

Metalogic, Qualia, and Identity on Neptune's Great Moon: Meaning and Mathematics in the Works of Joseph A. Goguen and Samuel R. Delany

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Abstract. The works of Joseph A. Goguen and Samuel R. Delany address wide arrays of "big" issues in philosophy: identity and qualitative experience, semiotic representation, and the divergence between meaning in formal systems of understanding and in everyday lived experience. This essay attempts to draw out some of the parallels between the works of these two authors, in particular regarding metalogic, qualia, and identity, using illustrative examples from the works of both authors. Their works exhibit parallel dual strands: (1) a desire to rigorously and precisely map out these fundamental issues, and (2) a desire to acknowledge and embrace the ambiguities of phenomenological experience and its divergence from any formalizable theory. In the end, addressing such a wide range of issues has required both authors to develop and adopt new discourse strategies ranging from rational argumentation to mathematics, from religious and philosophical commentary to speculative (science) fiction and poetry.

1 Introduction

A perusal of any dozen pages from the Summa reveals Slade's formal philosophical presentation falls into three, widely differing modes. There are the closely reasoned and crystallinely lucid arguments. There are the mathematical sections in which symbols predominate over words; and what words there are, are fairly restricted to: "... therefore we can see that...", "...we can take this to stand for...", "...from following these injunctions it is evident that...", and the like. The third mode comprises those sections of richly condensed (if not impenetrable) metaphor, in language more reminiscent of the religious mystic than the philosopher of logic. For even the more informed student, it is debatable which of these last modes, mathematical or metaphorical, is the more daunting. [8] – Samuel R. Delany, discussing the work of the fictitious metalogician Ashima Slade

I'm afraid that the reader may have found this paper rather a long strange trip, starting from the practice of software engineering, then going to category theory, and eventually ethics, passing through topics like equational deduction, various programming and specification paradigms, semiotics, theorem proving, requirements engineering and philosophy.

From another perspective, this paper can be considered a diary from a very personal journey moving from a mathematical view of computing, through a process of questioning why it wasn't working as hoped, to a wider view that tries to integrate the technical and social dimensions of computing. This journey has required a struggle to acquire and apply a range of skills that I could never have imagined would be relevant to computer science. Always I have sought to discover things of beauty – “flowers” - and present them in a way that could benefit all beings, though of course I don't expect that very many people will share my aesthetics or my ethics.

[15]

– Joseph Goguen, excerpts (slightly reordered) from an autobiographical essay tracing the trajectory of his research career

The aroma of algebraic flowers motivates this paper. Joseph Goguen has used the metaphor of flowers to describe the strivings of his own work because of the parsimonious beauty it is possible to evoke with elegant formalizations in mathematics. For him the essence of these “flowers” is rooted in compassion and a true desire to benefit humanity. Yet, Goguen’s metaphor for his work is also one of loss. His autobiographical essay “Tossing Algebraic Flowers Down the Great Divide,” [14] suggests that his beautiful work is tumbling downward into a dark crevasse between technical and social scientific or humanistic disciplines, perhaps only to be discovered at an unknown time, or perhaps never.

It is not so! Goguen's algebraic flowers garland a gossamer network of bridges between diverse fields: computing, mathematics, philosophy, sociology, semiotics, narratology, and more. Though perhaps more researchers are familiar with Goguen's work on the technical side of the divide, I intend to highlight the bridge his work builds from computing and mathematics to humanistic and artistic issues. Personally, this bridge has been a profound influence on my work. My academic training is in logic, interactive media art, and computer science. In the course of these studies, I became interested in new forms of interactive narrative that take advantage of the affordances provided by computing. I came to feel that a powerful direction in interactive artwork is to allow user interaction to affect meaning with narratives, and with Professor Goguen's guidance as my advisor this intuitive direction transformed into specific goals, for example generating new metaphors or constructing narratives as users provide input. Toward this end Goguen's algebraic semiotics and his approach to user-interface design were a revelation. He is an expert mathematician dealing with semiotic issues also addressed by art theory. He is a computer scientist who espouses the importance of narrative. Underneath this all is a concern for the social, ethical applications of his work. Because he has not compromised his work

toward either side of the divide, Goguen's feeling of loss regarding this work is probably due to the limited number of people on either side of the divide interested in seriously addressing the issues and methods of greatest import on the other side. I have described my own background only because I live directly in the center of the divide. For people like me, Goguen's work in these areas is of great importance both for its application and example. It can be used directly for artistic technical practices and it is an example of what is possible to achieve when combining methods from diverse fields with rigor and a careful attention to the values implicit in them. This essay is intended to convey this important aspect of Goguen's work by focusing on several particular topics in his oeuvre and contrasting them with the work of another author that has inspired me, Samuel R. Delany.

The title of this paper refers to my attempt to find sympathy in the works of these two eclectic and profound authors. The planet Neptune's largest moon is Triton, here alluding to the title of Delany's science fiction novel *Trouble on Triton: An Ambiguous Heterotopia*. The idea for the thesis of this paper was inspired by the character mentioned in the Delany quote above from that same novel. In the character Ashima Slade, using the idiosyncratic genre of "critical fiction" which allows meticulous commentary on his fictitious author, his lectures, and his theory, Delany has constructed an astounding parallel counterpart for Goguen. The parallel is astounding because of the amazing correspondence of topical concerns that exist between Delany's essay, and the content and style of his character Ashima Slade's Harbin-y Lecture *Shadows*¹ (on the topic of the "Modular Calculus," which grew out of "metalogic") [8] [10].

Goguen has never been one to shy away from "big" issues of human existence. Likewise, as a science fiction and fantasy author constructing civilizations, ancient and futuristic, in part to illuminate sociological points, Delany addresses major philosophical themes. Both are employed as university professors, Goguen in computer science and Delany in English and creative writing, yet the works of each extend well beyond their disciplinary boundaries. Indeed in the quote above Goguen expresses that his work has taken him on a journey through exotic disciplinary locales ranging from category theory to ethnomethodology, and his work also ranges to Buddhist thought and poetry and fiction writing on occasion. Similarly, Delany has commented on a wide range of concerns including semiotics, paraliterature, cultural theory, discourse analysis, gender studies, as well as producing meditations on mathematics and technology. These lists of interests of the two authors are not exhaustive, but they serve to highlight the difficulties, and pleasures for those sympathetic to deep interdisciplinary thought, in elucidating parallels in two prolific, singular authors.

There are many specific parallels in the works of Goguen and Delany. Mathematical metaphors are pervasive in Delany's oeuvre and metalogic takes a prominent role in

¹ Robert Elliot Fox tells us in his book *Conscientious Sorcerers* that "the title in the first lecture of the series, "Shadows," is one the Delany himself used for a speculative/critical essay. As Slade's fictitious editor tells us, Slade took the title 'from a nonfiction piece written in the twentieth century by an author of light, popular fictions.'" [10]

Trouble on Triton in particular. By the same token, identity and difference are major themes in Goguen's work. Often he addresses such concerns through very abstract mathematics such as the theory of institutions which allows for the comparison of logics (a type of metalogic). Though he is not as explicit about politicized social identity in the same sense as Delany, Goguen is also concerned with the relationship of these themes to everyday lived experience. This can be seen in his work on qualia. In phenomenology, philosophers use the term "qualia" to describe introspectively accessible feelings of everyday life that are irreducible to objective characteristics. [25] Goguen has carried out a set of experiments relating qualia to the issue of identity and difference. Similarly, while many artists are interested in exploring the qualitative experiences of life, Delany creates rigorous literary thought experiments that also seem to address the qualia of identity, in his case usually experiences of race, gender, sexual orientation, and similar issues of social identity. The care with which Delany constructs these detailed explorations is exemplified below in Section 2.1 as he uses the metaphor of metalogic to make very specific observations about the nature of race. Finally, both authors are brazenly concerned with mapping out meaning in all of its modularity and nuance. They are unified in this concern as they both draw upon a broad range of traditions from science, mathematics, literature, and social and cultural theories to comment upon some of the most fundamental issues we, as humans, experience in life.

The task of investigating the parallels above is quite worthwhile. It serves to highlight contributions of both Goguen and Delany that perhaps are less well-known than their main contributions to their fields, and more importantly because of the insights such an exercise provides to issues such as (1) identity and qualitative experience, (2) semiotic representation, and the (3) divergence between meaning in formal systems of understanding and everyday lived experience. These three issues are intended to focus this paper (as opposed to representing a comprehensive outline of shared concerns between the authors). This is not meant to be a complete survey of either author's work since I intend rather to highlight particularly salient parallels between them. Thus, the paper is structured as a series of two case studies followed by discussion and a conclusion.

The first case study is centered on Delany's description of "metalogic," and the "modular calculus" where appropriate, in his novel *Trouble on Triton: An Ambiguous Heterotopia*. The second case study is centered on the philosophical notion of qualia in Goguen's work in several papers [16] [19], and the theory of institutions where appropriate. [18] These case studies are unified by a concern with identity, though the starting points from which Goguen and Delany consider identity are quite different. The case studies are followed by a discussion that highlights the tension between both authors' desires to rigorously map meaning and representation (semiotic concerns), and both authors' realizations that this is a Sisyphean task when confronted with the immensity of the real world and human perception of it. The paper concludes with an account of the various discourse styles and strategies Goguen and Delany use to express their ideas – an account of the artistry of the authors. Their discourse styles can be seen as roughly fitting into the same three categories that Delany outlines for Ashima Slade's work: (1) well-reasoned rational argumentation, (2) mathematics (in Delany's case sometimes pseudomathematics used in a

metaphorical way), and (3) more esoteric, artistic, or even religious/spiritual discourse.

2 Metalogic, Qualia, and Identity

2.1 Delany on Metalogic and Identity

Trouble on Triton: An Ambiguous Heterotopia is a novel that tells the story of a self-described “reasonably happy man,” living in a futuristic society on Neptune’s moon Triton. [3] In truth, this man, a conflicted and pompous anti-hero named Bron Helstrom, is far from satisfied. He is ill at ease with his own social identity and relationships with others. He is not a likable or sympathetic character, perhaps meant to represent the pretentiousness often brought on by experience of the privileges accompanying dominant social status. In a world where physique, gender, religion, and race are nearly instantly reconfigurable, a world at war with our own planet Earth, Bron is constantly concerned with how he presents himself externally, and with compensating for his own insecurities. Though largely a meditation on identity, the novel features a robust metaphor of mathematics to address the qualitative experience of identity and the potential for transformation of identity.

At one point early in the novel Bron Helstrom takes about seven pages, and many elaborate analogies involving colored clouds as spaces of significance, hens and a half laying eggs and a half, and the grotte between the tiles of the Taj Mahal, to provide a brief description of the field of metalogic. [6] Though in the novel’s storyworld metalogic is meant to provide a rigorous theory and methodology for problem solving in the real world when rules of formal logic are inadequate, it becomes immediately clear that Delany’s discussion of metalogic has the issue of identity, and especially racial identity, as a subtext.

The reader is oriented to this subtext as the character Miriamne (to whom Bron is about to pontificate on metalogic) responds to Bron’s question on her preference for how she takes her coffee:

“Black,” she said from the sling chair, “as my old lady,” and laughed again...

“That’s what my father always used to say.” She put her hands on her knees. “My mother was from Earth – Kenya, actually; and I’ve been trying to live it down ever since.” [5]

Bron’s parents are soon to be revealed as “large, blond, diligent” and “like so many others it was embarrassing, laborers.” The discussion is then, at the level of nonfictional communication between Delany and the reader [22], a commentary on the social situation of a white male, possessed of a strong sense of entitlement and oriented primarily toward class distinctions, lecturing a woman of color. This commentary plays out metaphorically and metonymically as metalogic is explained

via several examples that are rich with terms that parallel racialized color such as “black,” “white,” “brown,” “pink,” “red,” “tan,” “colored,” and “nonwhite.”²

Specifically, Bron begins by posing a challenge to the “beginning tenet of practically every formal logic text ever written, ‘To deny P is true is to affirm P is false’.” The color consciousness comes into play when Miramne responds by mentioning that she recalls “something about denying the Taj Mahal is white ... is to affirm that it’s not white ... an idea that, just intuitively I’ve never felt comfortable with.” Delany goes on to explicate this discomfort by having his character Bron elaborate upon metalogic, with a series of arguments using the color of the Taj Mahal as an example. This series of arguments clearly could apply as easily to a discussion of the nuances of racial identity, moving from a simplistic system of finite (binary initially: white vs. nonwhite) classification to a much more complicated system, a “parametal model of language,” that stresses the metaphor to the breaking point as exemplified by the following quote:

...he used the fanciful analogy of “meanings” like colored clouds filling up the significance space, and words as homing balloons which, when strung together in a sentence, were tugged to various specific areas in their meaning clouds by the resultant syntax vectors but, when released, would drift back more or less to where, in their cloudy ranges, they’d started out. [7]

I now present a summary of the points that Bron makes in his informal discussion of metalogic and argument against the idea that to deny P is to affirm not-P:

- (1) Premise: denying the Taj Mahal is white is not to affirm that it is not white
- (2) the significance of ‘white’ is a range of possibilities
- (3) the significance of ‘white’ “fades imperceptibly” through grey to black and through pink to red, and even to some non-colors
- (4) accepting that ‘white(Taj Mahal) = F’ \rightarrow ‘ \neg white(Taj Mahal) = T’ means placing a boundary around an area in the range of significance and to call everything in this area white and everything outside of it not-white
- (5) this is already a distortion of what was already mentioned to exist, namely fading ranges of color and non-color
- (6) values on the boundary line are unaccounted for
- (7) objects that are piecewise white and not-white are unaccounted for, (e.g. the Taj Mahal is made of white tiles held to brown granite by tan grotte)

Notice that at this point the “Taj Mahal” in this discussion could have been substituted by “racial ambiguous individual” with no effect on Bron’s argument (besides making it more socially salient or politically charged). Furthermore, we have reached a point where a solution to the problem is to describe the Taj Mahal, or racialized person, piecewise as being ‘white’ and also being some other discrete color signifiers. This is how archaic (really still in practice, only sometimes less overtly) systems of racial identity functioned, with any number of arbitrary discrete color

² This is strikingly reminiscent of Duke Ellington’s “Black, Brown, and Beige” suite. [9]

categories often defined by quantified mixtures of identity³. Indeed I personally grew up well aware of the “one drop” rule that holds sway in the United States of America: any bit of “black blood” implies blackness (up to a practical limit of 1/16). It is common for individuals whose parents are identified as belonging to different racial groups to claim “biraciality,” or even more finely grained subdivisions of race. DNA testing technologies [2], along with contemporary sociological theories of classification admitting the arbitrary nature of race [1], have rendered these piecewise and discrete classifications of identity obsolete. With all this in mind, I present Miriamne’s response to Bron’s argument so far: “Wait a second: *Part* of the Taj Mahal is white, and *part* of the Taj Mahal is brown, and *part* of the Taj Mahal is – ” to which Bron responds by continuing his argument as follows:

- (8) the words ‘Taj Mahal’ also have a range of significance
- (9) the range of significance of ‘Taj Mahal’ is not discrete, is not unambiguous, and cannot be bounded in a simple two-dimensional model
- (10) the Taj Mahal must be described in terms of continuously valued parameters, not discrete perimeters. “Language is parametal, not perimetal. Areas of significance space intermesh and fade into one another like color-clouds in a three-dimensional spectrum.”
- (11) thus ‘logical’ bounding is dangerous because it implies that boundaries can be placed around significance spaces
- (12) natural language can overcome these problems and provide parametal descriptions
- (13) rigorous and precise modeling of such phenomena using mathematics requires extremely advanced tools of analysis (at minimum metalogicians have simple model with seven coordinates, in practice they often use twenty-one, and even this is just an abstract model for visualization that does not fully explain the real world, i.e. “real space”)

At this point, Bron’s argument is not yet complete. The problem is that “significance space” has been reified. That is, it is being treated as if it exists in the real world and there is such a thing as a “real” significance space to be modeled. Delany’s perspective here, as expressed through the character Bron, foreshadows recent directions in cognitive science. Bron’s explanation shifts to expressing “how what-there-is manages to accomplish what-it-does,” namely how the brain and sensory perception are the origins of complicated concepts such as “significance space” and other concepts in general. In short, it is almost an embodied perspective of cognition [26] (though Delany does not discuss motor operations). In this view “meaning”

³ The artist Betye Saar expresses this using real historical colorized terms for black people found in popular culture and works such as those of the author Langston Hughes. Some of these are: “bright/light, cream, fair, marinee, peola, pinky/pink toes, taffy, vanilla, banana, butterscotch, café au lait, ginger, golden, honey, peaches, yella/high yella/deep yella, almond, caramel, copper, red/red bone, rusty, bark, brownie, brown sugar, cocoa brown/high brown, low brown/seal brown/tobacco brown, chocolate/chocolate drop/deep chocolate, molasses, walnut, bronze, blackie, blackbird/blackberry, black/blue black/charcoal black/coal black/dark black/deep black/lamp black/stove black, crow jane, licorice, midnight/beyond midnight, nightblack boy, tar baby.”

depends upon the fact that humans exist “in a world that is inseparable from our bodies, our language, and our social history.” [26]

From here Bron continues to reformulate the problem, and to describe how metalogic allows us to address it.

- (14) the goals of metalogic are to delimit problems and to explore how elements in the significance space interpenetrate each other
- (15) metalogical delineation of significance space means examining specific human utterances or texts (syntax vectors) to dismiss some areas from consideration
- (16) the delimited area is then considered “metalogically valid”
- (17) to deny “meaningfully” that the Taj Mahal is not white does not imply, but suggests, that it is some color (and not, for example, “freedom,” “death,” “Halley’s comet,” or some other thing that is not relevant)
- (18) the topological representation of not-P can take any shape in the significance space, even contained within P (i.e. tangent to P at an infinite number of points – in this case it is said that it “shatters P”)
- (19) Summary: metalogic looks at cognitive activations triggered by linguistic parole (language as it is actually used) [24], selects a model of this in n-dimensional space, and looks at the interpenetration of truth values of relevant elements. Only in this context does it make (metalogical) sense to say that if the Taj Mahal is not white it is some other color, otherwise, the original premise is supported: denying the Taj Mahal is white is not to affirm that it is not white

The remainder of Bron’s lecture merely focuses on mathematical techniques to model the significance spaces and industry protocols for doing so. So, stepping back to look at what Bron has just explained, meaning in a metalogical framework is embodied and triggered via discourse. Modeling meaning requires looking at both its cognitive basis and its relationship to language as used in practice. Mathematical modeling does not reify meaning, but it allows for precise statements to be made given an abstraction, and this abstraction may be fairly complicated with the added advantage that it can be modeled computationally in order to get closer to a precise account of the fuzzy topic of human meaning. According to Bron, regarding the issue of identity, the metalogical framework is shown to be much better than simplistic logical formalizations and their simplistic underlying assumptions.

2.2 Goguen on Identity and Qualia

Goguen is also engaged in the business of metalogic. His paper with Rod Burstall on the theory of institutions begins:

There is a population explosion among the logical systems used in Computing Science. Examples include first order logic, equational logic, Horn clause logic, higher order logic, infinitary logic, dynamic logic, intuitionistic logic, order-sorted logic, and temporal logic; moreover, there is a tendency for each theorem prover to have its own idiosyncratic logical system. We introduce the concept of

institution to formalise the informal notation of "logical system.

[18]

He notes that some "exotic" logic systems have been proposed to handle various problems ranging from program construction to natural language. The theory of institutions allows comparison between various logics, translations between results in one logic and another, and an account of the fact that "many general results used in the applications are actually *completely independent* of what underlying logic is chosen." The notion of an "institution" was introduced to "formalize the informal notion of 'logical system'," with the requirement that there be "a satisfaction relation between models and sentences which is consistent under change of notation." Thus, the use of the prefix 'meta' in the case of Goguen and Burstall is traditional in that it abstracts to a higher level of generalization than model theory, which describes only the satisfaction relationship between syntax and semantics within a logical system. The theory of institutions allows logics themselves, many different vocabularies, to be compared. It is apparent that the theory of institutions is a rigorously formulated mathematical account with practical applications and wide theoretical implications.

[18]

In contrast, Delany's notion of metalogic is not 'meta' in the traditional sense, rather it is 'meta' in a socio-cultural sense. It begins by looking at formal logical reasoning and its relationship to everyday human thought and problem solving. The 'meta' level from this perspective is the issue of how "logical" reasoning and representation in cognitive, social, and cultural contexts diverges from formal logical systems. Needless to say, Delany does not present this work as rigorous mathematics (it is embedded in a science fiction novel!) and his use of the concept of a "logic" though primarily presented mathematically, is also largely meant metaphorically, without clear indication of where the boundaries between these two functions lie. This is not troublesome, however, because as seen above in Section 2.1 Delany's discussion of metalogic is multiveilant and is meant to comment upon the nuances of social identity relationships, to "ground" his novel (it is necessary in genre fiction to "mark" itself as conforming to conventions of the genre – in science fiction this is often done with detailed reference to mathematics and science) by postulating a well-thought out futuristic system of thought, and probably to explore some of his own thoughts as a philosopher and theoretician within the context of a fiction.

Goguen's work does address many overlapping issues with raised in Delany's account of metalogic, but rather than being found in Goguen's work on "metalogical" concerns (institutions), it can be found in his work on qualia and algebraic semiotics. In his paper "Time, Structure and Emotion in Music" [19], with Ryoko Goguen, it is stated that:

In formal logic the Law of Identity is stated as " $A = A$ " meaning that every object is equal (or identical) to itself...The Law of Identity may apply to objects of modern science or technology (e.g. numbers), but not to human experience. It appears that human senses have been optimized by evolution to find differences, in which case identity is the failure to find a significant difference.

This formulation of identity with regard to human experience also can provide commentary on sociological phenomena of identity such as prejudice, or even

politically topical issues such as racial profiling and gender discrimination. It positions these practices as grounded in failures of sensory perception to account for differences (physical or cultural, nuanced or overt) between individuals that undoubtedly exist (as attested to by victims of systematic discrimination or profiling!) and implicitly states that such practices are the results of failures to respect the individuality of humans (instead relying upon inadequate and coarse systems of generalization and classification). Furthermore, Goguen emphasizes that it is not only truth values of concepts that are important, but qualitative experience in human existence. Thus, Goguen is concerned with qualia, often described informally in philosophy as “what remains when all objective features are subtracted.” [19] Goguen would remark, however, that in lived experience subjective phenomena are often attributed at least as much “reality” as so-called “objective” phenomena.

Informal empirical experimentation and phenomenological analysis have moved Goguen to propose a different definition of qualia that avoids some of the vagueness of the traditional definition above. Goguen’s definition is: “Qualia are the hierarchically organized constituents of conscious experience, each with a saliency and an emotional tone.” To demonstrate qualia phenomena, he and Ryoko Goguen performed several musical experiments that yielded observations such as the following [19]:

- (1) added notes beneath a note can change the character of a top note
- (2) what comes before a note can greatly change its feeling
- (3) what comes after a note can greatly change its feeling
- (4) the apparent duration of a note can be changed by what comes before it
- (5) repetitive phrases are expected to take a role in a larger framework, are grouped, and with extreme repetition can become seen as background noise and ignored
- (6) a note can appear many times in a piece of music, but will not be interpreted merely as many instances of that note (the music is interpreted more holistically)

Clearly, though the subject matter is music, these experiments offer a strong commentary on the transitory and subjective nature of identity. It is easy to think of parallels with social identity such as: prejudices can influence dispositions from an individual toward another individual (quale 1 above), impressions of a person after meeting him or her can alter dispositions toward that person (quale 2 above), or the process of enculturation within a group can allow a shift from ignorance of social protocol to full fluency with social protocol, so that interaction becomes automatic (quale 5 above). While Goguen does not present such social experiments in his paper, probably introspection will allow the reader of this paper to agree with these phenomena. In fact, these phenomena are commonplace and not surprising at all. What is striking is that such everyday observations seem to illuminate inadequacies of common approaches to identity (prejudice and discrete classification), the limitations of “objectifying” identity, and the philosophically oft-overlooked importance of subjective experience and emotion when accounting for identity.

Since subjectivity phenomena rarely, if ever, occur in isolation, Goguen is also concerned with accounting for how qualia combine. He grounds this account in Gilles Fauconnier and Mark Turner’s theory of conceptual blending from cognitive

linguistics (along with Goguen's hierarchical information theory). Goguen and Goguen describe conceptual blending as the process

... in which relatively small, transient structures called conceptual spaces, combine or "blend" to yield a new space that may have emergent structure. Simple examples are words like "houseboat" and "roadkill," and phrases like "artificial life" and "computer virus." Blending is considered a basic human cognitive operation, invisible and effortless, but pervasive and fundamental, for example in grammar, reasoning, and combinations of text with music. [19]

Important here is the fact that conceptual blending theory has an embodied basis as discussed above in 2.1. Furthermore, Goguen has developed a theory of algebraic semiotics that uses algebraic specification from computer science to provide formal notation to describe sign systems and mappings between them that are capable of representing conceptual blends. Goguen and I have developed an algorithm that models some core aspects of conceptual blending theory. [20], [21] This means that despite the subjective nature of qualia, and the qualitative nature of identity, at least some aspects of these phenomena can be approached formally with the use of mathematics. Though Goguen is careful to claim that such work is not intended to reify the formal models (in parallel with Delany), it is clear that he seeks an account of qualia and identity that is precise and rigorous, and that corresponds with the daily realities of lived human experience.

3 Discussion

3.1 Goguen's Models and Realities

Goguen and Delany both seek rigorous accounts of social issues, and both take inspiration and ideas from logic and mathematics. Both also exhibit a tension in their work between a desire to account for social phenomena as carefully as possible, as enabled through construction of intricate models, and to acknowledge the inherent limitations of such approaches. In a very broad sense perhaps they are trying to reconcile the power of holistic accounts provided by structuralism with deeply felt postmodernist understandings of the inadequacies of such global models. The desire for rigorous modeling is exhibited as both authors offer semiotic foundations for their work.

In Goguen's algebraic semiotics the structure of complex signs, including signs in diverse media, and the blending of such structures are described using semiotic systems (also called sign systems) and semiotic morphisms. A sign system consists of [21]:

a loose algebraic theory composed of type declarations (called sorts) and operation declarations, usually including axioms and some constants), plus a **level ordering** on sorts (having a maximum element called the **top sort**) and a **priority ordering** on the constituents at each level. Loose sorts classify the parts of signs, while data sorts classify the values of attributes of signs (e.g., color

and size). **Signs** of a certain sort are represented by terms of that sort, including but not limited to constants. Among the operations in the signature, some are **constructors**, which build new signs from given sign parts as inputs. Levels express the whole-part hierarchy of complex signs, whereas priorities express the relative importance of constructors and their arguments; social issues play an important role in determining these orderings. Conceptual spaces are the special case where there are no operations except those representing constants and relations, and there is only one sort. Many details omitted here appear in [11].

A semiotic morphism is a mapping between sign systems. One very useful type of mapping discussed above is that between information and a representation of that information. A semiotic morphism maps sorts, constructors, predicates and functions of one sign system to sorts, constructors, predicates and functions of another sign system respectively. An example of how a sign system can be represented differently via different semiotic morphisms is presented in Figure 1 [11], which depicts representations of time as reported by different types of clocks.



A Strange “Unary” Clock

795

A Naive Digital Clock

13 15

A Military Time Clock

Fig. 1. Different representations of a clock

Goguen’s diagram depicts a unary clock that simply displays a character repeated a number of times equal to the number of elapsed minutes in a day, a simple digital clock that simply displays the same number of minutes in standard Arabic numerals, and a clock that displays military time. Semiotic morphisms from multiple conceptual spaces to a single conceptual space constitute a “blend.”

Using a basis in conceptual blending theory and algebraic semiotics, Goguen and I have also provided an account of “style,” another subjective and seemingly unformalizable topic. Still, we made modest claims that some notions of style can be captured by the principles by which concepts and signs are blended, though this is not to be seen as analogous to true, context dependent, qualitative human style. In [20], we proposed two dimensions of style (regarding computer mediated texts):

- (1) Construction of formal narrative (or other) elements of media structure, at different levels of granularity. At a large grain level these elements could be

narrative clauses, or scenes of a film, at a more fine grain they could be syntactic parameters of clauses, prosody of poems, or types of shots of a film, and at the smallest grain they could include character sprites or collectible items in games, specific metaphors in poems, or icons used in a user interface.

- (2) Selection of media and genres, selection of content, principles for how content elements can be combined, and controls for changing between media and genres.

Later, we even offer the following bold statement (though we mitigate both of these claims later):

Thus there are at least 12 dimensions of style in this approach, 4 at each level: choice of domain⁴, content of domain, optimality principles for blending, and controls for changing domains. [20]

The point here is not the particularities of this notion of style, but rather the desire for the “cake” of a formal model of style, while being “able to eat” the facts that we do not reify this formalization and we do realize its limitations.

Indeed, in another paper we make this value very explicit [21]:

Before briefly discussing algebraic semiotics, it may be helpful to be clear about its philosophical orientation. The reason for taking special case with this is that, in Western culture, mathematical formalisms are often given a status beyond what they deserve. For example, Euclid wrote, “The laws of nature are but the mathematical thoughts of God.” ... Somewhat less grandly, one might consider that conceptual spaces are somehow directly instantiated in the brain. However, the point of view of this paper is that such formalisms are constructed by researchers in the course of particular investigations, having the heuristic purpose of facilitating consideration of certain issues in that investigation.

Under this view, all theories are situated social entities, mathematical theories no less than others.

The varyingly humble and enthusiastic claims concerning the nature, and concrete applications, of algebraic semiotics illuminate what I assert is a rare attitude toward the integration of mathematics and social concern.

3.2 Delany’s Models and Realities

A rare attitude, but not unique. Delany’s “Informal Remarks Towards the Modular Calculus” display a similar impulse. Part one of the “remarks” consists of the body of the novel *Trouble on Triton* itself; other parts of the “remarks” are strewn throughout other novels Delany has written in a completely different genre. Thus, the literary theorist Robert Elliot Fox describes Delany’s “modular calculus” as a “mapping of culture” that “embraces both science fiction and fantasy, as well as critical/confessional modes.” [10] Using the vehicle of Ashima Slade’s Harbin-y

⁴ A “domain” here refers to a collection of knowledge regarding a particular idea or theme.

Lectures, Delany provides part two of his “informal remarks toward the modular calculus [3],” discussed below.

The character Ashima Slade uses the sentence “The hammer hit a nail” to provide an example of some core concepts of the modular calculus. In summing up the modeling accomplished by that sentence Slade offers:

We are modeling attitudes, objects, and various aspects of a relation between them; to do this job, we are using, among a large group of things and relations, various of those things and relations to stand for the objects, attitudes, and relations we wish to model.

Slade continues to explain that there are various ways to express the grammatical and semantic relationships evident in the sentence, and likewise there are various ways to describe the relationship between, for instance, “the three a’s in the sentence.” If the sentence is thought to be formed of only letters and spaces, the ways to describe the relationships that make up and describe the sentence are limited. Slade posits that if the letters in the sentence were instead made of lines in a matrix on a digital display



Fig. 2. Digital display flash-out from Delany’s *Trouble on Triton*

(see Figure 2), the ways of describing a list of relations in the sentence would be quite different, especially considering that letters can be made in multiple forms (see Figure 3).



Fig. 3. Digital letter forms from Delany’s *Trouble on Triton*

In explicating the modular calculus⁵, Slade distinguishes between modular and non-modular descriptions. A modular description “preserves *some* of the modular properties of the sentence in a list that describes the sentence.” A non-modular description “preserves *none* of the modular relations of the sentence in a list that describes the sentence.” Thus, Slade asserts that the digital display is modular whereas mere letters and spaces are nonmodular. The modular calculus, then, translates between a grammar (a list of sentences about how to compose sentences – an inherently nonmodular description even if it is complete), and a modular description. Slade concludes with the following remarks about the modular calculus:

Now the advantages of a modular description of either a modeling object, like a sentence, or a modeling process, like a language, are

⁵ And distinguishing it from the “modular algebra,” which sadly Delany does not have Slade explain in depth in the same essay.

obvious vis-à-vis a nonmodular description. A modular description allows us reference routes back to the elements in the situation which is being modeled. A nonmodular description is nonmodular precisely because, complete or incomplete as it may be, it destroys those reference routes: it is, in effect, a cipher.

...

The problem that still remains to the calculus, despite my work, and that will be discussed in later lectures, is the generation of formal algorithms for distinguishing incoherent modular descriptive systems from coherent modular descriptive systems. Indeed, the calculus has already given us partial descriptions of many such algorithms, as well as generating ones for determining completeness, partiality, coherence, and incoherence—processes which till now had to be considered, as in literature, matters of taste.

The parallel between the two authors' ideas described above goes far beyond the fact that both use figures depicting digital displays, Goguen and Delany share a concern for the various ways to represent a particular sign system, and the fact (following Saussure) that "signs come in systems." [11] Both also are interested in mapping the complex ways that sign systems are composed. But recall that Ashima Slade is naught but a character in Delany's "Informal Remarks Towards a Modular Calculus," and that the informal remarks are written in the fictional mode. Slade's remarks and their mathematical timbre serve a metaphorical purpose (though their contents also express and reinforce that purpose) which is to express the complexities of meaning and identity formations (at the very least Delany raises many other social and philosophical issues) with fiction rather than formal modeling and the epistemological problems formalisms present. This decision to employ a fictional mode provides an advantage outlined observation of his other character, Bron Helstrom: "Ordinary, informal, nonrigorous language overcomes all these problems, however, with a bravura, panache and elegance that leave the formal logician panting and applauding."

Like Goguen does with algebraic semiotics, Delany mitigates the modular calculus. Slade's fictitious biographer informs us that the modular calculus grew out of Slade's earlier work in metalogic. But Bron Helstrom's lecture on metalogic was completely undermined by his unsympathetic persona. Bron is a pompous "white" male who speaks with dominant cultural authority and in fact is filled with insecurities. At one point he angrily berates a worker on the telephone (or some futuristic version of a telephone) whose department had mistakenly placed Miriamne, a cybralogician, in the metalogics division. It becomes clear that Bron's performance is only displayed in the hopes of impressing Miriamne (Bron continues pretending to yell at the worker even after he is hung up on). He exhibits an inability to relate to the woman in front of him, and is completely bewildered by his own identity, revealing the limited utility of his ability to pontificate on the subtly nuanced metalogical identity of the Taj Mahal. And in the end, the discussion of formally modeling the color of the Taj Mahal faded out in the face of lived reality as Bron's lecture veered toward "muzzy eloquence": "...the thought struck: Somewhere in real space was the real Taj Mahal. He had never seen it: He had never been to Earth."

And the discussion of metalogic itself flashes out as Miriamne changes the subject to mention that earlier she had run into a female acquaintance that Bron was interested in. “What happened next was that his heart began to pound.”

4 Conclusions

Composing this paper has been a satisfying exercise that brought into conjunction the works of two people whom I admire a great deal. This process raised important issues about topics as diverse as social identity, qualia, semiotics, and consciousness, but perhaps as importantly, an unifying aesthetic was formed. Both authors offer a type of groundless [12] work with audacity in approaching “big” issues of life. In order to locate the ambiguities and consistencies of representation and meaning, Delany and Goguen each use a diving rod that bifurcates in two seemingly opposite directions: (1) a desire to rigorously map and exploit regularities of the world(s) we inhabit, and (2) a desire to acknowledge and embrace the ambiguities of lived human experience and its divergence from any idealized theory. The feelings, sometimes tension, sometimes cool detachment, most times deep compassion, the authors evoke come in part from the subject matters of their inquiries, and in part from their methods and discourse strategies used in their explorations, meditations. I conclude with a few remarks on a final parallel between the two authors.

Delany, in a pair of quotations above, through the characters Bron Helstrom and Ashima Slade, expressed the “bravura, panache, and elegance” of informal language, and the ability of literature to formulate the modular calculus. Goguen, though cognizant of the limitations of formal methods, writes that his early formal mathematical work “may have an austere kind of beauty from its abstraction and generality,” and coined the metaphor of “Tossing Algebraic Flowers Down the Great Divide” to describe his life’s work in a biographical paper [14]. In the end, Goguen and Delany exhibit aesthetically motivated craftsmanship in their work. They both utilize a range of discourse styles, indeed all three that are exhibited in the fictitious work of Ashima Slade which are, once again: (1) rational argumentation, (2) logic and mathematics, and (3) more esoteric, artistic, or even religious/spiritual discourse. Samuel R. Delany’s three modes can be exemplified in:

- (1) the genre of critical fiction as in part two of “The Informal Remarks Towards the Modular Calculus,
- (2) the exposition of metalogic,
- (3) and contrasting descriptions of subcultures, both self-indulgent:

Really, breast-bangles on a man? (even a very young man.) Just aesthetically: weren’t breast bangles more or less predicated on breasts that, a) protruded and, b) bobbed...?” [4],

and acetic:

Seven years ago, he’d actually attended a meeting of the Poor Children of the Avestal Light and Changing Secret Name; over three instruction sessions he’d learned the first of the Nintey-Seven

Sayable mantras/mumbles: Mimimomomizo-
lalilamialomuelamironoriminos... [4]
along with a lyrical beauty, now sparse, now dense, in his prose style.

Joseph A. Goguen's three modes can be exemplified in:

- (1) his philosophical discussion of qualia with some grounding in the work of Martin Heidegger and Edmund Husserl [16]
- (2) a great deal of his work in mathematics, a mild example is the introduction of the notion of an institution:

...an institution consists of an abstract category Sign , the objects of which are signatures, a functor $\text{Sen}: \text{Sign} \rightarrow \text{Set}$, and a contravariant functor $\text{Mod}: \text{Sign} \rightarrow \text{Setop}$ (more technically, we might use classes instead of sets here). Satisfaction is then a parameterized relation \models_S between $\text{Mod}(S)$ and $\text{Sen}(S)$, such that the following satisfaction condition holds, for any signature morphism $f: S \rightarrow S'$, any S -model M , and any S' -sentence e :

$$M \models_S f(e) \text{ iff } f(M) \models_{S'} e$$

This condition expresses the invariance of truth under change of notation. [18]

- (3) his Buddhism based explorations of phenomenological and even metaphysical concerns:

However, if Heidegger and the Buddhists are right, it is the possibility of non-being which gives beings their character of luminosity, and hence the nothing, i.e., shunyata, is not only prior to negation, but also to things.

The effect of this, as Heidegger says, is to rob logic of its claim to supremacy, and in particular, to rob it of its claim to provide foundations for science and even for mathematics. Indeed, we must conclude that foundations in the sense sought by logicians are simply not possible. The judgements that we make, and in particular any negative judgements, are necessarily grounded in our being-in-the-world, and not in any pre-existing unshakable truths, or eternal world of ideal things. [17]

And finally his poetry:

6:41 am

Clear leaf cloud masses
motionlessly moving
past the static gray road -
almost too lovely to bear. [13]

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Samuel “Chip” Delany I have only met in passing moments, as a fan. In New York City he graciously provided me his address so that I could mail him a correspondence regarding one of his stories – my favorite short story in existence: “The Tale of Rumor and Desire” – I never could find the right words to write him. In San Diego, he offered bit of encouragement on publishing my novel. I thank Delany for forging a trail in the combination of fantasy and sociology that is my passion.

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