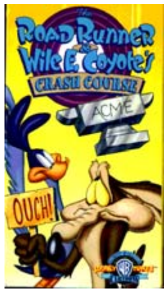
Computer Animation III

Quaternions

Dynamics

Some slides courtesy of Leonard McMillan and Jovan Popovic

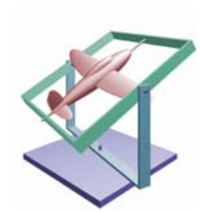


Recap: Euler angles

3 angles along 3 axis

Poor interpolation, lock

But used in flight simulation, etc. because natural

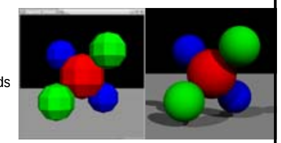


http://www.fho-emden.de/~hoffmann/gimbal09082002.pdf

Assignment 5: OpenGL

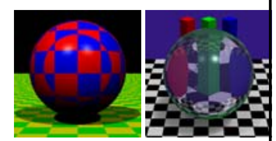
Interactive previsualization

- OpenGL API
- Graphics hardware
- Jusr send rendering commands
- State machine



Solid textures

- New Material subclass
- Owns two Material*
- Chooses between them
- "Shader tree"



Final project

First brainstorming session on Thursday

Groups of three

Proposal due Monday 10/27

- A couple of pages
- Goals
- Progression

Appointment with staff

Final project

Goal-based

- Simulate a visual effect
- Natural phenomena
- Small animation
- Game
- Reconstruct an existing scene

Technique-based

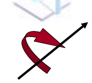
- Monte-Carlo Rendering
- Radiosity
- Fluid dynamics

Overview

Interpolation of rotations, quaternions

- Euler angles

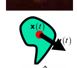
- Quaternions



Dynamics

- Particles

- Rigid body



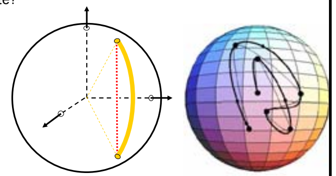
- Deformable objects

Quaternion principle

A quaternion = point on unit 3-sphere in 4D = orientation.
We can apply it to a point, to a vector, to a ray
We can convert it to a matrix

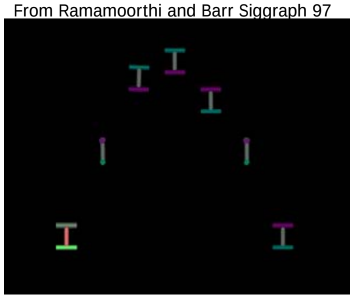
We can interpolate in 4D and project back onto sphere

- How do we interpolate?
- How do we project?



Demo

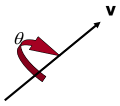
From Ramamoorthi and Barr Siggraph 97



Quaternion recap 1 (wake up)

4D representation of orientation

$\mathbf{q} = \{\cos(\theta/2);\ \mathbf{v}\sin(\theta/2)\}$



Inverse is $\mathbf{q}^{-1} = (s, -\mathbf{v})$

Multiplication rule

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1 s_2 - \left(\vec{v}_1 \cdot \vec{v}_2\right),\ s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

- Consistent with rotation composition

How do we apply rotations?
How do we interpolate?

Quaternion Algebra

Two general quaternions are multiplied by a special rule:

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1 s_2 - \left(\vec{v}_1 \cdot \vec{v}_2\right),\ s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

Sanity check : $\{\cos(\alpha/2);\ \mathbf{v}\sin(\alpha/2)\}\ \{\cos(\beta/2);\ \mathbf{v}\sin(\beta/2)\}$

$\{\cos(\alpha/2)\cos(\beta/2) - \sin(\alpha/2)\mathbf{v}.\ \sin(\beta/2)\}\ \mathbf{v}$,
$\cos(\beta/2)\sin(\alpha/2)\ \mathbf{v} + \cos(\alpha/2)\sin(\beta/2)\ \mathbf{v} + \mathbf{v} \times \mathbf{v}\}$

$\{\cos(\alpha/2)\cos(\beta/2) - \sin(\alpha/2)\sin(\beta/2)$,
$\mathbf{v}(\cos(\beta/2)\sin(\alpha/2) + \cos(\alpha/2)\sin(\beta/2))\}$

$\{\cos((\alpha+\beta)/2),\ \mathbf{v}\sin((\alpha+\beta)/2)\ \}$

Quaternion Algebra

Two general quaternions are multiplied by a special rule:

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1 s_2 - \left(\vec{v}_1 \cdot \vec{v}_2\right),\ s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

To rotate 3D point/vector **p** by **q**, compute

- $\mathbf{q}\ \{0;\ \mathbf{p}\}\ \mathbf{q}^{-1}$

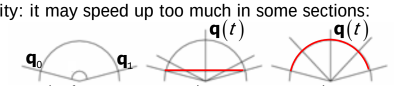
$p = (x,y,z)\ \mathbf{q} = \{\cos(\theta/2), 0,0,\sin(\theta/2)\} = \{c, 0,0,s\}$

$\mathbf{q}\ \{0,\mathbf{p}\} = \{c, 0, 0, s\}\ \{0, x, y, z\}$
$= \{c.0 - zs,\ \ c\mathbf{p} + 0(0,0,s) + (0,0,s) \times \mathbf{p}\}$
$= \{-zs,\ c\,\mathbf{p} + (-sy,sx,0)\}$

$\mathbf{q}\ \{0,\mathbf{p}\}\ \mathbf{q}^{-1} = \{-zs,\ c\,\mathbf{p} + (-sy,sx,0)\}\ \ \{c, 0,0,-s\}$
$= \{-zsc - (c\mathbf{p} + (-sy,sx,0)).(0,0,-s),$
$-zs(0,0,-s) + c(c\mathbf{p} + (-sy, sx,0)) + (c\,\mathbf{p} + (-sy,sx,0))\ x\ (0,0,-s)\}$
$= \{0,\ \ (0,0,zs^2) + c^2\mathbf{p} + (-csy, csx,0) + (-csy, csx, 0) + (s^2x, s^2y, 0)\}$
$= \{0,\ \ (c^2x - 2csy - s^2x,\ c^2y + 2csx - s^2y,\ zs^{2+}sc^2)\}$
$= \{0,\ x\cos(\theta) - y\sin(\theta),\ x\sin(\theta) + y\cos(\theta),\ z\}$

Quaternion Interpolation (velocity)

The only problem with **l**inear int**erp**olation (lerp) of quaternions is that it interpolates the straight line (the secant) between the two quaternions and not their spherical distance. As a result, the interpolated motion does not have smooth velocity: it may speed up too much in some sections:



keyframes lerp slerp

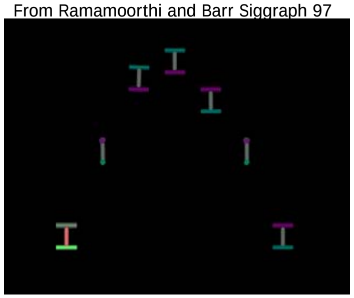
Spherical **l**inear int**erp**olation (slerp) removes this problem by interpolating along the arc lines instead of the secant lines.

$$\operatorname{slerp}\left(\mathbf{q}_0, \mathbf{q}_1, t\right) = \mathbf{q}(t) = \frac{\mathbf{q}_0 \sin\left((1-t)\omega\right) + \mathbf{q}_1 \sin\left(t\omega\right)}{\sin\left(\omega\right)},$$

where $\omega = \cos^{-1}\left(\mathbf{q}_0 \cdot \mathbf{q}_1\right)$

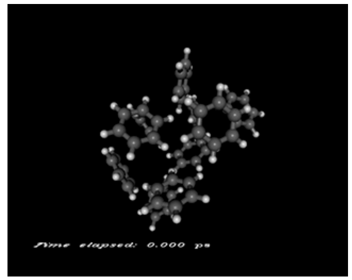
Demo

From Ramamoorthi and Barr Siggraph 97



Demo

From Ramamoorthi and Barr Siggraph 97



Quaternions

Can also be defined like complex numbers

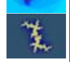
a+bi+cj+dk

Multiplication rules

- $i^2=j^2=k^2=-1$
- $ij=k=-ji$
- $jk=i=-kj$
- $ki=j=-ik$

...

Fun:Julia Sets in Quaternion space



Mandelbrot set: $Z_{n+1}=Z_n^2+Z_0$

Julia set $Z_{n+1}=Z_n^2+C$

http://aleph0.clarku.edu/~djoyce/julia/explorer.html

Do the same with Quaternions!

Rendered by Skal (Pascal Massimino) http://skal.planet-d.net/

See also http://www.chaospro.de/gallery/gallery.php?cat=Anim

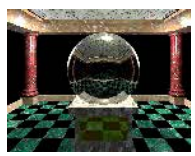
Fun:Julia Sets in Quaternion space

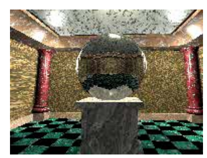
Julia set $Z_{n+1}=Z_n^2+C$

Do the same with Quaternions!

Rendered by Skal (Pascal Massimino) http://skal.planet-d.net/

This is 4D, so we need the time dimension as well

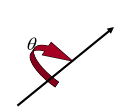




Recap: quaternions

3 angles represented in 4D

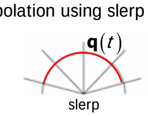
$\mathbf{q} = \{\cos(\theta/2);\ \mathbf{v}\sin(\theta/2)\}$



Weird multiplication rules

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1s_2 - (\vec{v}_1 \cdot \vec{v}_2),\ s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

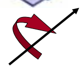
Good interpolation using slerp



Overview

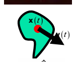
Interpolation of rotations, quaternions

- Euler angles
- Quaternions



Dynamics

- Particles
- Rigid body
- Deformable objects

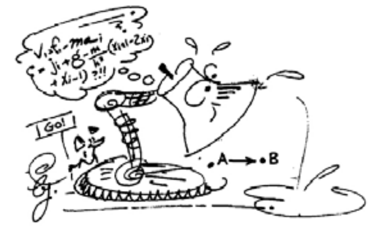


Break: movie time

Pixar *For the Bird*

Now

Dynamics

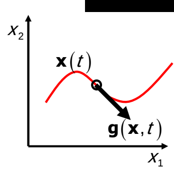


ACM© 1988 "Spacetime Constraints"

Particle

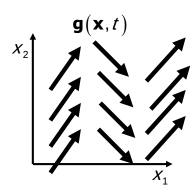
A single particle in 2-D moving in a flow field

- Position $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- Velocity $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \mathbf{v} = \frac{d\mathbf{x}}{dt}$
- The flow field function dictates particle velocity $\mathbf{v} = \mathbf{g}(\mathbf{x}, t)$



Vector Field

The flow field $\mathbf{g}(\mathbf{x}, t)$ is a vector field that defines a vector for any particle position $\mathbf{x}$ at any time t.



How would a particle move in this vector field?

Differential Equations

The equation $\mathbf{v} = \mathbf{g}(\mathbf{x}, t)$ is a first order differential equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{g}(\mathbf{x}, t)$$

Position is computed by integrating the differential equation:

$$\mathbf{x}(t) = \mathbf{x}(t_0) + \int_{t_0}^{t} \mathbf{g}(\mathbf{x}, t)\, dt$$

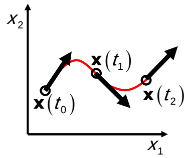
Usually, no analytical solution

Numeric Integration

Instead we use numeric integration:

- Start at initial point $\mathbf{x}(t_0)$
- Step along vector field to compute the position at each time

This is called an **initial value problem**.



Euler's Method

Simplest solution to an initial value problem.

- Starts from initial value
- Take small time steps along the flow:

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\,\mathbf{g}(\mathbf{x}, t)$$

Why does this work?

Consider Taylor series expansion of $\mathbf{x}(t)$:

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t \frac{d\mathbf{x}}{dt} + \frac{\Delta t^2}{2}\frac{d^2\mathbf{x}}{dt^2} + \cdots$$

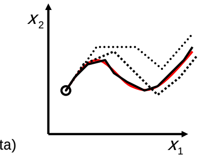
Disregarding higher-order terms and replacing the first derivative with the flow field function yields the equation for the Euler's method.

Other Methods

Euler's method is the simplest numerical method.
The error is proportional to Δt^2.

For most cases, it is inaccurate and unstable

- It requires very small steps.



Other methods:

- Midpoint (2nd order Runge-Kutta)
- Higher order Runge-Kutta (4th order, 6th order)
- Adams
- Adaptive Stepsize

Particle in a Force Field

What is a motion of a particle in a force field?

The particle moves according to Newton's Law:

$$\frac{d^2\mathbf{x}}{dt^2} = \frac{\mathbf{f}}{m} \qquad (\mathbf{f} = ma)$$

The mass m describes the particle's inertial properties:

Heavier particles are easier to move than lighter particles.

In general, the force field $\mathbf{f}(\mathbf{x}, \mathbf{v}, t)$ may depend on the time t and particle's position $\mathbf{x}$ and velocity $\mathbf{v}$.

Second-Order Differential Equations

Newton's Law => ordinary differential equation of 2nd order:

$$\frac{d^2\mathbf{x}(t)}{dt^2} = \frac{\mathbf{f}(\mathbf{x}, \mathbf{v}, t)}{m}$$

A clever trick allows us to reuse the numeric solvers for 1st-order differential equations.

Define new phase vector $\mathbf{y}$:

- Concatenate position $\mathbf{x}$ and velocity $\mathbf{v}$,

Then construct a new 1st-order differential equation whose solution will also solve the 2nd-order differential equation.

$$\mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \mathbf{v} \end{bmatrix}, \quad \frac{d\mathbf{y}}{dt} = \begin{bmatrix} d\mathbf{x}/dt \\ d\mathbf{v}/dt \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{f}/m \end{bmatrix}$$

Particle Animation

```
AnimateParticles(n, y0, t0, tf)
{
    y = y0
    t = t0
    DrawParticles(n, y)
    while(t != tf) {
        f = ComputeForces(y, t)
        dydt = AssembleDerivative(y, f)
        {y, t } = ODESolverStep(6n, y, dy/dt)
        DrawParticles(n, y)
    }
}
```

Particle Animation [Reeves et al. 1983]

Start Trek, The Wrath of Kahn

Particle Modeling [Reeves et al. 1983]

Overview

Interpolation of rotations, quaternions

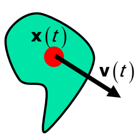
- Euler angles
- Quaternions

Dynamics

- Particles
- Rigid body
- Deformable objects

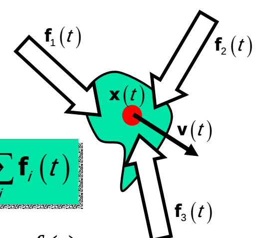
Rigid-Body Dynamics

Could use particles for all points
But rigid body does not deform
Few degrees of freedom
Start with only one particle at center of mass



$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{x}(t) \\ ? \\ \mathbf{v}(t) \\ ? \end{bmatrix}$$

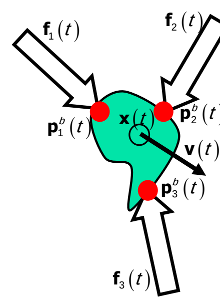
Net Force



$$\mathbf{f}(t) = \sum_i \mathbf{f}_i(t)$$

$$\frac{dMv(t)}{dt} = f(t)$$

Net Torque



$$\boldsymbol{\tau}(t) = \sum_i \left(\mathbf{p}_i^b - \mathbf{x}(t)\right) \times \mathbf{f}_i(t)$$

Rigid-Body Equation of Motion

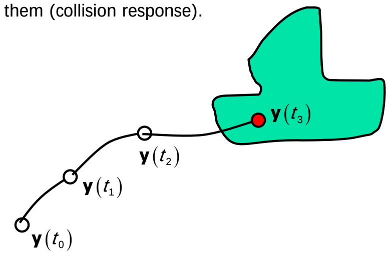
$$\frac{d}{dt}\mathbf{y}(t) = \frac{d}{dt}\begin{bmatrix} \mathbf{x}(t) \\ \mathbf{R}(t) \\ M\mathbf{v}(t) \\ \mathbf{I}(t)\boldsymbol{\omega}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{v}(t) \\ \omega(t) \times \mathbf{R}(t) \\ \mathbf{f}(t) \\ \boldsymbol{\tau}(t) \end{bmatrix}$$

$M\mathbf{v}(t) \rightarrow$ linear momentum

$\mathbf{I}(t)\boldsymbol{\omega}(t) \rightarrow$ angular momentum

Simulations with Collisions

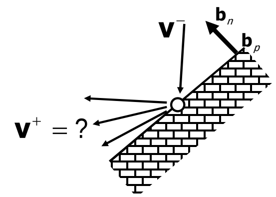
Simulating motions with collisions requires that we detect them (collision detection) and fix them (collision response).



Collision Response

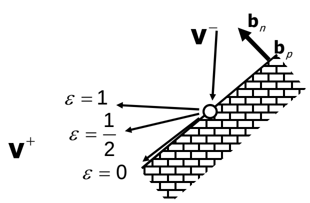
The mechanics of collisions are complicated

Simple model: assume that the two bodies exchange collision *impulse* instantaneously.



Frictionless Collision Model

$$\mathbf{b}_n \cdot \mathbf{v}^+ = -\varepsilon\left(\mathbf{b}_n \cdot \mathbf{v}^-\right)$$



Overview

Interpolation of rotations, quaternions

- Euler angles 
- Quaternions 

Dynamics

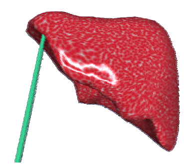
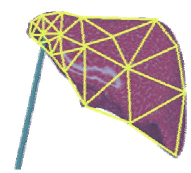
- Particles 
- Rigid body 
- Deformable objects 

Deformable models

Shape deforms due to contact

Discretize the problem

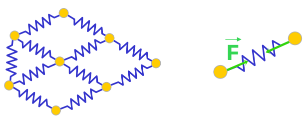
Animation runs with smaller time steps than rendering (between 1/10,000s and 1/100s)

Images from Debunne et al. 2001

Mass-Spring system

Network of masses and springs
Express forces
Integrate
Deformation of springs simulates deformation of objects



Images from Debunne et al. 2001

[Dorsey 1996]

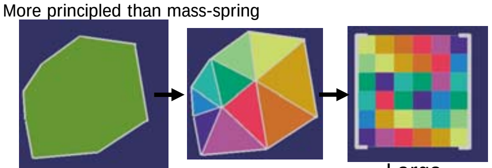
Explicit Finite Elements

Discretize the problem
Solve locally
Simpler but less stable than implicit

Slide from Debunne et al. 2001

Implicit Finite Elements

Discretize the problem
Express the interrelationship
Solve a big system
More principled than mass-spring



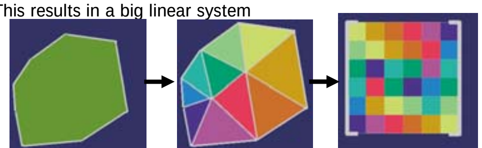
Slide from Debunne et al. 2001

Formally: Finite Elements

We are trying to solve a continuous problem
- Deformation of all points of the object
- Infinite space of functions

We project to a finite set of basis functions
- E.g. piecewise linear, piecewise constant

We project the equations governing the problem
This results in a big linear system



Cloth animation

Discretize cloth
Write physical equations
Integrate
Collision detection

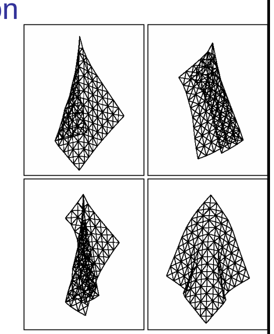


Image from Meyer et al. 2001

Fluid simulation

Discretize volume of fluid
- Exchanges and velocity at voxel boundary

Write Navier Stokes equations
- Incompressible, etc.

Numerical integration
- Finite elements, finite differences

Challenges:
- Robust integration, stability
- Speed
- Realistic surface

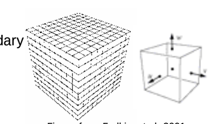


Figure from Fedkiw et al. 2001

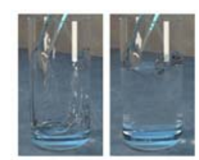
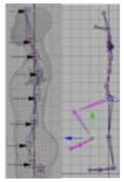


Figure from Enright et al. 2002

How do they animate movies?

- Keyframing mostly
- Articulated figures, inverse kinematics
- Skinning
  - Complex deformable skin
  - Muscle, skin motion
- Hierarchical controls
  - Smile control, eye blinking, etc.
  - Keyframes for these higher-level controls
- A huge time is spent building the 3D models, its skeleton and its controls
- Physical simulation for secondary motion
  - Hair, cloths, water
  - Particle systems for "fuzzy" objects



Images from the Maya tutorial

Final project

- First brainstorming session on Thursday
- Groups of three
- Large programming content
- Proposal due Monday 10/27
  - A couple of pages
  - Goals
  - Progression
- Appointment with staff

Final project

- Goal-based
  - Render some class of object (leaves, flowers, CDs)
  - Natural phenomena (plants, terrains, water)
  - Weathering
  - Small animation of articulated body, explosion, etc.
  - Visualization (explanatory, scientific)
  - Game
  - Reconstruct an existing scene
- Technique-based
  - Monte-Carlo Rendering
  - Radiosity
  - Finite elements/differences (fluid, cloth, deformable objects)
  - Display acceleration
  - Model simplification
  - Geometry processing

Based on your ray tracer

- Global illumination
  - Distribution ray tracing (depth of field, motion blur, soft shadows)
  - Monte-Carlo rendering
  - Caustics
- Appearance modeling
  - General BRDFS
  - Subsurface scattering