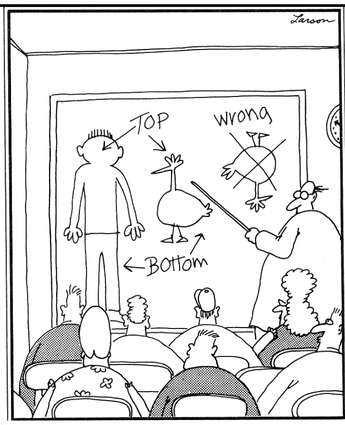
Transformations & Local Illumination



People who don't know which end is up.

Last Time?

- Transformations
  - Rigid body, affine, similitude, linear, projective
- Linearity
  - f(x+y)=f(x)+f(y); f(ax) = a f(x)
- Homogeneous coordinates
  - (x, y, z, w) ~ (x/w, y/w, z/w)
  - Translation in a matrix
  - Projective transforms
- Non-commutativity
- Transformations in modeling

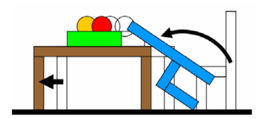
Today

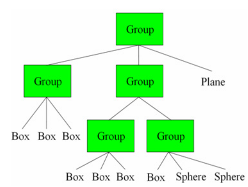
- Intro to Transformations
- Classes of Transformations
- Representing Transformations
- Combining Transformations
- Transformations in Modeling
- Adding Transformations to our Ray Tracer

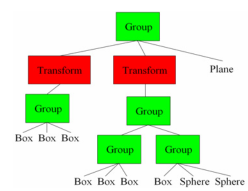
- Local illumination and shading

Why is a Transform an Object3D?

- To position the logical groupings of objects within the scene

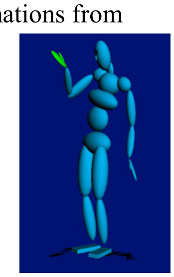






Recursive call and composition

- Recursive call tree: leaves are evaluated first
- Apply matrix from right to left
- Natural composition of transformations from object space to world space
  - First put finger in hand frame
  - Then apply elbow transform
  - Then shoulder transform
  - etc.



Questions?

Today

- Intro to Transformations
- Classes of Transformations
- Representing Transformations
- Combining Transformations
- Transformations in Modeling
- Adding Transformations to our Ray Tracer

Incorporating Transforms

1. Make each primitive handle any applied transformations

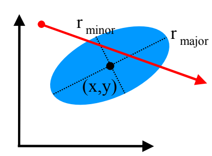
```
Sphere {
    center 1 0.5 0
    radius 2
}
```

2. Transform the Rays

```
Transform {
    Translate { 1 0.5 0 }
    Scale { 2 2 2 }
    Sphere {
        center 0 0 0
        radius 1
    }
}
```


Primitives handle Transforms

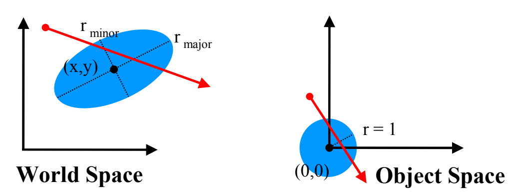
```
Sphere {
    center 3 2 0
    z_rotation 30
    r_major 2
    r_minor 1
}
```



- Complicated for many primitives

Transform the Ray

- Move the ray from *World Space* to *Object Space*

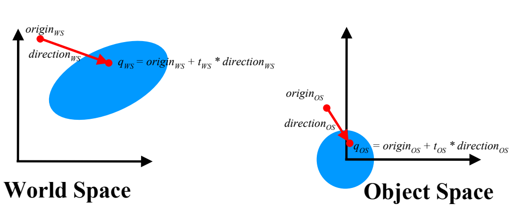


$$p_{WS} = \mathbf{M} \; p_{OS}$$

$$p_{OS} = \mathbf{M}^{-1} \; p_{WS}$$

Transform Ray

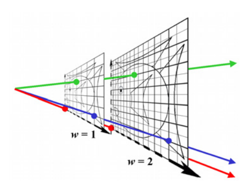
- New origin:
  $origin_{OS} = \mathbf{M}^{-1}\ origin_{WS}$
- New direction:
  $direction_{OS} = \mathbf{M}^{-1}\ (origin_{WS} + 1 * direction_{WS}) \ - \ \mathbf{M}^{-1}\ origin_{WS}$
  $direction_{OS} = \mathbf{M}^{-1}\ direction_{WS}$



Transforming Points & Directions

- Transform point

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} ax+by+cz+d \\ ex+fy+gz+h \\ ix+jy+kz+l \\ 1 \end{bmatrix}$$



- Transform direction

$$\begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ ex+fy+gz \\ ix+jy+kz \\ 0 \end{bmatrix}$$

Homogeneous Coordinates: *(x,y,z,w)*
w = 0 is a point at infinity (direction)

• With the usual storage strategy (no *w*) you need different routines to apply M to a point and to a direction

What to do about the depth, t

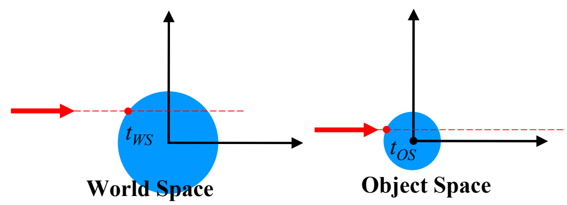
If **M** includes scaling, $direction_{OS}$ will NOT be normalized

1. Normalize the direction

2. Don't normalize the direction

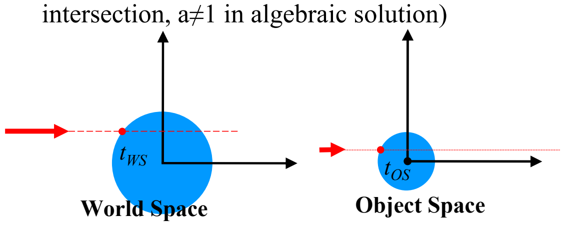
1. Normalize direction

- $t_{OS} \neq t_{WS}$
  and must be rescaled after intersection



2. Don't normalize direction

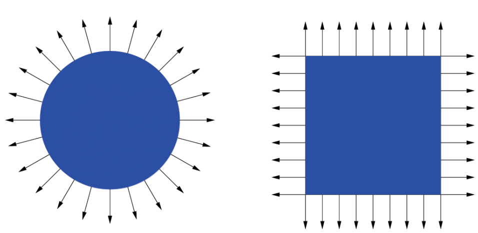
- $t_{OS} = t_{WS}$
- Don't rely on t_{OS} being true distance during intersection routines (e.g. geometric ray-sphere intersection, a≠1 in algebraic solution)



Questions?

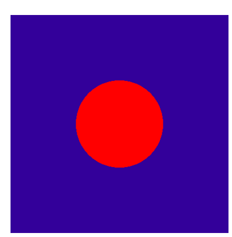
New component of the Hit class

- Surface Normal: unit vector that is locally perpendicular to the surface

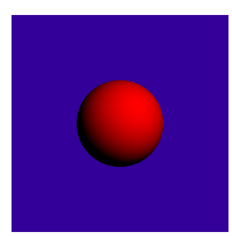


Why is the Normal important?

- It's used for shading — makes things look 3D!

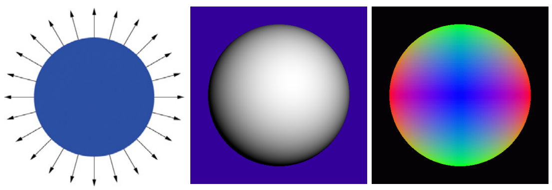


object color only
(Assignment 1)



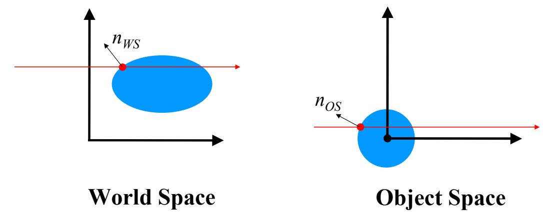
Diffuse Shading
(Assignment 2)

Visualization of Surface Normal



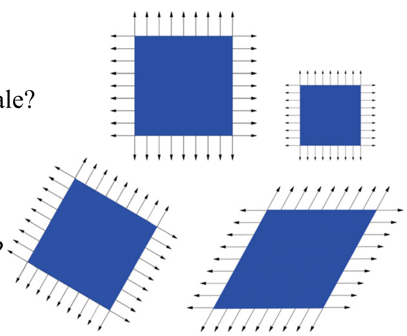
$\pm x$ = Red
$\pm y$ = Green
$\pm z$ = Blue

How do we transform normals?



Transform the Normal like the Ray?

- translation?
- rotation?
- isotropic scale?
- scale?
- reflection?
- shear?
- perspective?



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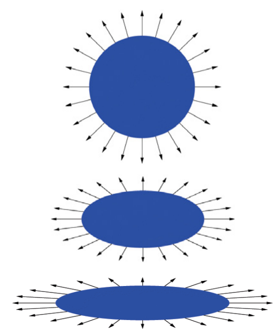
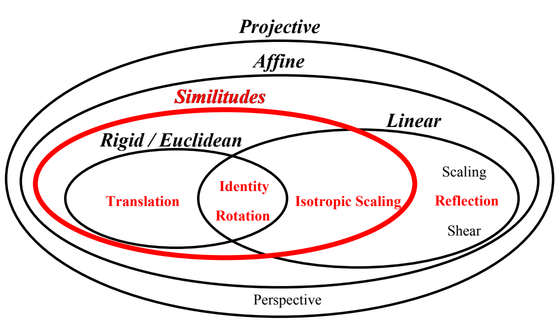
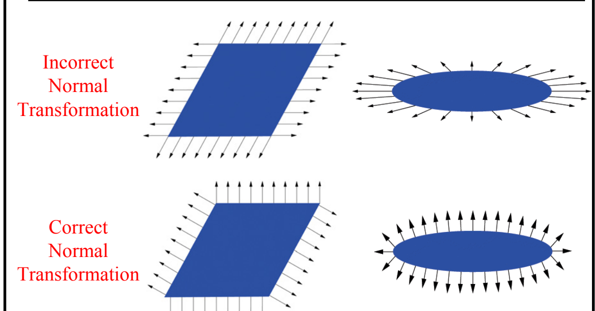


What class of transforms?

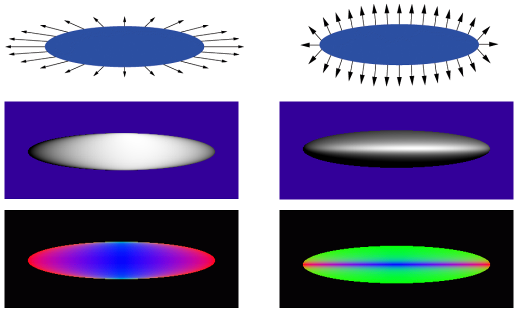


a.k.a. Orthogonal Transforms

Transformation for shear and scale



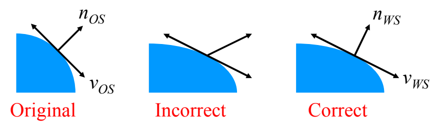
More Normal Visualizations



Incorrect Normal Transformation   Correct Normal Transformation

So how do we do it right?

- Think about transforming the *tangent plane* to the normal, not the normal *vector*



Pick any vector v_{OS} in the tangent plane, how is it transformed by matrix **M**?

$$v_{WS} = \mathbf{M}\ v_{OS}$$

Transform tangent vector v

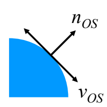
v is perpendicular to normal n:

Dot product $\quad n_{OS}^{\mathbf{T}}\ v_{OS} = 0$

$$n_{OS}^{\mathbf{T}}\ (\mathbf{M}^{-1}\ \mathbf{M})\ v_{OS} = 0$$

$$(n_{OS}^{\mathbf{T}}\ \mathbf{M}^{-1})\ (\mathbf{M}\ \ v_{OS}) = 0$$

$$(n_{OS}^{\mathbf{T}}\ \mathbf{M}^{-1})\ v_{WS} = 0$$

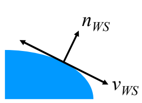


v_{WS} is perpendicular to normal n_{WS}:

$$n_{WS}^{\mathbf{T}} = n_{OS}^{\mathbf{T}}\ (\mathbf{M}^{-1})$$

$$\boxed{n_{WS} = (\mathbf{M}^{-1})^{\mathbf{T}}\ n_{OS}}$$

$$n_{WS}^{\mathbf{T}}\ v_{WS} = 0$$



Comment

- So the correct way to transform normals is:

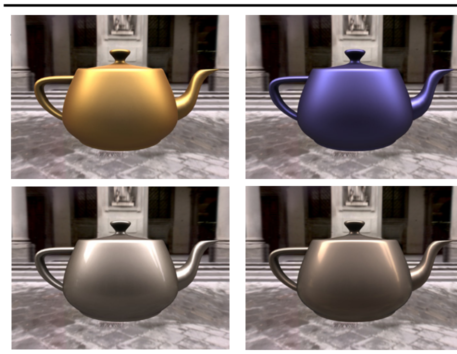
  $n_{WS} = (\mathbf{M}^{-1})^{\mathbf{T}}\ n_{OS}$ Sometimes noted $\mathbf{M}^{-\mathbf{T}}$
- But why did $n_{WS} = \mathbf{M}\ n_{OS}$ work for similitudes?
- Because for similitude / similarity transforms,

  $$(\mathbf{M}^{-1})^{\mathbf{T}} = \lambda\ \mathbf{M}$$
- e.g. for orthonormal basis:

$$\mathbf{M} = \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ n_x & n_y & n_z \end{pmatrix} \qquad \mathbf{M}^{-1} = \begin{pmatrix} x_u & x_v & x_n \\ y_u & y_v & y_n \\ z_u & z_v & z_n \end{pmatrix}$$

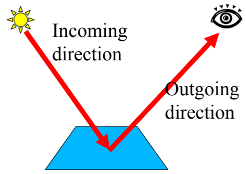
Questions?

Local Illumination



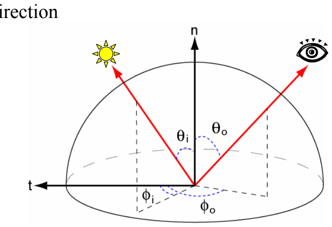
BRDF

- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function



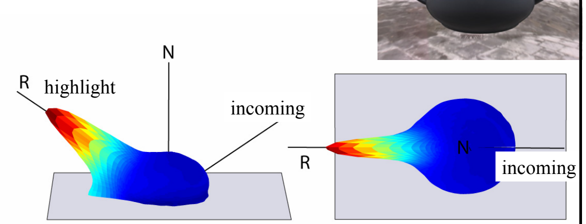
BRDF

- Bidirectional Reflectance Distribution Function
  - 4D
    - 2 angles for each direction
  - $R(\theta_i, \phi_i; \theta_o, \phi_o)$



Slice at constant incidence

- 2D spherical function



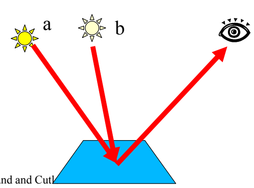
Example: Plot of "PVC" BRDF at 55° incidence

Unit issues - radiometry

- We will not be too formal in this lecture
- Typical issues:
  - Directional quantities vs. integrated over all directions
  - Differential terms: per solid angle, per area, per time
  - Power, intensity, flux

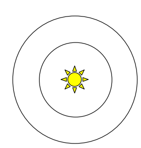
Light sources

- Today, we only consider point light sources
- For multiple light sources, use linearity
  - We can add the solutions for two light sources
    - I(a+b)=I(a)+I(b)
  - We simply multiply the solution when we scale the light intensity
    - I(s a) = s I(a)



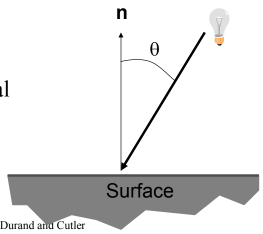
Light intensity

- $1/r^2$ falloff
  - Why?
  - Same power in all concentric circles



Incoming radiance

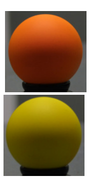
- The amount of light received by a surface depends on incoming angle
  - Bigger at normal incidence
    - Similar to Winter/Summer difference
- By how much?
  - Cos θ law
  - Dot product with normal
  - This term is sometimes included in the BRDF, sometimes not

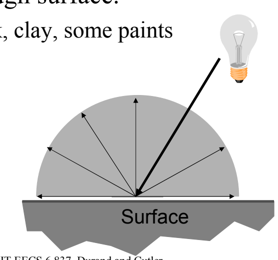


Questions?

Ideal Diffuse Reflectance

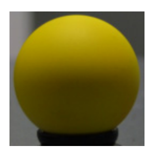
- Assume surface reflects equally in all directions.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
  - Example: chalk, clay, some paints

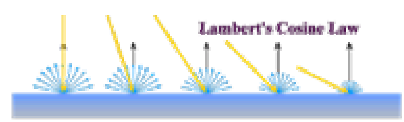




Ideal Diffuse Reflectance

- Ideal diffuse reflectors reflect light according to Lambert's cosine law.

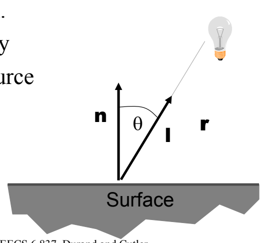




Ideal Diffuse Reflectance

- Single Point Light Source
  - k_d: diffuse coefficient.
  - **n**: Surface normal.
  - **l**: Light direction.
  - L_i: Light intensity
  - r: Distance to source

$$L_o = k_d (\mathbf{n} \cdot \mathbf{l}) \frac{L_i}{r^2}$$



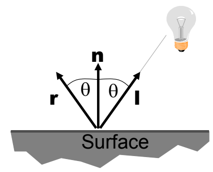
Ideal Diffuse Reflectance – More Details

- If **n** and **l** are facing away from each other, **n • l** becomes negative.
- Using max((**n • l**),0) makes sure that the result is zero.
  - From now on, we mean max() when we write •.
- Do not forget to **normalize your vectors** for the dot product!

Questions?

Ideal Specular Reflectance

- Reflection is only at mirror angle.
  - View dependent
  - Microscopic surface elements are usually oriented in the same direction as the surface itself.
  - Examples: mirrors, highly polished metals

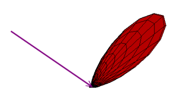


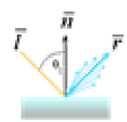
Non-ideal Reflectors

- Real materials tend to deviate significantly from ideal mirror reflectors.
- Highlight is blurry
- They are not ideal diffuse surfaces either …

Non-ideal Reflectors

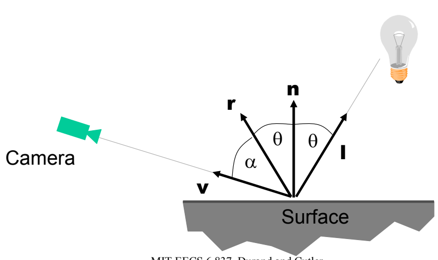
- Simple Empirical Model:
  - We expect most of the reflected light to travel in the direction of the ideal ray.
  - However, because of microscopic surface variations we might expect some of the light to be reflected just slightly offset from the ideal reflected ray.
  - As we move farther and farther, in the angular sense, from the reflected ray we expect to see less light reflected.





The Phong Model

- How much light is reflected?
  - Depends on the angle between the ideal reflection direction and the viewer direction α.

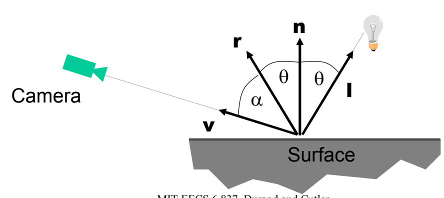


The Phong Model

- Parameters
  - k_s: specular reflection coefficient
  - q : specular reflection exponent

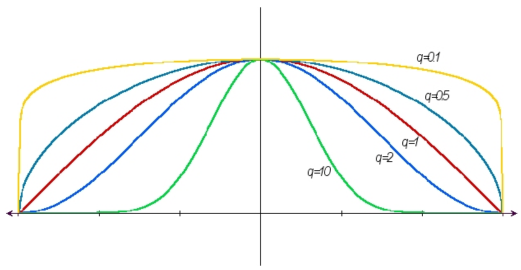
$$L_o = k_s (\cos\alpha)^q \frac{L_i}{r^2}$$

$$L_o = k_s (\mathbf{v} \cdot \mathbf{r})^q \frac{L_i}{r^2}$$



The Phong Model

- Effect of the q coefficient



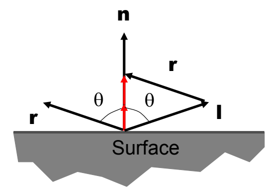
How to get the mirror direction?

$$\mathbf{r} + \mathbf{l} = 2\ \cos\theta\, \mathbf{n}$$

$$\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$$

$$L_o = k_s (\mathbf{v} \cdot \mathbf{r})^q \frac{L_i}{r^2} =$$

$$= k_s (\mathbf{v} \cdot (2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}))^q \frac{L_i}{r^2}$$

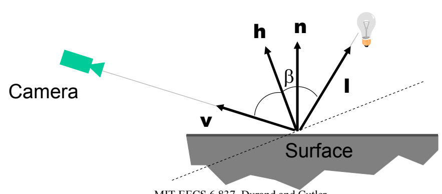


Blinn-Torrance Variation

- Uses the halfway vector **h** between **l** and **v**.

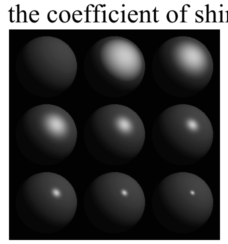
$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$

$$L_o = k_s (\cos\beta)^q \frac{L_i}{r^2} = k_s (\mathbf{n} \cdot \mathbf{h})^q \frac{L_i}{r^2}$$

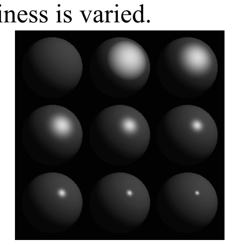


Phong Examples

- The following spheres illustrate specular reflections as the direction of the light source and the coefficient of shininess is varied.



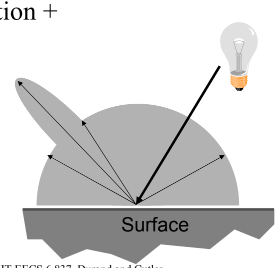
Phong



Blinn-Torrance

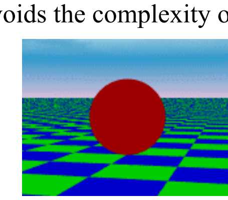
The Phong Model

- Sum of three components:
  diffuse reflection +
  specular reflection +
  "ambient".



Ambient Illumination

- Represents the reflection of all indirect illumination.
- This is a total hack!
- Avoids the complexity of global illumination.

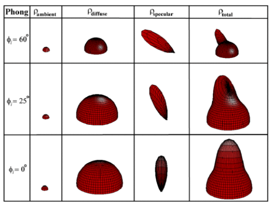


$$L(\omega_r) = k_a$$

Putting it all together

- Phong Illumination Model

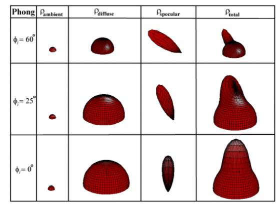
$$L_o = k_a + \left(k_d(\mathbf{n}\cdot\mathbf{l}) + k_s(\mathbf{v}\cdot\mathbf{r})^q\right)\frac{L_i}{r^2}$$



For Assignment 3

- Variation on Phong Illumination Model

$$L_o = k_a L_a + \left(k_d(\mathbf{n}\cdot\mathbf{l}) + k_s(\mathbf{v}\cdot\mathbf{r})^q\right)\frac{L_i}{r^2}$$



Adding color

- Diffuse coefficients:
  – $k_{d\text{-}red}$, $k_{d\text{-}green}$, $k_{d\text{-}blue}$
- Specular coefficients:
  – $k_{s\text{-}red}$, $k_{s\text{-}green}$, $k_{s\text{-}blue}$
- Specular exponent:
  q

Questions?

Shaders (`Material` class)

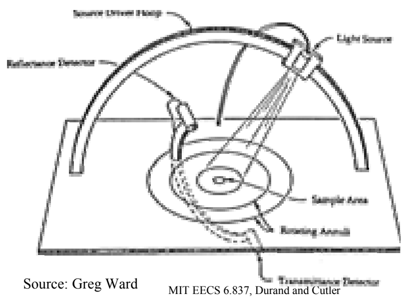
- Functions executed when light interacts with a surface
- Constructor:
  – set shader parameters
- Inputs:
  – Incident radiance
  – Incident & reflected light directions
  – surface tangent (anisotropic shaders only)
- Output:
  – Reflected radiance

BRDFs in the movie industry

- http://www.virtualcinematography.org/publications/acrobat/BRDF-s2003.pdf
- For the Matrix movies
- Clothes of the agent Smith are CG, with measured BRDF

How do we obtain BRDFs?

- Gonioreflectometer
  - 4 degrees of freedom



Source: Greg Ward

BRDFs in the movie industry

- http://www.virtualcinematography.org/publications/acrobat/BRDF-s2003.pdf
- For the Matrix movies

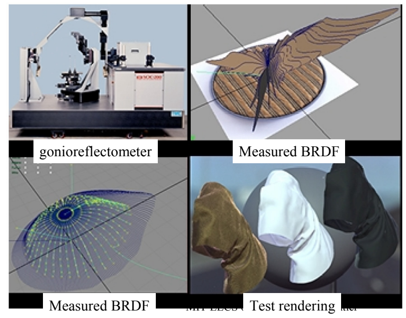
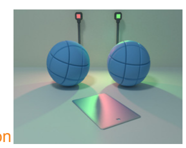


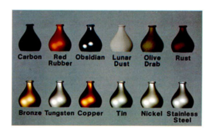
Photo CG Photo CG

BRDF Models

- Phenomenological
  - Phong [75]
    - Blinn-Phong [77]
  - Ward [92]
  - Lafortune et al. [97]
  - Ashikhmin et al. [00]
- Physical
  - Cook Torrance [81]
  - He et al. [91]

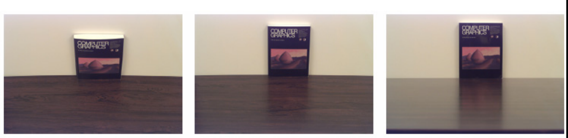
Roughly increasing computation time

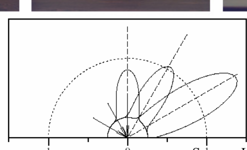




Fresnel Reflection

- Increasing specularity near grazing angles.

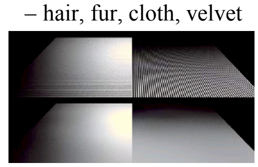




Source: Lafortune et al. 97

Anisotropic BRDFs

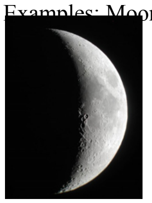
- Surfaces with strongly oriented microgeometry elements
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet



Source: Westin et.al 92

Off-specular & Retro-reflection

- Off-specular reflection
  - Peak is not centered at the reflection direction
- Retro-reflection:
  - Reflection in the direction of incident illumination
  - Examples: Moon, road markings



Questions?