# Matching features

Computational Photography, 6.882

Prof. Bill Freeman April 11, 2006

Image and shape descriptors: Harris corner detectors and SIFT features.

Suggested readings: Mikolajczyk and Schmid, David Lowe IJCV.

# Matching with Invariant Features

Darya Frolova, Denis Simakov The Weizmann Institute of Science March 2004

ttp://www.wisdom.weizmann.ac.il/~deniss/vision\_spring04/files/InvariantFeatures.pr

# Building a Panorama



M. Brown and D. G. Lowe. Recognising Panoramas. ICCV 2003

# How do we build panorama?

• We need to match (align) images





# Matching with Features

•Detect feature points in both images





# Matching with Features

- •Detect feature points in both images
- •Find corresponding pairs





# Matching with Features

- •Detect feature points in both images
- •Find corresponding pairs
- •Use these pairs to align images



# Matching with Features

- Problem 1:
  - Detect the *same* point *independently* in both images





no chance to match

We need a repeatable detector

# Matching with Features

- Problem 2:
  - For each point correctly recognize the corresponding one



We need a reliable and distinctive descriptor

# More motivation...

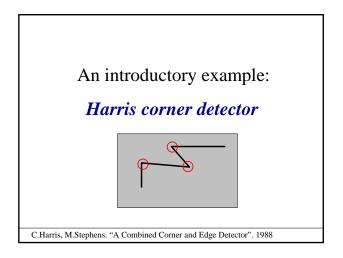
- Feature points are used also for:
  - Image alignment (homography, fundamental matrix)
  - 3D reconstruction
  - Motion tracking
  - Object recognition
  - Indexing and database retrieval
  - Robot navigation
  - ... other

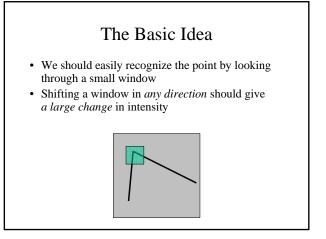
# **Selecting Good Features**

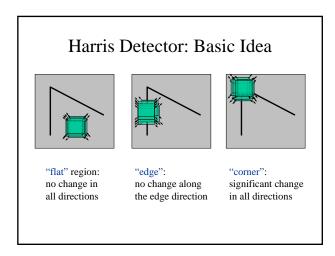
- What's a "good feature"?
  - Satisfies brightness constancy
  - Has sufficient texture variation
  - Does not have too much texture variation
  - Corresponds to a "real" surface patch
  - Does not deform too much over time

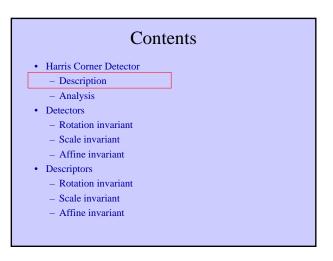
## Contents

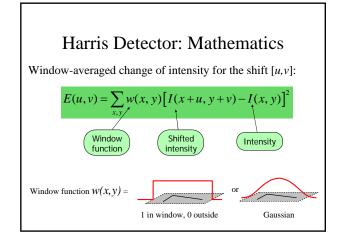
- Harris Corner Detector
  - Description
  - Analysis
- Detectors
  - Rotation invariant
  - Scale invariant
  - Affine invariant
- Descriptors
  - Rotation invariant
  - Scale invariant
  - Affine invariant











Go through 2<sup>nd</sup> order Taylor series expansion on board

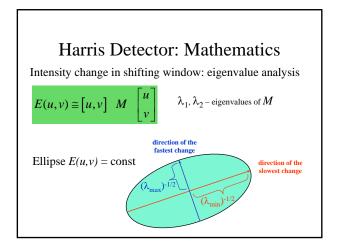
# Harris Detector: Mathematics

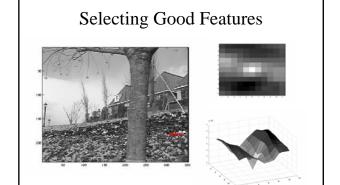
Expanding E(u,v) in a  $2^{nd}$  order Taylor series expansion, we have, for small shifts [u,v], a *bilinear* approximation:

$$E(u,v) \cong \begin{bmatrix} u,v \end{bmatrix} \ M \ \begin{bmatrix} u \\ v \end{bmatrix}$$

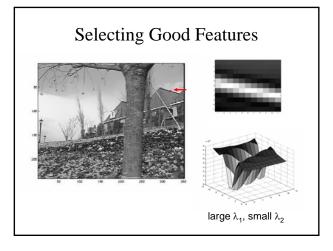
where M is a  $2\times 2$  matrix computed from image derivatives:

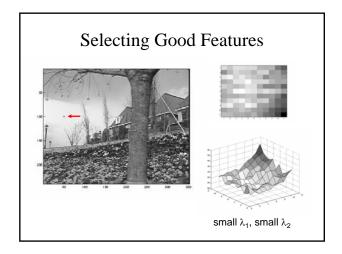
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

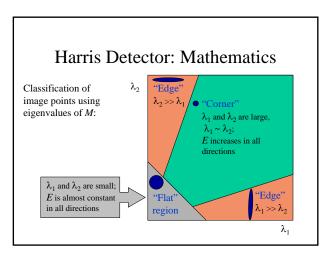




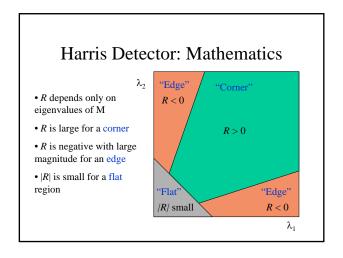
 $\lambda_1$  and  $\,\lambda_2$  are large





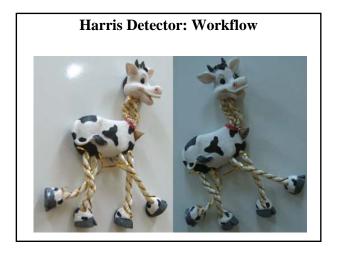


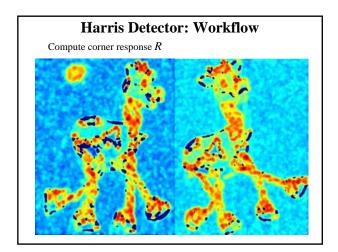
# Harris Detector: Mathematics Measure of corner response: $R = \det M - k \left( \operatorname{trace} M \right)^{2}$ $\det M = \lambda_{1} \lambda_{2}$ $\operatorname{trace} M = \lambda_{1} + \lambda_{2}$ (k - empirical constant, k = 0.04-0.06)

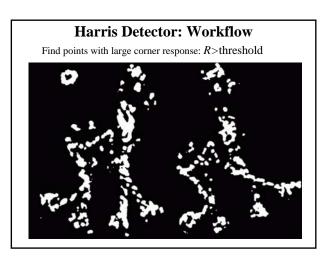


# Harris Detector

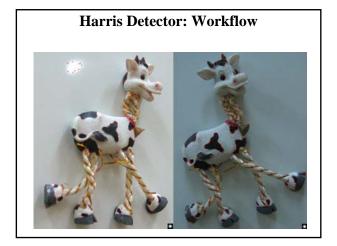
- The Algorithm:
  - Find points with large corner response functionR (R > threshold)
  - Take the points of local maxima of R







# Harris Detector: Workflow Take only the points of local maxima of R



# Harris Detector: Summary

• Average intensity change in direction [*u*,*v*] can be expressed as a bilinear form:

$$E(u,v) \cong [u,v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

• Describe a point in terms of eigenvalues of *M*: *measure of corner response* 

$$R = \lambda_1 \lambda_2 - k \left( \lambda_1 + \lambda_2 \right)^2$$

• A good (corner) point should have a *large intensity change* in *all directions*, i.e. *R* should be large positive

# Contents

- Harris Corner Detector
  - Description
  - Analysis
- Detectors
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  - Scale invariant
  - Affine invariant
- Descriptors
  - Rotation invariant
  - Scale invariant
  - Affine invariant

# Harris Detector: Some Properties

• Rotation invariance?







# Harris Detector: Some Properties

· Rotation invariance









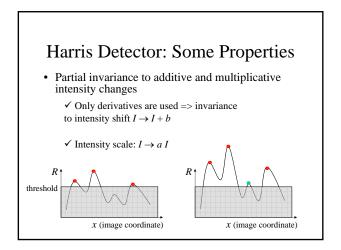


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

# Harris Detector: Some Properties

• Invariance to image intensity change?

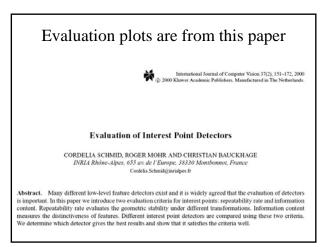


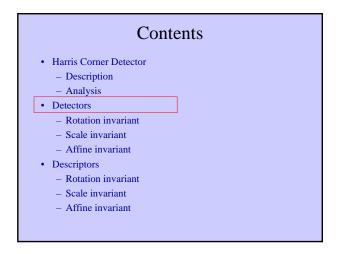
# Harris Detector: Some Properties

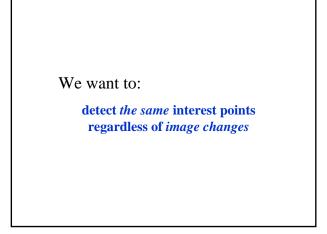
• Invariant to image scale?

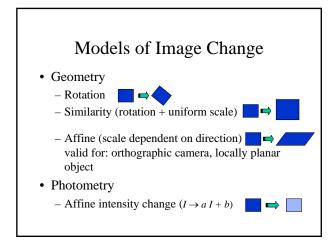
# Harris Detector: Some Properties • Not invariant to image scale! All points will be classified as edges Corner!

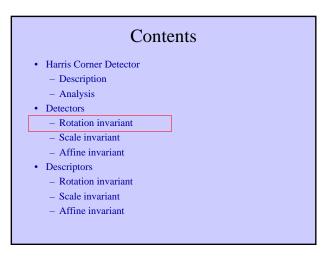
# Harris Detector: Some Properties • Quality of Harris detector for different scale changes Repeatability rate: # correspondences # possible correspondences # Detectors of the possible correspondences # C.Schmid et.al. "Evaluation of Interest Point Detectors". IJCV 2000

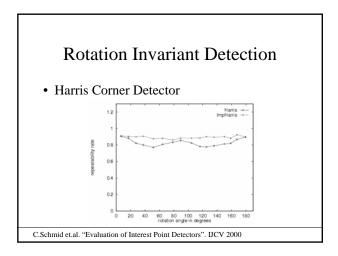


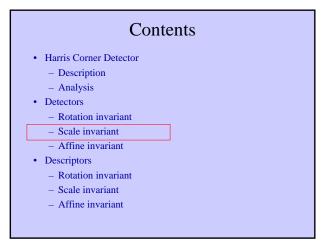




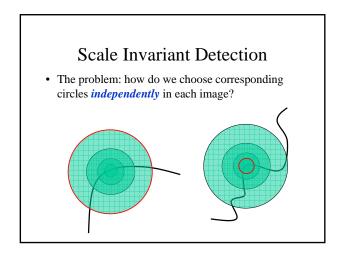




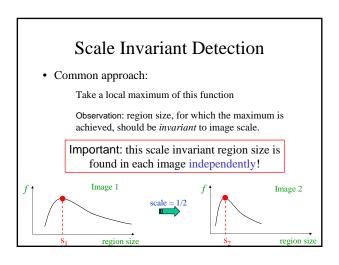


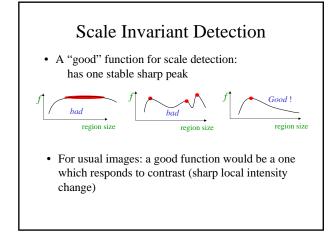


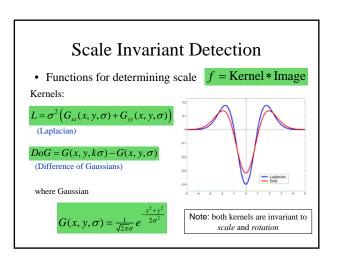
# Scale Invariant Detection Consider regions (e.g. circles) of different sizes around a point Regions of corresponding sizes will look the same in both images



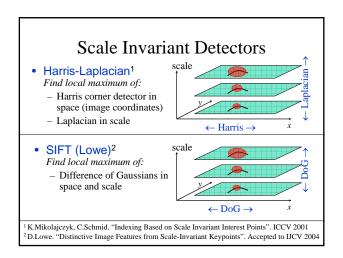
# Scale Invariant Detection • Solution: - Design a function on the region (circle), which is "scale invariant" (the same for corresponding regions, even if they are at different scales) Example: average intensity. For corresponding regions (even of different sizes) it will be the same. - For a point in one image, we can consider it as a function of region size (circle radius) Image 1 Image 2 region size

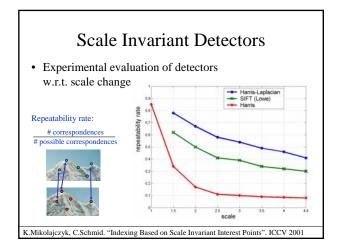






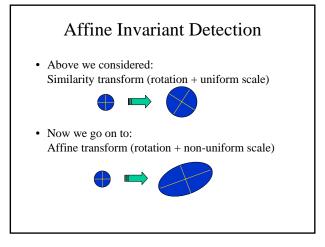
# Scale Invariant Detection • Compare to human vision: eye's response Shimon Ullman, Introduction to Computer and Human Vision Course, Fall 2003





# Scale Invariant Detection: Summary Given: two images of the same scene with a large scale difference between them Goal: find the same interest points independently in each image Solution: search for maxima of suitable functions in scale and in space (over the image) Methods: Harris-Laplacian [Mikolajczyk, Schmid]: maximize Laplacian over scale, Harris' measure of corner response over the image SIFT [Lowe]: maximize Difference of Gaussians over scale and space

# Contents • Harris Corner Detector - Description - Analysis • Detectors - Rotation invariant - Scale invariant - Affine invariant • Descriptors - Rotation invariant - Scale invariant - Affine invariant - Affine invariant - Scale invariant - Scale invariant - Affine invariant



## Affine Invariant Detection

- · Take a local intensity extremum as initial point
- Go along every ray starting from this point and stop when extremum of function f is reached



 $f(t) = \frac{\left|I(t) - I_0\right|}{\frac{1}{t} \int_0^t \left|I(t) - I_0\right| dt}$ 

We will obtain approximately corresponding regions

Remark: we search for scale in every direction





T.Tuytelaars, L.V.Gool. "Wide Baseline Stereo Matching Based on Local,

## **Affine Invariant Detection**

- The regions found may not exactly correspond, so we approximate them with ellipses
- Geometric Moments:

 $m_{pq} = \int_{a}^{b} x^{p} y^{q} f(x, y) dx dy$ 

Fact: moments  $m_{pq}$  uniquely determine the function f

Taking f to be the characteristic function of a region (1 inside, 0 outside), moments of orders up to 2 allow to approximate the region by an ellipse



This ellipse will have the same moments of orders up to 2 as the original region

# **Affine Invariant Detection**

· Covariance matrix of region points defines an ellipse:







 $p^T \Sigma_1^{-1} p = 1$ 

 $\sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{1}{n} \right)^{n}$ 

 $q^T \Sigma_2^{-1} q = 1$ 

 $\Sigma_2 = \left\langle q q^T \right\rangle_{\text{region 2}}$ 

(  $p = [x, y]^T$  is relative to the center of mass)

 $\Sigma_2 = A \Sigma_1 A^T$ 

Ellipses, computed for corresponding regions, also correspond!

# **Affine Invariant Detection**

- · Algorithm summary (detection of affine invariant region):
  - Start from a local intensity extremum point
  - Go in every direction until the point of extremum of some function f
  - Curve connecting the points is the region boundary
  - Compute geometric moments of orders up to 2 for this region
  - Replace the region with ellipse





T.Tuytelaars, L.V.Gool. "Wide Baseline Stereo Matching Based on Local, Affinely Invariant Regions". BMVC 2000.

# **Affine Invariant Detection**

- Maximally Stable Extremal Regions
  - Threshold image intensities:  $I > I_0$
  - Extract connected components ("Extremal Regions")
  - Find a threshold when an extremal region is "Maximally Stable",
     i.e. *local minimum* of the relative growth of its square
  - Approximate a region with an *ellipse*



J.Matas et.al. "Distinguished Regions for Wide-baseline Stereo". Research Report of CMP, 2001.

# Affine Invariant Detection : Summary

- Under affine transformation, we do not know in advance shapes of the corresponding regions
- Ellipse given by geometric covariance matrix of a region robustly approximates this region
- $\bullet \ \ \text{For corresponding regions ellipses also correspond}$

### Methods

- 1. Search for extremum along rays [Tuytelaars, Van Gool]:
- 2. Maximally Stable Extremal Regions [Matas et.al.]



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# Point Descriptors

- We know how to detect points
- Next question:

## How to match them?



Point descriptor should be:

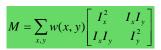
- 1. Invariant
- 2. Distinctive

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# **Descriptors Invariant to Rotation**

• Harris corner response measure: depends only on the eigenvalues of the matrix M











C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988

# **Descriptors Invariant to Rotation**

• Image moments in polar coordinates

$$m_{kl} = \iint r^k e^{-i\theta l} I(r,\theta) dr d\theta$$

Rotation in polar coordinates is translation of the angle:

 $\theta \! \to \theta \! + \theta_0$  This transformation changes only the phase of the moments, but

Rotation invariant descriptor consists of magnitudes of moments:



Matching is done by comparing vectors  $[|m_{kl}|]_{k,l}$ 

J.Matas et.al. "Rotational Invariants for Wide-baseline Stereo". Research Report of CMP, 2003

# **Descriptors Invariant to Rotation**

• Find local orientation

Dominant direction of gradient





· Compute image derivatives relative to this orientation

 $^{\rm I}$  K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001  $^{\rm 2}$  D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". Accepted to IJCV 2004

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# Descriptors Invariant to Scale

• Use the scale determined by detector to compute descriptor in a normalized frame

For example:

- moments integrated over an adapted window
- ullet derivatives adapted to scale:  $sI_x$

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# **Affine Invariant Descriptors**

• Affine invariant color moments

$$m_{pq}^{abc} = \int_{region} x^p y^q R^a(x, y) G^b(x, y) B^c(x, y) dx dy$$

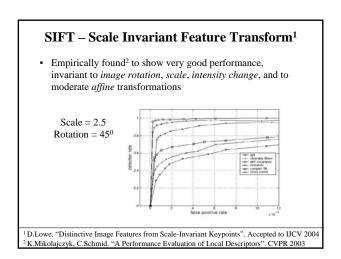
Different combinations of these moments are fully affine invariant

Also invariant to affine transformation of intensity  $I \rightarrow a \ I + b$ 

F.Mindru et.al. "Recognizing Color Patterns Irrespective of Viewpoint and Illumination". CVPR99

# Affine Invariant Descriptors • Find affine normalized frame $\Sigma_1 = \langle pp^T \rangle$ $\Sigma_2 = \langle qq^T \rangle$ $\Sigma_1^{-1} = A_1^T A_1$ $A_1$ $\Sigma_2^{-1} = A_2^T A_2$ • Compute rotational invariant descriptor in this normalized frame

J.Matas et.al. "Rotational Invariants for Wide-baseline Stereo". Research Report of CMP, 2003



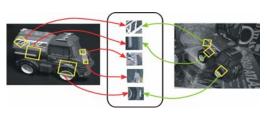
# **CVPR 2003 Tutorial**

# Recognition and Matching Based on Local Invariant Features

David Lowe Computer Science Department University of British Columbia

## **Invariant Local Features**

• Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



SIFT Features

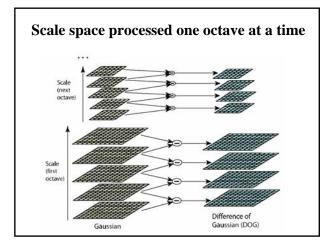
# Advantages of invariant local features

- Locality: features are local, so robust to occlusion and clutter (no prior segmentation)
- **Distinctiveness:** individual features can be matched to a large database of objects
- Quantity: many features can be generated for even small objects
- Efficiency: close to real-time performance
- Extensibility: can easily be extended to wide range of differing feature types, with each adding robustness

## Scale invariance

# Requires a method to repeatably select points in location and scale:

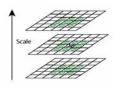
- The only reasonable scale-space kernel is a Gaussian (Koenderink, 1984; Lindeberg, 1994)
- An efficient choice is to detect peaks in the difference of Gaussian pyramid (Burt & Adelson, 1983; Crowley & Parker, 1984 – but examining more scales)
- Difference-of-Gaussian with constant ratio of scales is a close approximation to Lindeberg's scale-normalized Laplacian (can be shown from the heat diffusion equation)

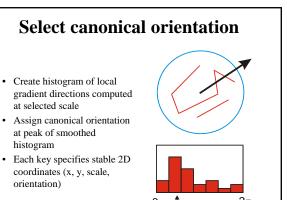


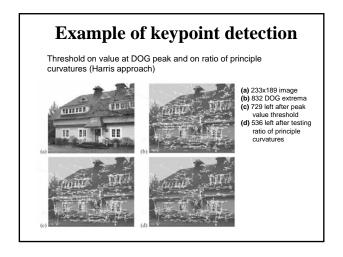
# **Key point localization**

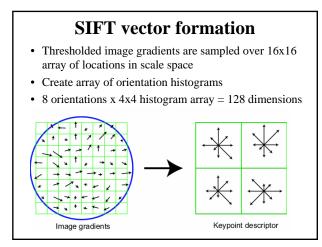
- Detect maxima and minima of difference-of-Gaussian in scale space
- Fit a quadratic to surrounding values for sub-pixel and sub-scale interpolation (Brown & Lowe, 2002)
- Taylor expansion around point:

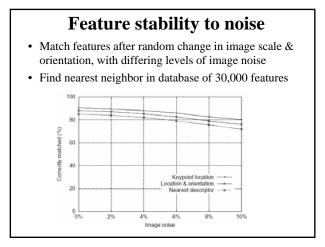
$$\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$$











# 

Feature stability to affine change

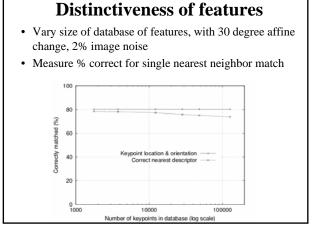
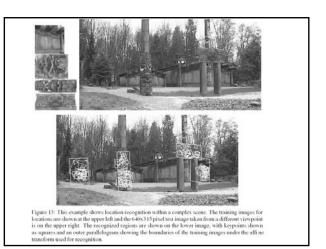




Figure 12: The training images for two objects are shown on the left. These can be recognized in a cluttered image with extensive occlusion, shown in the middle. The results of recognition are shown on the right. A parallelogram is drawn around each recognized object showing the boundaries of the original training image under the affine transformation solved for during recognition. Smaller squares indicate the keypoints that were used for recognition.



# A good SIFT features tutorial

http://www.cs.toronto.edu/~jepson/csc2503/tutSIFT04.pdf

By Estrada, Jepson, and Fleet.

# Talk Resume

- Stable (repeatable) feature points can be detected regardless of image changes
  - Scale: search for correct scale as maximum of appropriate function
  - Affine: approximate regions with *ellipses* (this operation is affine invariant)
- Invariant and distinctive descriptors can be computed
  - Invariant moments
  - Normalizing with respect to scale and affine transformation

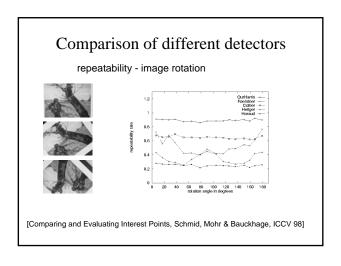
# Evaluation of interest points and descriptors

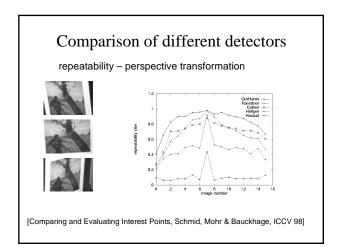
Cordelia Schmid
CVPR'03 Tutorial

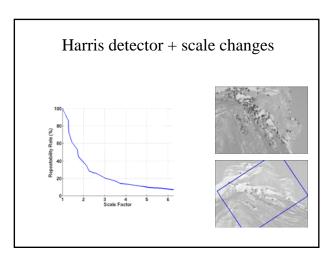
# Introduction

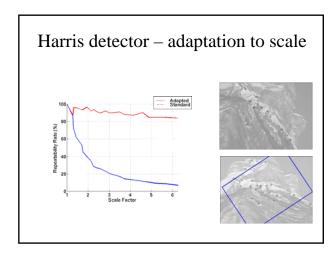
- Quantitative evaluation of interest point detectors
  - points / regions at the same relative location
  - => repeatability rate
- Quantitative evaluation of descriptors
  - distinctiveness
  - => detection rate with respect to false positives

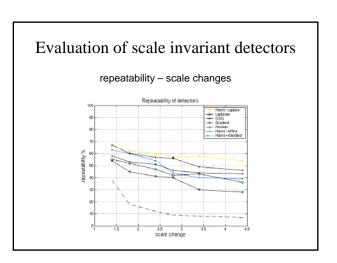
# Quantitative evaluation of detectors Repeatability rate: percentage of corresponding points homography Two points are corresponding if The location error is less than 1.5 pixel The intersection error is less than 20%

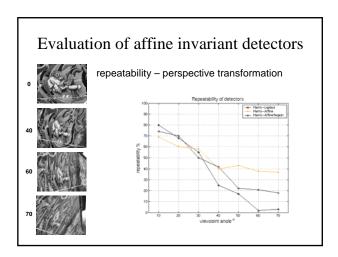








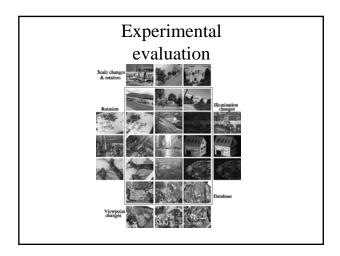


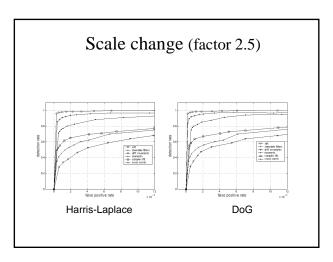


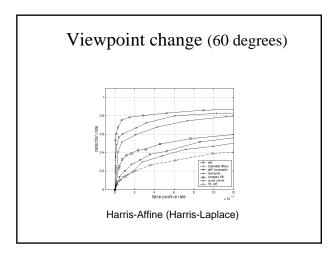
# Quantitative evaluation of descriptors

- Evaluation of different local features
  - SIFT, steerable filters, differential invariants, moment invariants, cross-correlation
- · Measure : distinctiveness
  - receiver operating characteristics of detection rate with respect to false positives
  - detection rate = correct matches / possible matches
  - false positives = false matches / (database points \* query points)

[A performance evaluation of local descriptors, Mikolajczyk & Schmid, CVPR'03]







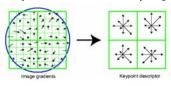
# Descriptors - conclusion

- SIFT + steerable perform best
- Performance of the descriptor independent of the detector
- Errors due to imprecision in region estimation, localization

# end

### SIFT - Scale Invariant Feature Transform

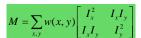
- · Descriptor overview:
  - Determine scale (by maximizing DoG in scale and in space), local orientation as the dominant gradient direction.
     Use this scale and orientation to make all further computations invariant to scale and rotation.
  - Compute gradient orientation histograms of several small windows (128 values for each point)
  - Normalize the descriptor to make it invariant to intensity change



D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". Accepted to IJCV 2004

# **Affine Invariant Texture Descriptor**

- Segment the image into regions of different textures (by a non-invariant method)
- Compute matrix *M* (the same as in Harris detector) over these regions



· This matrix defines the ellipse





- Regions described by these ellipses are
- invariant under affine transformations
   Find affine normalized frame
- Compute rotation invariant descriptor

F.Schaffalitzky, A.Zisserman. "Viewpoint Invariant Texture Matching and Wide Baseline Stereo". ICCV 2003

# **Invariance to Intensity Change**

- Detectors
  - mostly invariant to affine (linear) change in image intensity, because we are searching for maxima
- Descriptors
  - Some are based on derivatives => invariant to intensity shift
  - Some are normalized to tolerate intensity scale
  - Generic method: pre-normalize intensity of a region (eliminate shift and scale)