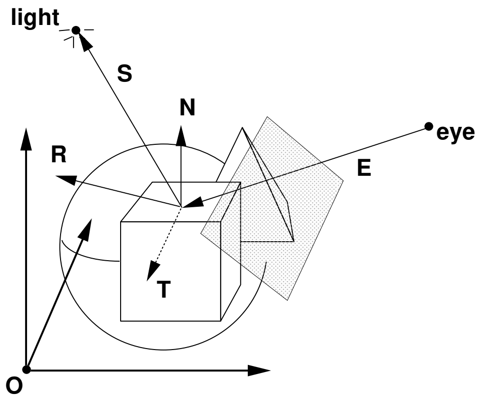
Lecture 14: October 21, 1997

Final project topics!
  Email feedback
  Team formation (only 20?)
  Proposals by Friday
    Put on web (plaintext, simple HTML)
    Send staff email, with proposal URL
      Make sure to `Cc:` all team members !
Today:
  Radiosity (continuation of Global Illumination)
Thursday:
  Animation, Physically-Based Modeling

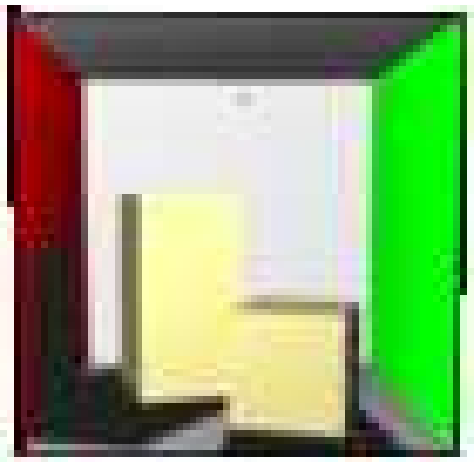
Radiosity Solvers

One possibility: modify ray-tracer to sample area light sources, hemispherical irradiance:



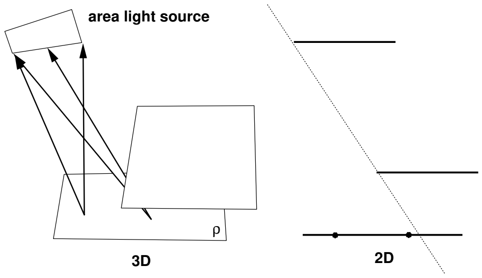
this requires lots of CPU for ray casting
any other disadvantages ?

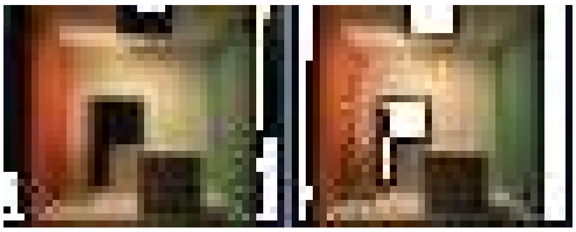
Hard shadows



Abrupt changes in illumination arise in ray tracing
  "Point" light source is non-physical; it goes
    abruptly from visible to invisible

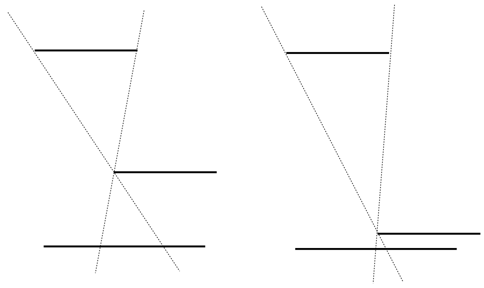
Soft shadows





Illumination Gradients

What if occluder is relatively far from receiver?
What if occluder is relatively close to receiver?



Radiosity Methods

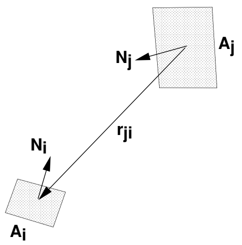
Arose from thermal radiation literature
(Analyzing heat thrown from blast furnaces)
Find equilibrium distribution of radiant energy
Suitable for ideal *diffuse* environments

Radiosity Equation - Intuitive Form

Radiosity equation:

$$\begin{aligned}\text{Exitant Power} &= \text{Emitted Power} + \text{Reflected Power}\\ B_iA_i &= E_iA_i + \rho_i(\text{Total Incident Power})\\ &= E_iA_i + \rho_i \sum_j (\text{Incident Power from } j)\\ &= E_iA_i + \rho_i \sum_j B_jA_jF_{ji}\end{aligned}$$

Where F_{ji} is the **fraction of radiant power leaving j that arrives at i**



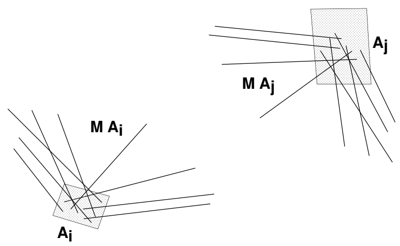
(Or, fraction of j's hemisphere occupied by i, integrated over all points on j)
F_{ji} called the "Form Factor" between j and i
Note order, interpretation of subscripts!

Radiosity Equation: Computational Form

Geometric fact:

$$A_jF_{ji} = A_iF_{ij}$$

Proof: lines can be counted under some measure



# of lines through A_i is MA_i
# of lines through A_j is MA_j
Why? What is M?
So # of lines leaving A_i arriving at A_j is MA_iF_{ij},
and # of lines leaving A_j arriving at A_i is MA_jF_{ji}
But both quantities describe the same set of lines!

Radiosity Equation: Computational Form

Relate total power as follows

$$\begin{aligned} B_i A_i &= E_i A_i + \rho_i \sum_j B_j A_j F_{ji} \\ &= E_i A_i + \rho_i \sum_j B_j F_{ij} A_i \end{aligned}$$

This is the "power equation"

Dividing power equation through by A_i yields

$$B_i = E_i + \rho_i \sum_j B_j F_{ij}$$

This is the "radiosity equation"
  More efficient, but more confusing
  Be careful to get subscripts right!

Solving the Radiosity Equation

Can't be done in closed form (in general)
  so use *numerical methods* (what are they?)
Set up initial conditions (for all i):
  Assign E_i:
    $E_i > 0$ (light sources)
    $E_i = 0$ (all other surfaces)
  Assign B_i, initial guess for i^{th} surface's radiosity
    What is a good heuristic for initial guess?
  Determine **geometric** coupling:
    Compute F_{ij} for all i, j
    (How long does this take?)

Solving the Radiosity Equation

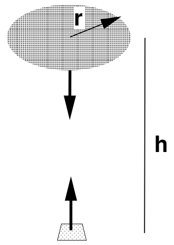
Iterate system until convergence
  Improve "guess" for each B_i based on the B_j, $j \neq i$
  In practice, can't capture fine-scale illumination
    effects unless original surfaces are "meshed"
Terminology (somewhat arbitrary):
  Surfaces: boundaries of scene objects
  Patches: surface portion acting as illumination source
  Elements: patch portion receiving illumination

Computing Form Factors

What is the form factor? (Intuition.)
  If patches are back-facing?
  If patches are front-facing, but occluded?
  If patches are front-facing, and intervisible?
    What is a good approximation?

Computing VISIBLE Form Factors

Approximating front-facing, intervisible F_{ij}



For i differential, j a parallel disk of radius r
Then solid angle subtended by j from i is

$$\frac{r^2}{h^2 + r^2}$$

If non-parallel, incorporate a cosine term
Closed-form expressions exist for other cases

Computing PARTIAL Form Factors

Front-facing, but only **partially** intervisible?
  Very complex; no known closed-form sol'n
  What to do?

Radiosity Solvers

Numerical iteration of radiosity values

At equilibrium:

$$B_i = E_i + \rho_i \sum_j B_j F_{ij}$$

So, given estimates at step t, produce estimates at step $t+1$ as:

$$B_i^{t+1} = E_i + \rho_i \sum_j B_j^t F_{ij}$$