within hot Jupiter: A new appraisal. Astron. Astrophys. 462, L5–L8.

Mignard, F., 1980. The evolution of the lunar orbit revisited, II. Moon Planets 23, 185–201.

Peale, S.J., Cassen, P., 1978. Contribution of tidal dissipation to lunar thermal history. Icarus 36, 245–269.

Peale, S.J., Gold, T., 1965. Rotation of the planet Mercury. Nature 206, 1240–1241.

Peale, S.J., Cassen, P., Reynolds, R., 1979. Melting of Io by tidal dissipation. Science 203, 892–894.

Plummer, H.C., 1960. An Introductory Treatise on Dynamical Astronomy. Dover, New York.

Winn, J.N., Holman, M.J., 2005. Obliquity tides on hot Jupiters. Astrophys. J. 628, L159–L162.

Wisdom, J., 2004. Spin–orbit secondary resonance dynamics of Enceladus. Astron. J. 128, 484–491.