Computer Animation II

Orientation interpolation

Dynamics

Some slides courtesy of Leonard McMillan and Jovan Popovic

Review from Thursday

Interpolation
- Splines

Articulated bodies
- Forward kinematics
- Inverse Kinematics
- Optimization
  - Gradient descent
    - Following the steepest slope
  - Lagrangian multipliers
    - Turn optimization with constraints into no constraints

Debugging

Debug all sub-parts as you write them

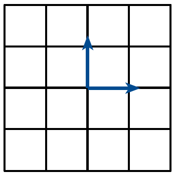
Print as much information as possible

Use simple test cases

When stuck, use step-by-step debugging

Grid acceleration

Debug all these steps:
- Sphere rasterization
- Ray initialization
- Marching
- Object insertion
- Acceleration



Final project

First brainstorming session on Thursday

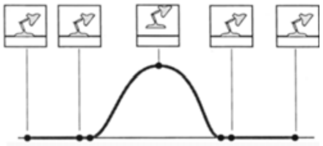
Groups of three

Proposal due Monday 10/27
- A couple of pages
- Goals
- Progression

Appointment with staff

Computer-Assisted Animation

Keyframing
- automate the inbetweening
- good control
- less tedious
- creating a good animation still requires considerable skill and talent



ACM © 1987 “Principles of traditional animation applied to 3D computer animation”

Procedural animation
- describes the motion algorithmically
- express animation as a function of small number of parameteres
- Example: a clock with second, minute and hour hands
  - hands should rotate together
  - express the clock motions in terms of a “seconds” variable
  - the clock is animated by varying the seconds parameter
- Example 2: A bouncing ball
  - $\mathrm{Abs}(\sin(\omega t+\theta_0))*e^{-kt}$

Computer-Assisted Animation

Physically Based Animation

- Assign physical properties to objects (masses, forces, inertial properties)
- Simulate physics by solving equations
- Realistic but difficult to control

ACM© 1988 "Spacetime Constraints"

Motion Capture

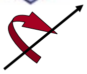
- Captures style, subtle nuances and realism
- You must observe someone do something

Overview

Interpolation of rotations, quaternions

- Euler angles

- Quaternions



Dynamics

- Particles

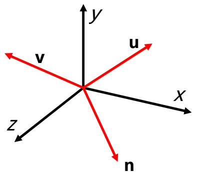
- Rigid body

- Deformable objects

Interpolating Orientations in 3-D

Rotation matrices

Given rotation matrices M_i and time t_i, find M(t) such that $M(t_i)=M_i$.



$$M = \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ n_x & n_y & n_z \end{pmatrix}$$

Flawed Solution

Interpolate each entry independently

Example: M_0 is identity and M_1 is 90^o around x-axis

$$\text{Interpolate}\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

Is the result a rotation matrix?

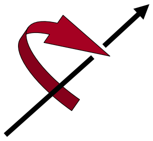
NO:
For example, RR^T does not equal identity.
This interpolation does not preserve rigidity
(angles and lengths)

3D rotations

How many degrees of freedom for 3D orientations?

3 degrees of freedom:

e.g. direction of rotation and angle

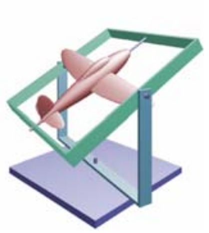


Or 3 Euler angles

Euler Angles

An Euler angle is a rotation about a single axis. Any orientation can be described composing three rotation around each coordinate axis.

Roll, pitch and yaw



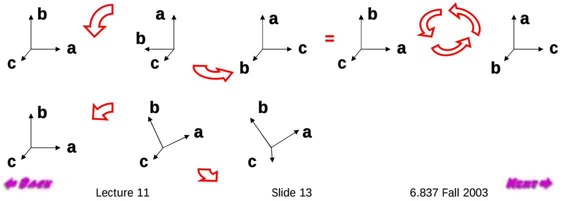
http://www.fho-emden.de/~hoffmann/gimbal09082002.pdf

Interpolating Euler Angles

Natural orientation representation:
3 angles for 3 degrees of freedom

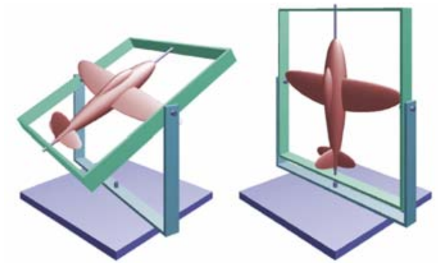
Unnatural interpolation:
A rotation of 90º first around Z and then around Y
= 120º around (1, 1, 1).

But 30º around Z then Y
differs from 40º around (1, 1, 1).



Gimbal lock

Gimbal lock: two or more axis align resulting in a loss of rotation degrees of freedom.



http://www.fho-emden.de/~hoffmann/gimbal09082002.pdf

Demo

By Sobeit void

from
http://www.gamedev.net/reference/programming/features/qpowers/page7.asp

See also
http://www.cgl.uwaterloo.ca/GALLERY/image_html/gimbal.jpg.html

Euler angles in the real world

Apollo inertial measurement unit

To prevent lock, they had to add a fourth Gimbal!

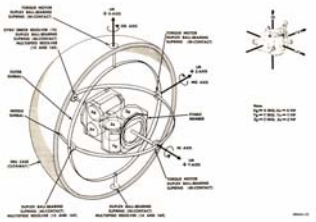
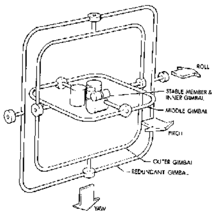


Figure 2.1-24. IMU Gimbal Assembly



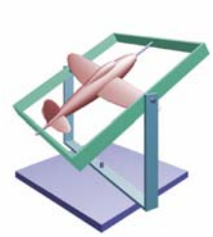
http://www.hq.nasa.gov/office/pao/History/alsj/gimbals.html

Recap: Euler angles

3 angles along 3 axis

Poor interpolation, lock

But used in flight simulation, etc. because natural



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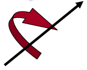
Overview

Interpolation of rotations, quaternions

- Euler angles

- Quaternions



Dynamics

- Particles

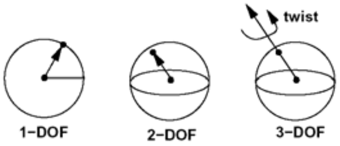
- Rigid body

- Deformable objects

Solution: Quaternion Interpolation

Interpolate orientation on the unit sphere

By analogy: 1-, 2-, 3-DOF rotations as constrained points on 1, 2, 3-spheres



1D sphere and complex plane

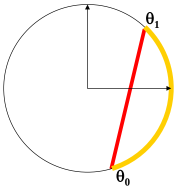
Interpolate orientation in 2D

1 angle

- But messy because modulo 2π

Use interpolation in (complex) 2D plane

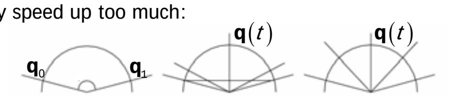
Orientation = complex argument of the number



Velocity issue & slerp

linear int**erp**olation (lerp) in the plane interpolates the straight line between the two orientations and not their spherical distance. $\text{lerp}(\mathbf{q}_0, \mathbf{q}_1, t) = \mathbf{q}(t) = \mathbf{q}_0(1-t) + \mathbf{q}_1 t$

=> The interpolated motion does not have uniform velocity: it may speed up too much:



keyframes lerp slerp

Solution: Spherical **l**inear int**erp**olation (slerp): interpolate along the arc lines by adding a sine term.

$$\text{slerp}(\mathbf{q}_0, \mathbf{q}_1, t) = \mathbf{q}(t) = \frac{\mathbf{q}_0 \sin((1-t)\omega) + \mathbf{q}_1 \sin(t\omega)}{\sin(\omega)},$$

where $\omega = \cos^{-1}(\mathbf{q}_0 \cdot \mathbf{q}_1)$

2-angle orientation

Embed 2-sphere in 3D

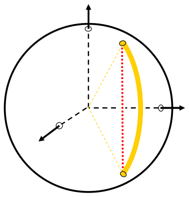
2 angles

- Messy because modulo 2π and pole

Use linear interpolation in 3D space

Orientation = projection onto the sphere

Same velocity correction



3 angles

Use the same principle

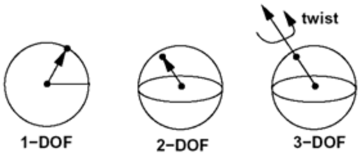
- interpolate in higher-dimensional space
- Project back to unit sphere

Probably need the 3-sphere embedded in 4D

More complex, harder to visualize

Use the so-called Quaternions

- Due to Hamilton (1843); also Shoemake, Siggraph '85



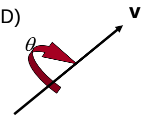
Quaternions

Quaternions are unit vectors on 3-sphere (in 4D)

Right-hand rotation of θ radians about **v** is

q = $\{\cos(\theta/2);\ \mathbf{v}\sin(\theta/2)\}$

- Often noted (s, **v**)
- What if we use –**v** ?



What is the quaternion of Identity rotation?

-

Is there exactly one quaternion per rotation?

-

What is the inverse $\mathbf{q}^{-1}$ of quaternion **q** {a, b, c, d}?

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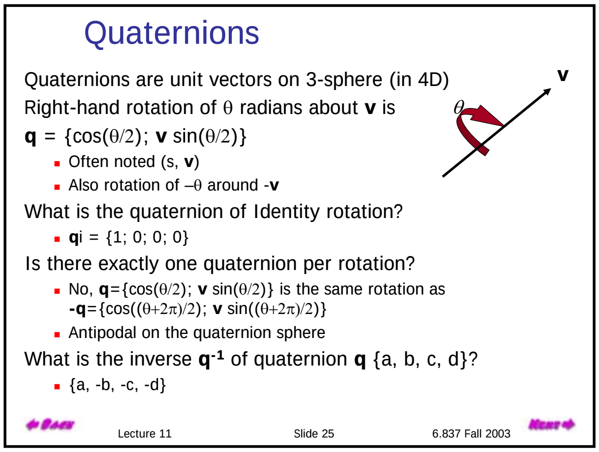

Quaternions

Quaternions are unit vectors on 3-sphere (in 4D)

Right-hand rotation of θ radians about **v** is

q = {cos(θ/2); **v** sin(θ/2)}

- Often noted (s, **v**)
- Also rotation of –θ around -**v**

What is the quaternion of Identity rotation?

- **q**i = {1; 0; 0; 0}

Is there exactly one quaternion per rotation?

- No, **q**={cos(θ/2); **v** sin(θ/2)} is the same rotation as -**q**={cos((θ+2π)/2); **v** sin((θ+2π)/2)}
- Antipodal on the quaternion sphere

What is the inverse $\mathbf{q}^{-1}$ of quaternion **q** {a, b, c, d}?

- {a, -b, -c, -d}

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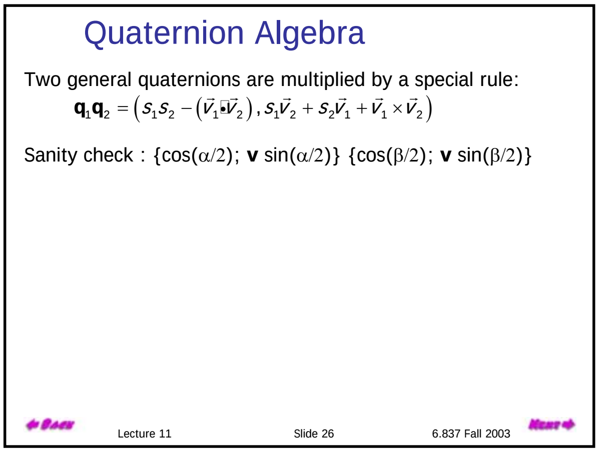

Quaternion Algebra

Two general quaternions are multiplied by a special rule:

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1 s_2 - \left(\vec{v}_1 \bullet \vec{v}_2\right), s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

Sanity check : {cos(α/2); **v** sin(α/2)} {cos(β/2); **v** sin(β/2)}

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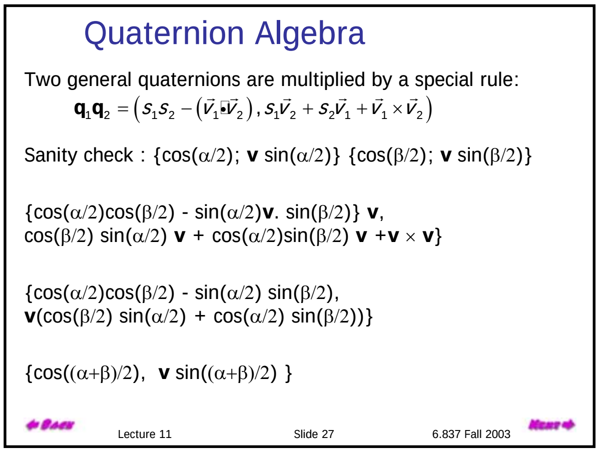

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Sanity check : {cos(α/2); **v** sin(α/2)} {cos(β/2); **v** sin(β/2)}

{cos(α/2)cos(β/2) - sin(α/2)**v**. sin(β/2)} **v**,
cos(β/2) sin(α/2) **v** + cos(α/2)sin(β/2) **v** +**v** × **v**}

{cos(α/2)cos(β/2) - sin(α/2) sin(β/2),
v(cos(β/2) sin(α/2) + cos(α/2) sin(β/2))}

{cos((α+β)/2), **v** sin((α+β)/2) }

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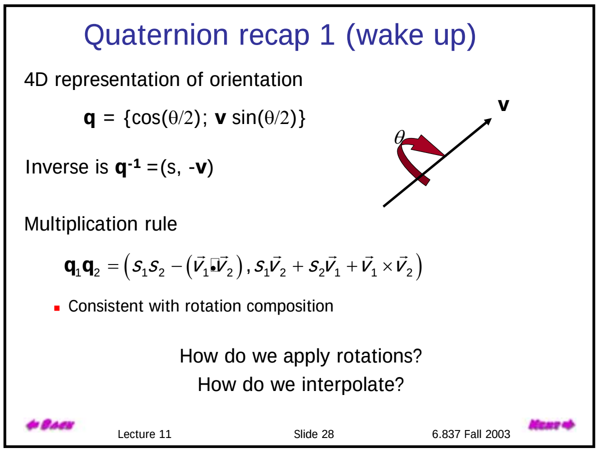

Quaternion recap 1 (wake up)

4D representation of orientation

q = {cos(θ/2); **v** sin(θ/2)}

Inverse is $\mathbf{q}^{-1}$ =(s, -**v**)

Multiplication rule

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1 s_2 - \left(\vec{v}_1 \bullet \vec{v}_2\right), s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

- Consistent with rotation composition

How do we apply rotations?
How do we interpolate?

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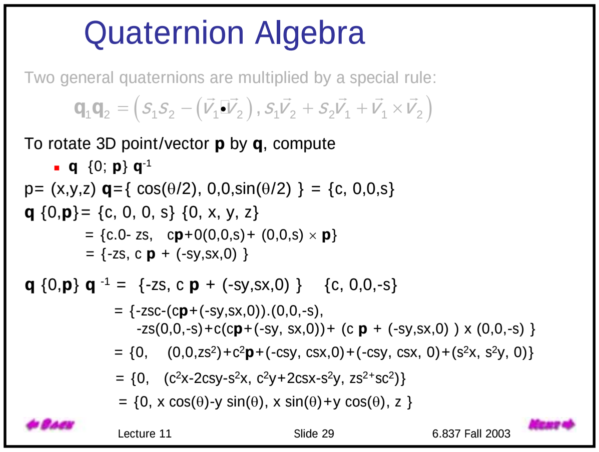

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To rotate 3D point/vector **p** by **q**, compute

- **q** {0; **p**} $\mathbf{q}^{-1}$

p= (x,y,z) **q**={ cos(θ/2), 0,0,sin(θ/2) } = {c, 0,0,s}

q {0,**p**}= {c, 0, 0, s} {0, x, y, z}

= {c.0- zs, c**p**+0(0,0,s)+ (0,0,s) × **p**}
= {-zs, c **p** + (-sy,sx,0) }

q {0,**p**} $\mathbf{q}^{-1}$ = {-zs, c **p** + (-sy,sx,0) } {c, 0,0,-s}

= {-zsc-(c**p**+(-sy,sx,0)).(0,0,-s),
-zs(0,0,-s)+c(c**p**+(-sy, sx,0))+ (c **p** + (-sy,sx,0)) x (0,0,-s) }

= {0, (0,0,zs^2)+c^2**p**+(-csy, csx,0)+(-csy, csx, 0)+(s^2x, s^2y, 0)}

= {0, (c^2x-2csy-s^2x, c^2y+2csx-s^2y, $zs^{2+}sc^2$)}

= {0, x cos(θ)-y sin(θ), x sin(θ)+y cos(θ), z }

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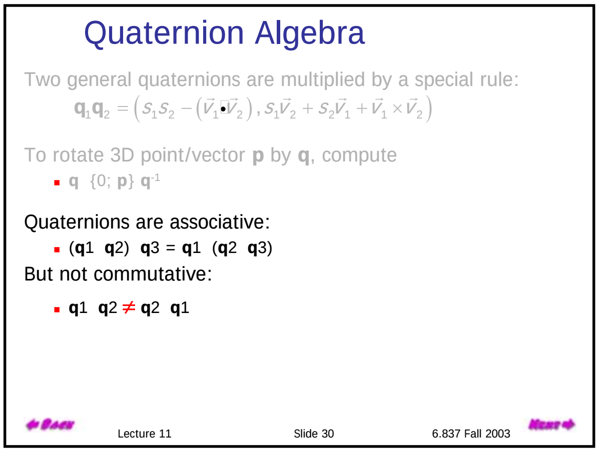

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To rotate 3D point/vector **p** by **q**, compute

- **q** {0; **p**} $\mathbf{q}^{-1}$

Quaternions are associative:

- (**q**1 **q**2) **q**3 = **q**1 (**q**2 **q**3)

But not commutative:

- **q**1 **q**2 ≠ **q**2 **q**1

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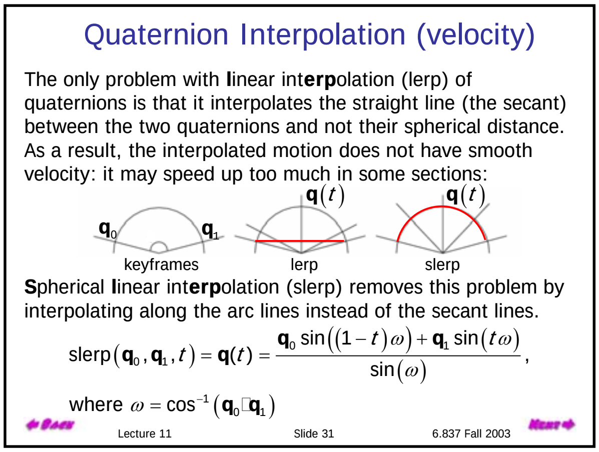


Quaternion Interpolation (velocity)

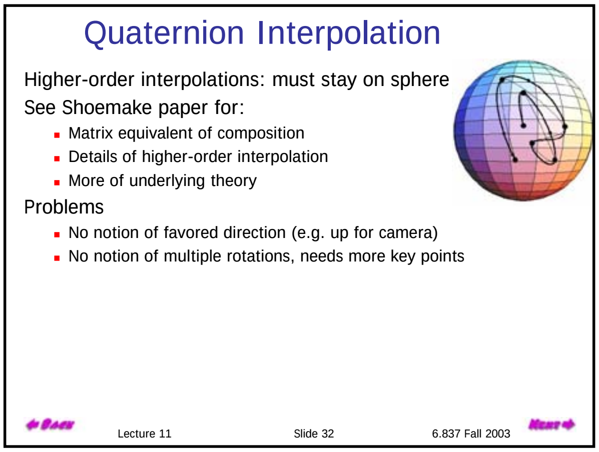
The only problem with **l**inear int**erp**olation (lerp) of quaternions is that it interpolates the straight line (the secant) between the two quaternions and not their spherical distance. As a result, the interpolated motion does not have smooth velocity: it may speed up too much in some sections:

$\mathbf{q}_0$
$\mathbf{q}_1$
$\mathbf{q}(t)$
$\mathbf{q}(t)$
keyframes
lerp
slerp

Spherical **l**inear int**erp**olation (slerp) removes this problem by interpolating along the arc lines instead of the secant lines.

$$\text{slerp}(\mathbf{q}_0, \mathbf{q}_1, t) = \mathbf{q}(t) = \frac{\mathbf{q}_0 \sin((1-t)\omega) + \mathbf{q}_1 \sin(t\omega)}{\sin(\omega)},$$

where $\omega = \cos^{-1}(\mathbf{q}_0 \square \mathbf{q}_1)$



Quaternion Interpolation

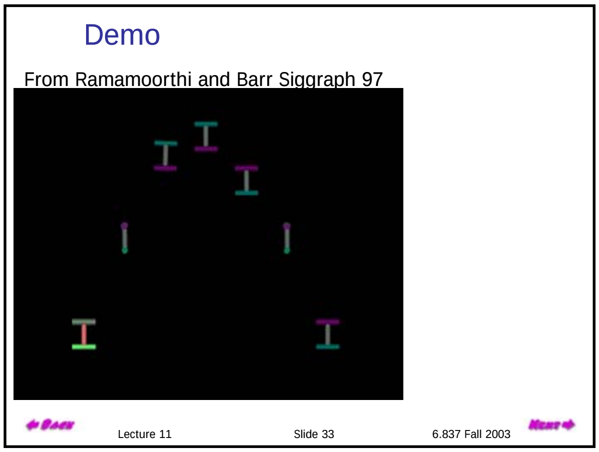
Higher-order interpolations: must stay on sphere

See Shoemake paper for:

- Matrix equivalent of composition
- Details of higher-order interpolation
- More of underlying theory

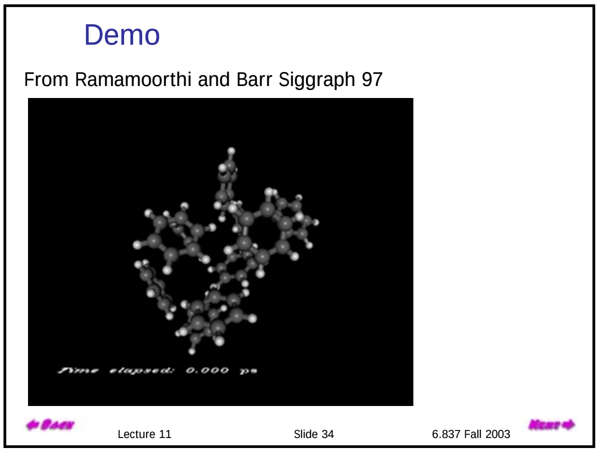
Problems

- No notion of favored direction (e.g. up for camera)
- No notion of multiple rotations, needs more key points



Demo

From Ramamoorthi and Barr Siggraph 97



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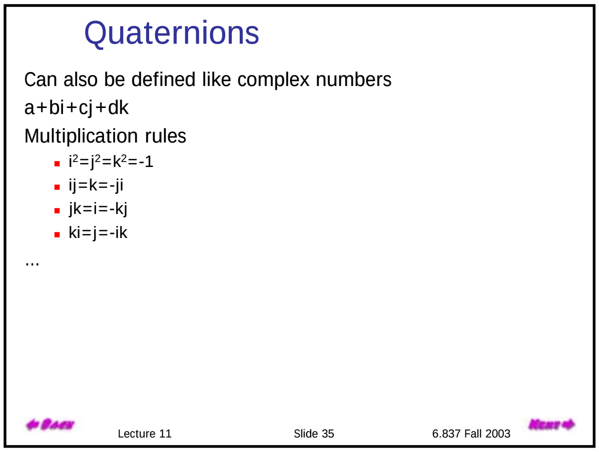


Quaternions

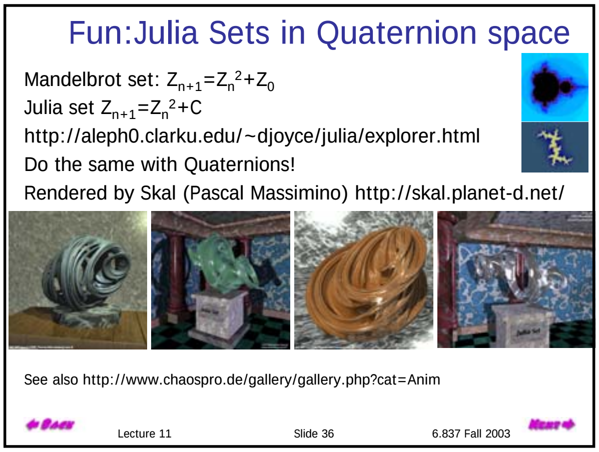
Can also be defined like complex numbers

a+bi+cj+dk

Multiplication rules

- $i^2=j^2=k^2=-1$
- ij=k=-ji
- jk=i=-kj
- ki=j=-ik

…



Fun:Julia Sets in Quaternion space

Mandelbrot set: $Z_{n+1}=Z_n^2+Z_0$

Julia set $Z_{n+1}=Z_n^2+C$

http://aleph0.clarku.edu/~djoyce/julia/explorer.html

Do the same with Quaternions!

Rendered by Skal (Pascal Massimino) http://skal.planet-d.net/

See also http://www.chaospro.de/gallery/gallery.php?cat=Anim

Fun:Julia Sets in Quaternion space

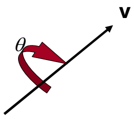
Julia set $Z_{n+1}=Z_n^2+C$

Do the same with Quaternions!

Rendered by Skal (Pascal Massimino) http://skal.planet-d.net/

This is 4D, so we need the time dimension as well

Recap: quaternions



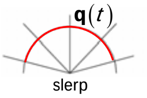
3 angles represented in 4D

$\mathbf{q} = \{\cos(\theta/2);\ \mathbf{v}\sin(\theta/2)\}$

Weird multiplication rules

$$\mathbf{q}_1\mathbf{q}_2 = \left(s_1 s_2 - \left(\vec{v}_1 \cdot \vec{v}_2\right), s_1\vec{v}_2 + s_2\vec{v}_1 + \vec{v}_1 \times \vec{v}_2\right)$$

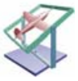
Good interpolation using slerp

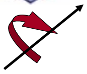


Overview

Interpolation of rotations, quaternions

- Euler angles
- Quaternions





Dynamics

- Particles
- Rigid body
- Deformable objects

Break: movie time

Pixar *For the Bird*