Ray Casting

Last Time?

- Luxo Jr.
- Applications of Computer Graphics
- Overview of the semester
- IFS
  - Assignment 0 due tomorrow @ 11:59pm
- Questions?





Notes on Assignments

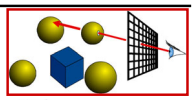
- Make sure you turn in a ***linux*** or ***windows*** executable (so we can test your program)
- Don't use athena dialup
- In your `README.txt`
  - time spent, collaborators, known bugs, extensions
- 6.837-staff@graphics.csail.mit.edu

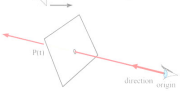
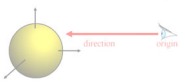
Administrivia: Lab & Office Hours

- Barb
  - Mondays 6-8pm in W20-575
- Fredo
  - Tuesdays 6-7pm in W20-575
- Rob
  - Wednesdays 8-11pm in W20-575
- Send email to make an appointment for some other time

Overview of Today

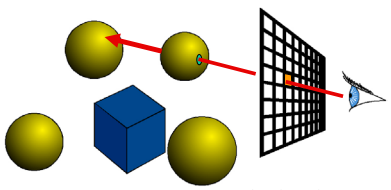
- Ray Casting Basics
- Camera and Ray Generation
- Ray-Plane Intersection
- Ray-Sphere Intersection



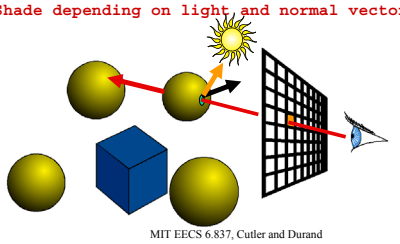
Ray Casting

```
For every pixel
  Construct a ray from the eye
  For every object in the scene
      Find intersection with the ray
      Keep if closest
```



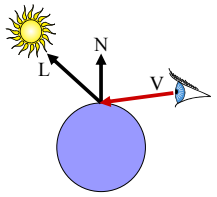
Shading

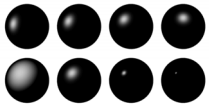
```
For every pixel
  Construct a ray from the eye
  For every object in the scene
      Find intersection with the ray
      Keep if closest
  Shade depending on light and normal vector
```



A Note on Shading

- Surface/Scene Characteristics:
  - surface normal
  - direction to light
  - viewpoint
- Material Properties
  - Diffuse (matte)
  - Specular (shiny)
  - …
- Much more next Thursday!

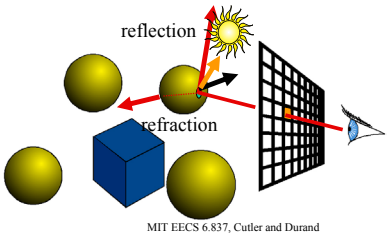




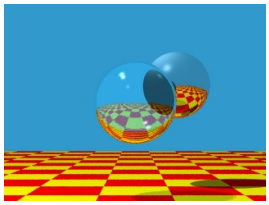
Diffuse sphere *Specular spheres*

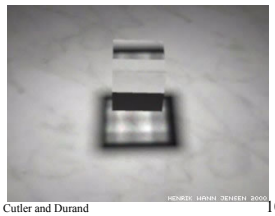
Ray Tracing

- Secondary rays (shadows, reflection, refraction)
- In a couple of weeks



Ray Tracing





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  Construct a ray from the eye
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   Shade depending on light and normal vector
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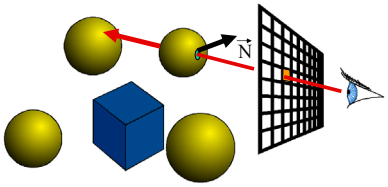
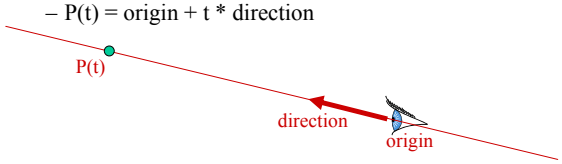


Finding the intersection and normal is the central part of ray casting

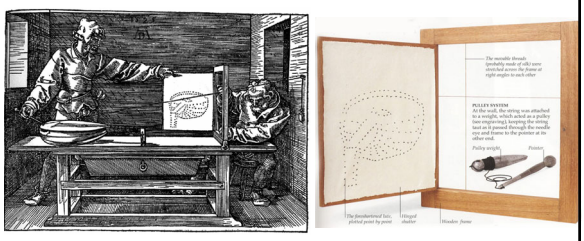
Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line
  - P(t) = origin + t * direction



Durer's Ray Casting Machine

- Albrecht Durer, 16th century



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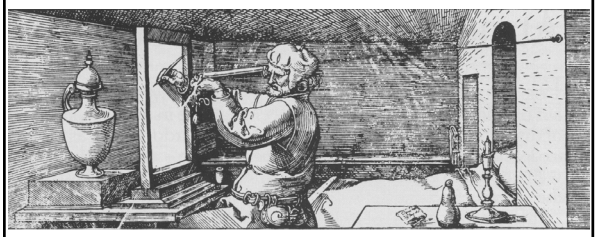

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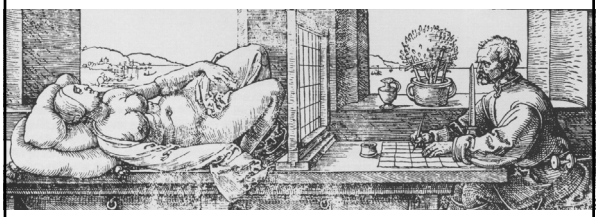
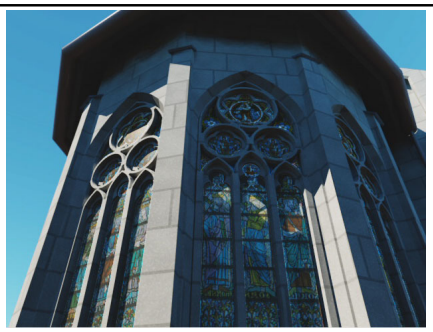


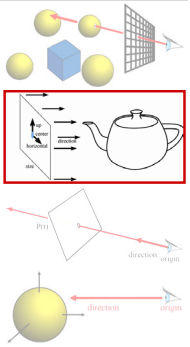
Questions?



Henrik Wann Jensen & Stephen Duck

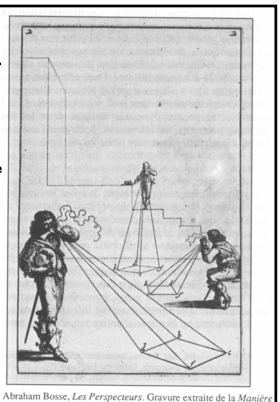
Overview of Today

- Ray Casting Basics
- Camera and Ray Generation
- Ray-Plane Intersection
- Ray-Sphere Intersection



Cameras

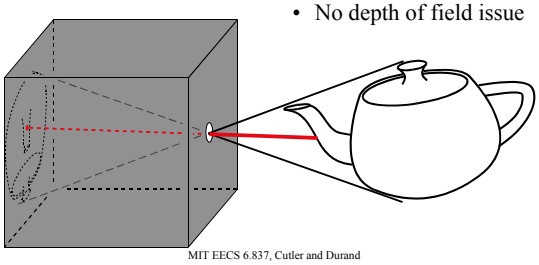
```
For every pixel
  Construct a ray from the eye
  For every object in the scene
    Find intersection with ray
      Keep if closest
```



Abraham Bosse, *Les Perspecteurs*. Gravure extraite de la *Manière*

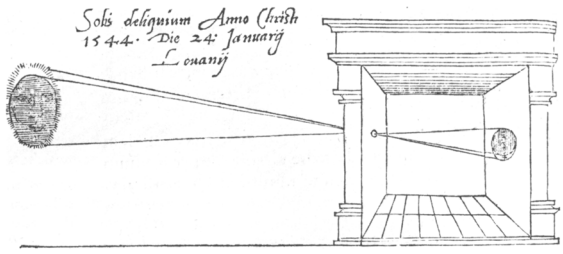
Pinhole Camera

- Box with a tiny hole
- Inverted image
- Similar triangles
- Perfect image if hole infinitely small
- Pure geometric optics
- No depth of field issue

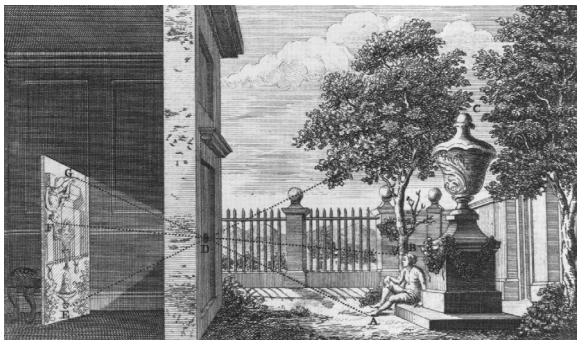


Oldest Illustration

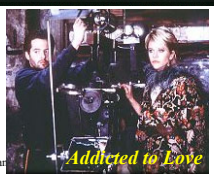
- From. R. Gemma Frisius, 1545



Camera Obscura

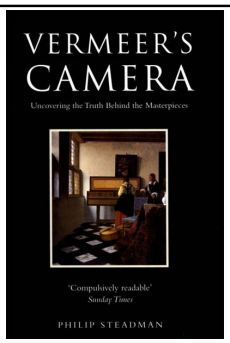


Camera Obscura Today



Abelardo Morell
www.abelardomorell.net

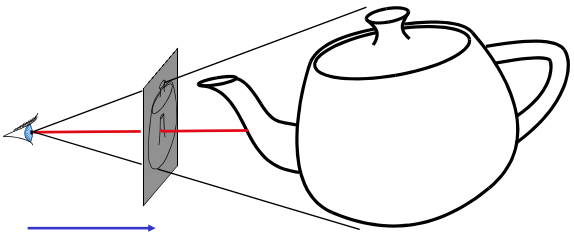
Camera Obscura in Art



Johannes Vermeer ~1665

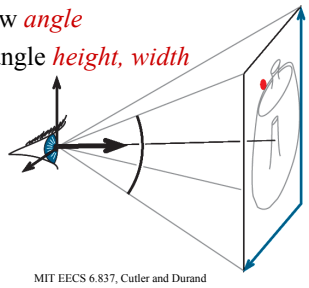
Simplified Pinhole Camera

- Eye-image pyramid (frustum)
- Note that the distance/size of image are arbitrary

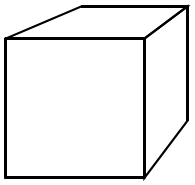


Camera Description?

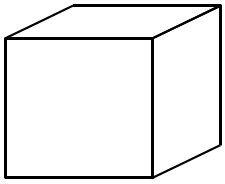
- Eye point *e (center)*
- Orthobasis *u, v, w (horizontal, up, -direction)*
- Field of view *angle*
- Image rectangle *height, width*



Perspective vs. Orthographic



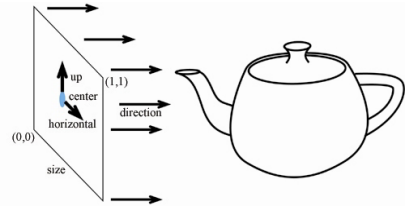
perspective



orthographic

- Parallel projection
- No foreshortening
- No vanishing point

Orthographic Camera

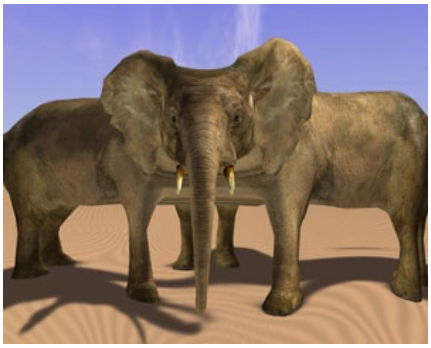


- Ray Generation?
  - Origin = center + (x-0.5)*size*horizontal + (y-0.5)*size*up
  - Direction is constant

Other Weird Cameras

- E.g. fish eye, omnimax, panorama

Questions?



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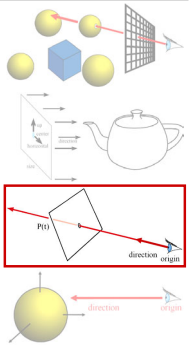
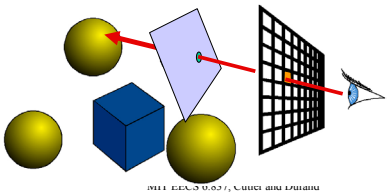


Ray Casting

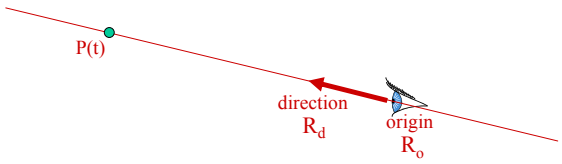
```
For every pixel
  Construct a ray from the eye
  For every object in the scene
      Find intersection with the ray
      Keep if closest
```

First we will study ray-plane intersection



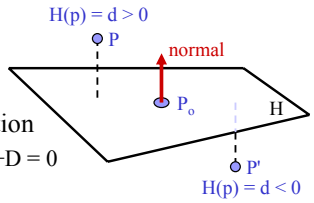
Recall: Ray Representation

- Parametric line
- $P(t) = R_o + t * R_d$
- Explicit representation



3D Plane Representation?

- Plane defined by
  - $P_o = (x,y,z)$
  - $n = (A,B,C)$
- Implicit plane equation
  - $H(P) = Ax+By+Cz+D = 0$
    $= n \cdot P + D = 0$
- Point-Plane distance?
  - If n is normalized, distance to plane, $d = H(P)$
  - d is the *signed distance*!

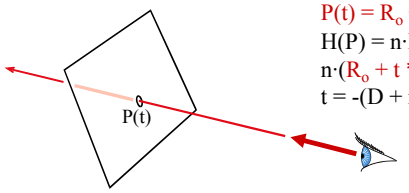


Explicit vs. Implicit?

- Ray equation is explicit $P(t) = R_o + t * R_d$
  - Parametric
  - Generates points
  - Hard to verify that a point is on the ray
- Plane equation is implicit $H(P) = n \cdot P + D = 0$
  - Solution of an equation
  - Does not generate points
  - Verifies that a point is on the plane
- Exercise: Explicit plane and implicit ray

Ray-Plane Intersection

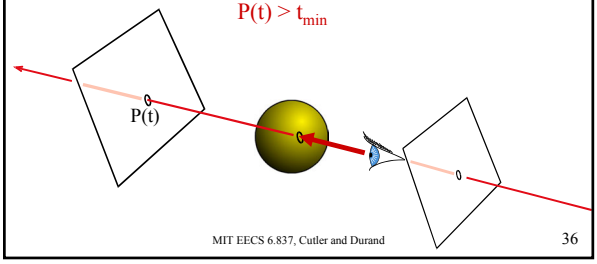
- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane & solve for t



$P(t) = R_o + t * R_d$
$H(P) = n \cdot P + D = 0$
$n \cdot (R_o + t * R_d) + D = 0$
$t = -(D + n \cdot R_o) / n \cdot R_d$

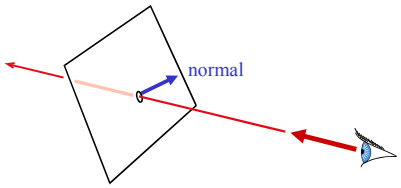
Additional Housekeeping

- Verify that intersection is closer than previous
  $P(t) < t_{current}$
- Verify that it is not out of range (behind eye)
  $P(t) > t_{min}$



Normal

- For shading
  - diffuse: dot product between light and normal
- Normal is constant



Questions?

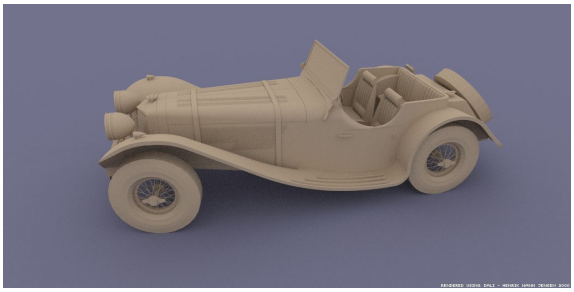


Image by Henrik Wann Jensen

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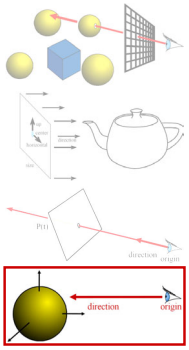
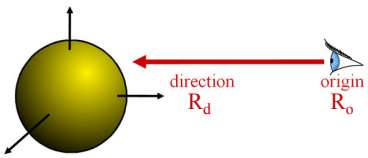


Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - $H(P) = P \cdot P - r^2 = 0$



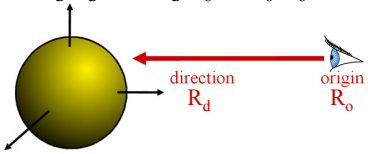
Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for t

$$P(t) = R_o + t*R_d \qquad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$

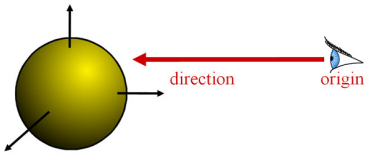


Ray-Sphere Intersection

- Quadratic: $at^2 + bt + c = 0$
  - $a = 1$ (remember, $||R_d|| = 1$)
  - $b = 2R_d \cdot R_o$
  - $c = R_o \cdot R_o - r^2$
- with discriminant $d = \sqrt{b^2 - 4ac}$
- and solutions $t_{\pm} = \dfrac{-b \pm d}{2a}$

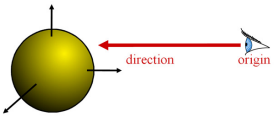
Ray-Sphere Intersection

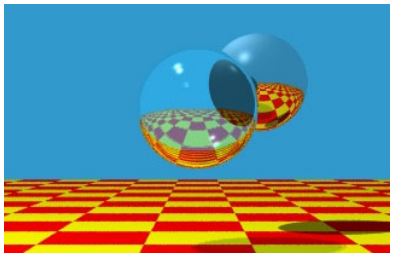
- 3 cases, depending on the sign of $b^2 - 4ac$
- What do these cases correspond to?
- Which root (t+ or t-) should you choose?
  - Closest positive! (usually t-)



Ray-Sphere Intersection

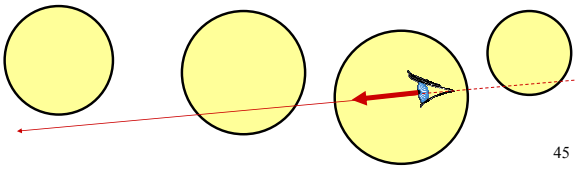
- It's so easy that all ray-tracing images have spheres!





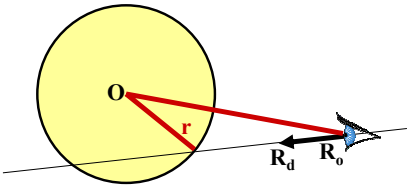
Geometric Ray-Sphere Intersection

- Shortcut / easy reject
- What geometric information is important?
  - Ray origin inside/outside sphere?
  - Closest point to ray from sphere origin?
  - Ray direction: pointing away from sphere?



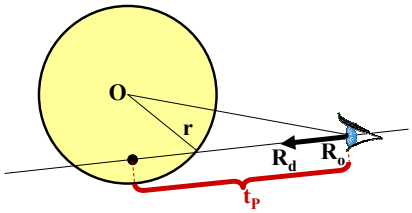
Geometric Ray-Sphere Intersection

- Is ray origin **inside/outside/on** sphere?
  - $(R_o \cdot R_o < r^2$ / $R_o \cdot R_o > r^2$ / $R_o \cdot R_o = r^2)$
  - If origin on sphere, be careful about degeneracies…



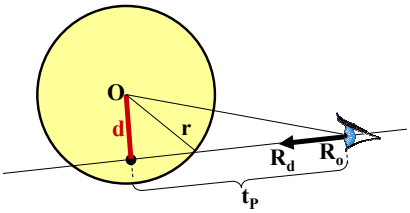
Geometric Ray-Sphere Intersection

- Is ray origin **inside/outside/on** sphere?
- Find closest point to sphere center, $t_P = - R_o \cdot R_d$
  - If origin outside & $t_p < 0$ → **no hit**



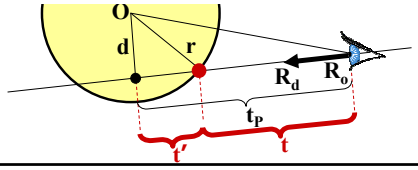
Geometric Ray-Sphere Intersection

- Is ray origin **inside/outside/on** sphere?
- Find closest point to sphere center, $t_P = - R_o \cdot R_d$
- Find squared distance, $d^2 = R_o \cdot R_o - t_P^2$
  - If $d^2 > r^2$ → **no hit**



Geometric Ray-Sphere Intersection

- Is ray origin **inside/outside/on** sphere?
- Find closest point to sphere center, $t_P = - R_o \cdot R_d$
- Find squared distance: $d^2 = R_o \cdot R_o - t_P^2$
- Find distance (t') from closest point (t_P) to correct intersection: $t'^2 = r^2 - d^2$
  - If origin outside sphere $\rightarrow t = t_P - t'$
  - If origin inside sphere $\rightarrow t = t_P + t'$

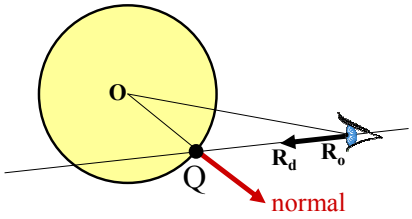


Geometric vs. Algebraic

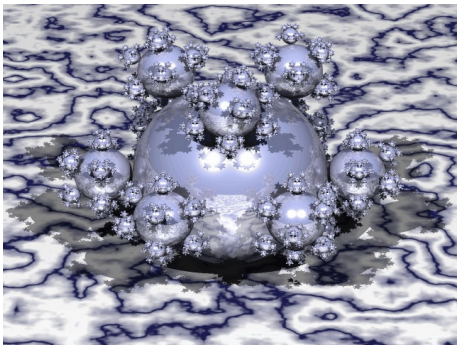
- Algebraic is simple & generic
- Geometric is more efficient
  - Timely tests
  - In particular for rays outside and pointing away

Sphere Normal

- Simply Q/||Q||
  - Q = P(t), intersection point
  - (for spheres centered at origin)

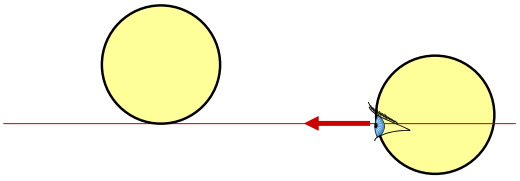


Questions?



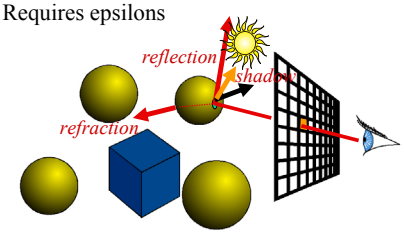
Precision

- What happens when
  - Origin is on an object?
  - Grazing rays?
- Problem with floating-point approximation



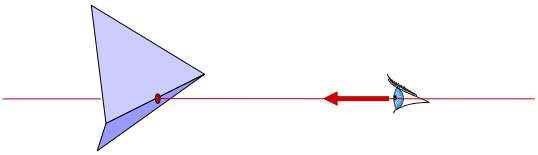
The evil ε

- In ray tracing, do NOT report intersection for rays starting at the surface (no false positive)
  - Because secondary rays
  - Requires epsilons



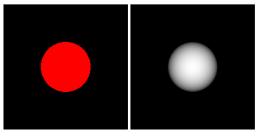
The evil ε: a hint of nightmare

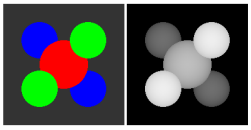
- Edges in triangle meshes
  - Must report intersection (otherwise not watertight)
  - No false negative



Assignment 1: Ray Casting

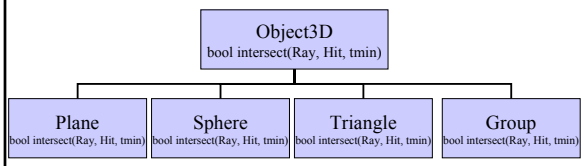
- Write a basic ray caster
  - Orthographic camera
  - Sphere Intersection
  - Main loop rendering
  - 2 Display modes: color and distance
- We provide:
  - Ray: origin, direction
  - Hit: t, Material, (*normal*)
  - Scene Parsing





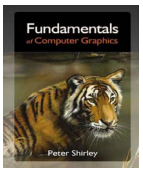
Object-Oriented Design

- We want to be able to add primitives easily
  - Inheritance and virtual methods
- Even the scene is derived from Object3D!

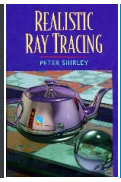


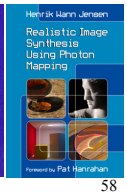
Graphics Textbooks

- Recommended for 6.837:
  Peter Shirley
  Fundamentals of Computer Graphics
  AK Peters



- Ray Tracing





Questions?

Next Time: More Ray Casting

- Other primitives
  - Boxes
  - Polygons
  - Triangles
  - IFS?