Supplementary Material

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1. Speckle Image Formation Equation

1.1. Speckle formation

As shown in Fig. 1, speckle image formation can be modeled by five steps: (a) the incident laser is first scattered by the object surface, and then (b) the field propagates to the lens. Next, (c) the field is modulated by the lens, (d) propagates to the camera senor, and then (e) is recorded by the camera sensor as a speckle image. The propagation steps, (a) and (d), are basically the same operation with different parameters. Here we consider the 1D case. The extension to 2D can be derived by following the same reasoning. The notation and coordinate systems are shown in Fig. 1.

Scattering The incident laser field, denoted as $A_{in}(x)$, is modulated by a surface modulation function $e^{jkh(x)}$, where $h(x)$ is the surface height at x, and $k = \frac{2\pi}{\lambda}$ is the wave number for a laser with wavelength λ. The physical meaning of factor $kh(x)$ is the phase change caused by surface height. The scattered field, denoted as $A_s(x)$, is

$$A_s(x) = A_{in}(x)e^{jkh(x)} \ . \tag{1}$$

Propagation The field at the lens plane, denoted as $A_l(u)$, receives contributions from every point on the surface,

$$A_l(u) = \int_S A_{x\to u}(x,u)dx \ , \tag{2}$$

where S is the surface and $A_{x_0\to u_0}$ represents the field propagated from x_0 on the surface to u_0 on the lens. Each surface point can be regarded as a point light source, whose propagation results in a phase change and attenuation,

$$A_{x_0\to u_0} = A_s(x_0)\frac{e^{jkd_{x_0,u_0}}}{d_{x_0,u_0}} \ , \tag{3}$$

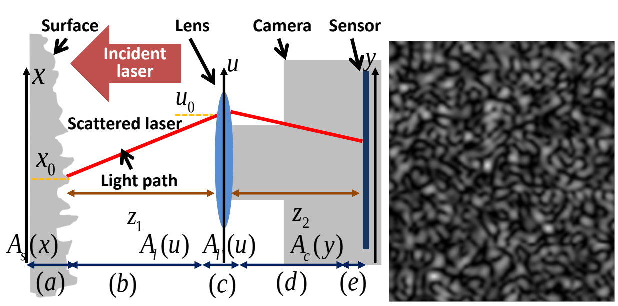

Figure 1: Left: Speckle image formation can be decomposed as 5 main steps: (a) scattering, (b) propagation between surface and camera lens, (c) lens modulation, (d) propagation between lens and sensor, and (e) image formation at sensor. Right: A speckle image simulated using the speckle image formation equation (Eq. (7)), with random surface height and constant incident laser amplitude.

where d_{x_0,u_0} is the distance between x_0, u_0. Assuming that the lens-surface distance is large relative to the extent of the imaged area, $z \gg x$, we can write $d_{x,u} \simeq z + \frac{x^2+u^2-2xu}{2z}$ (Fraunhofer far field regime). Together with Eqs. (3) and (2), this gives us

$$A_l(u) = \frac{e^{j\frac{ku^2}{2z}}}{z} \int_S A_s(x) e^{j\frac{kx^2}{2z}} e^{-jkxu} dx \ , \tag{4}$$

ignoring the constant phase term e^{jkz}. This equation shows that the propagation can be decomposed into three basic operations: (1) multiply the attenuated scattered field $A_s(x)$ by a phase factor $e^{j\frac{kx^2}{2z}}$, (2) apply the Fourier transform to the output, and then (3) multiply the output by another phase factor $e^{j\frac{ku^2}{2z}}$ and an attenuation factor $\frac{1}{z}$.

Lens modulation The field modulated by the lens, denoted as $A_{l'}(u)$, is

$$A_{l'}(u) = A_l(u) P(u) e^{j\frac{ku^2}{2f}} \ , \tag{5}$$

where $P(u)$ is the aperture function ($P(u) = 1$ if u is on the lens, and $P(u) = 0$ otherwise), and f is the focal length. For the field that passes through the aperture, lens modulation is equivalent to multiplying by a phase factor quadratic in u.

Image formation The relation between the speckle image $I(y)$ and the field propagated to the sensor $A_c(y)$ is

$$I(y) = \|A_c(y)\|^2 \ . \tag{6}$$

This describes the fact that the sensor measures the power of the field and does not record phase information. Rather, phase from the surface results in interference patterns in the speckle image.

Speckle equation Combining the above operations, the speckle image can be derived from surface height directly. By assuming that the lens is focused on the surface plane, i.e. $\frac{1}{f} = \frac{1}{z_1} + \frac{1}{z_2}$, and that the incident wave is planar, $A_{in}(x) = A_0$, the speckle image is

$$I(y) = \left\| f\left(-\frac{z_1}{z_2} y\right) \otimes \tilde{g}\left(\frac{y}{z_2 \lambda}\right) \right\|^2 \ , \tag{7}$$

where $f(t) = \frac{A_0}{z_1} e^{jk\left(h(t) + \frac{t^2}{2z_1}\right)}$ is the input function, $\otimes$ denotes convolution, and $\tilde{g}(\omega) = \mathbb{F}\{P\}$ is the Fourier transform of the aperture function $P(u)$.