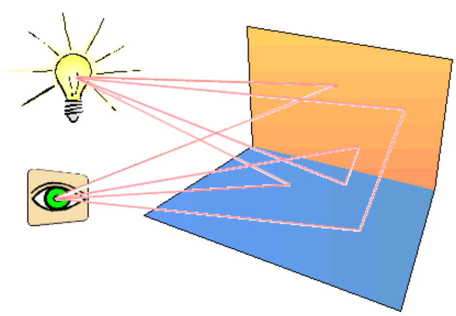
Local Illumination

Outline

- Introduction
- Radiometry
- Reflectance
- Reflectance Models

MIT EECS 6.837, Durand and Cutler – Guest lecture by Matusik

The Big Picture



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Radiometry

- Energy of a photon

$$e_\lambda = \frac{hc}{\lambda} \qquad h \approx 6.63 \cdot 10^{-34} J \cdot s \quad c \approx 3 \cdot 10^{8} m/s$$

- Radiant Energy of *n photons*

$$Q = \sum_{i=1}^{n} \frac{hc}{\lambda_i}$$

- Radiation flux (electromagnetic flux, radiant flux)
  Units: Watts

$$\Phi = \frac{dQ}{dt}$$

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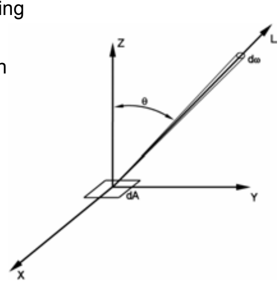
Radiometry

- **Radiance** – radiant flux per unit solid angle per unit projected area
  - Number of photons arriving per time at a small area from a particular direction

$$L(\omega) = \frac{d^2\Phi}{\cos\theta \, dA \, d\omega}$$

$$Units: \frac{Watt}{meter^2 \; steradian}$$



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Radiometry

- **Irradiance** – differential flux falling onto differential area

$$E = \frac{d\Phi}{dA} \qquad Units: \frac{Watt}{meter^2}$$

- Irradiance can be seen as a density of the incident flux falling onto a surface.
- It can be also obtained by integrating the radiance over the solid angle.

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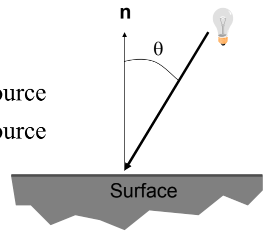
Light Emission

- Light sources: sun, fire, light bulbs etc.
- Consider a point light source that emits light uniformly in all directions

$$E = \frac{\Phi_s \cos\theta}{4\pi d^2} \qquad L = \frac{\Phi_s}{4\pi d^2}$$

Φ_s – power of the light source

d – distance to the light source



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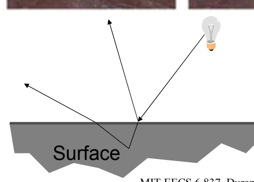
Reflection & Reflectance

- Reflection - the process by which electromagnetic flux incident on a surface leaves the surface without a change in frequency.
- Reflectance – a fraction of the incident flux that is reflected
- We do not consider:
  - absorption, transmission, fluorescence
  - diffraction

Reflectance

- Bidirectional scattering-surface distribution Function (BSSRDF)

Source: Jensen et.al 01

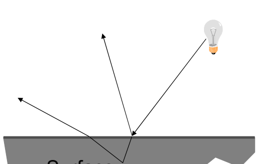


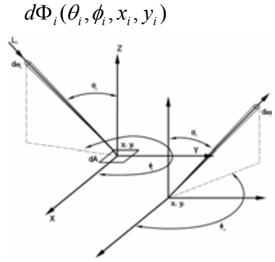
Reflectance

- Bidirectional scattering-surface distribution Function (BSSRDF)

$$S(\theta_i,\phi_i,\theta_r,\phi_r,x_i,y_i,x_r,y_r) = \frac{dL_r(\theta_r,\phi_r,x_r,y_r)}{d\Phi_i(\theta_i,\phi_i,x_i,y_i)}$$

$$Units: \frac{1}{meter^2\ steradian}$$

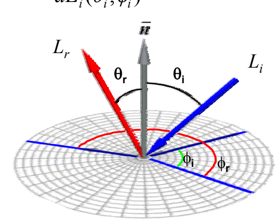




Reflectance

- Bidirectional Reflectance Distribution Function (BRDF)

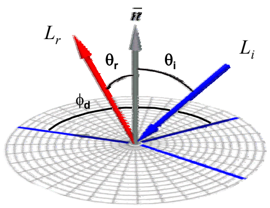
$$f_r(\theta_i,\phi_i,\theta_r,\phi_r) = \frac{dL_r(\theta_r,\phi_r)}{dE_i(\theta_i,\phi_i)} \qquad Units: \frac{1}{steradian}$$



Isotropic BRDFs

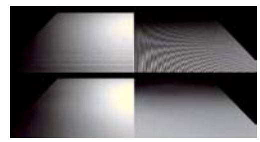
- Rotation along surface normal does not change reflectance

$$f_r(\theta_i,\theta_r,\phi_r-\phi_i)=f_r(\theta_i,\theta_r,\phi_d)=\frac{dL_r(\theta_r,\phi_d)}{dE_i(\theta_i,\phi_d)}$$

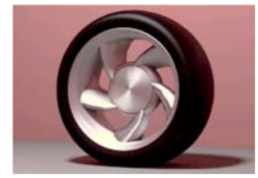


Anisotropic BRDFs

- Surfaces with strongly oriented microgeometry elements
- Examples:
  - brushed metals,
  - hair, fur, cloth, velvet



Source: Westin et.al 92



Properties of BRDFs

- Non-negativity

$$f_r(\theta_i,\phi_i,\theta_r,\phi_r)\ge 0$$

- Energy Conservation

$$\int_\Omega f_r(\theta_i,\phi_i,\theta_r,\phi_r)d\mu(\theta_r,\phi_r)\le 1 \quad \text{for all } (\theta_i,\phi_i)$$

- Reciprocity

$$f_r(\theta_i,\phi_i,\theta_r,\phi_r)=f_r(\theta_r,\phi_r,\theta_i,\phi_i)$$

How to compute reflected radiance?

- Continuous version

$$L_r(\omega_r)=\int_\Omega f_r(\omega_i,\omega_r)dE_i(\omega_i)=$$

$$=\int_\Omega f_r(\omega_i,\omega_r)\,dL_i(\omega_i)\cos(\omega_i\cdot n)\,d\omega_i \qquad \omega=(\theta,\phi)$$

- Discrete version – *n* point light sources

$$L_r(\omega_r)=\sum_{j=1}^{n} f_r(\omega_{ij},\omega_r)E_j=$$

$$=\sum_{j=1}^{n} f_r(\omega_{ij},\omega_r)\cos\theta_j\frac{\Phi_{sj}}{4\pi\, d_j^{\,2}}$$

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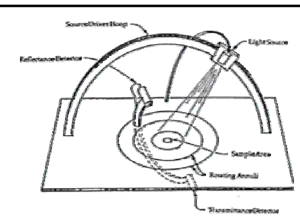
How do we obtain BRDFs?

- Measure BRDF values directly

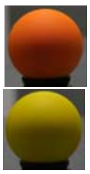


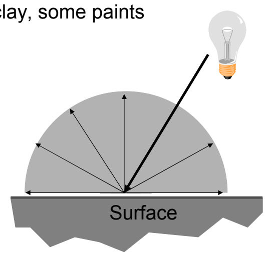
Source: Greg Ward

- Analytic Reflectance Models
  - Physically-based models
    - based on laws on physics
  - Empirical models
    - “ad hoc” formulas that work

Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
  - Example: chalk, clay, some paints





Ideal Diffuse Reflectance

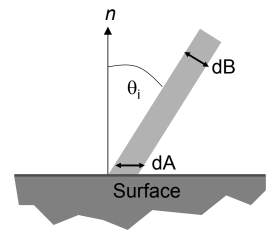
- BRDF value is constant

$$L_r(\omega_r) = \int_\Omega f_r(\omega_i, \omega_r) dE_i(\omega_i) =$$

$$= f_r \int_\Omega dE_i(\omega_i) =$$

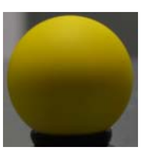
$$= f_r E_i$$

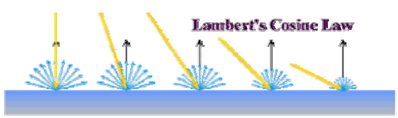
$$dB = dA \cos\theta_i$$



Ideal Diffuse Reflectance

- Ideal diffuse reflectors reflect light according to Lambert's cosine law.

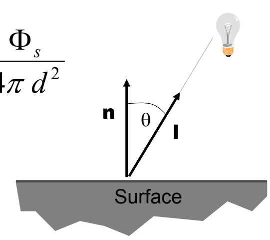




Ideal Diffuse Reflectance

- Single Point Light Source
  - k_d: The diffuse reflection coefficient.
  - **n**: Surface normal.
  - **l**: Light direction.

$$L(\omega_r) = k_d (\mathbf{n} \cdot \mathbf{l}) \frac{\Phi_s}{4\pi d^2}$$

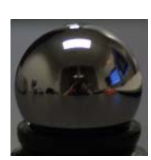


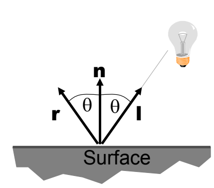
Ideal Diffuse Reflectance – More Details

- If **n** and **l** are facing away from each other, **n** • **l** becomes negative.
- Using max((**n** • **l**),0) makes sure that the result is zero.
  - From now on, we mean max() when we write •.
- Do not forget to normalize your vectors for the dot product!

Ideal Specular Reflectance

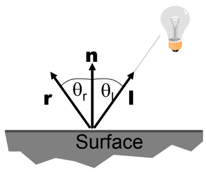
- Reflection is only at mirror angle.
  - View dependent
  - Microscopic surface elements are usually oriented in the same direction as the surface itself.
  - Examples: mirrors, highly polished metals.





Ideal Specular Reflectance

- Special case of Snell's Law
  - The incoming ray, the surface normal, and the reflected ray all lie in a common plane.



$$n_l \sin\theta_l = n_r \sin\theta_r$$

$$n_l = n_r$$

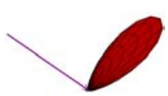
$$\theta_l = \theta_r$$

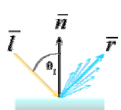
Non-ideal Reflectors

- Snell's law applies only to ideal mirror reflectors.
- Real materials tend to deviate significantly from ideal mirror reflectors.
- They are not ideal diffuse surfaces either …

Non-ideal Reflectors

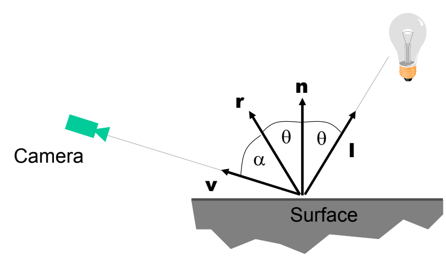
- Simple Empirical Model:
  - We expect most of the reflected light to travel in the direction of the ideal ray.
  - However, because of microscopic surface variations we might expect some of the light to be reflected just slightly offset from the ideal reflected ray.
  - As we move farther and farther, in the angular sense, from the reflected ray we expect to see less light reflected.





The Phong Model

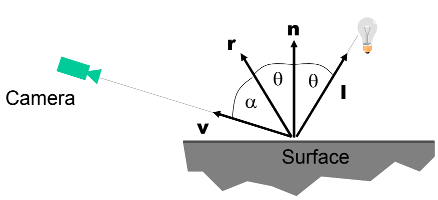
- How much light is reflected?
  - Depends on the angle between the ideal reflection direction and the viewer direction α.



The Phong Model

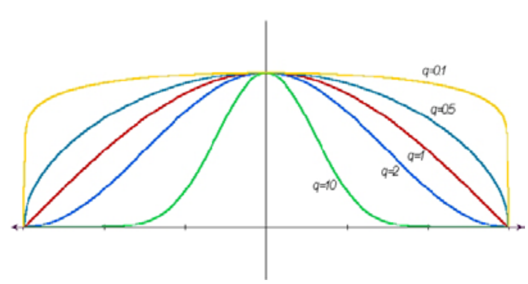
- Parameters
  - k_s: specular reflection coefficient
  - q : specular reflection exponent

$$L(\omega_r) = k_s (\cos\alpha)^q \frac{\Phi_s}{4\pi\, d^2} = k_s (\mathbf{v} \cdot \mathbf{r})^q \frac{\Phi_s}{4\pi\, d^2}$$



The Phong Model

- Effect of the q coefficient



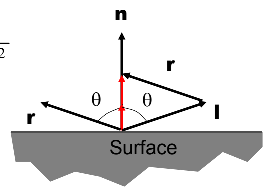
The Phong Model

$$\mathbf{r} + \mathbf{l} = 2\ \cos\theta\,\mathbf{n}$$

$$\mathbf{r} = 2(\mathbf{n}\cdot\mathbf{l})\mathbf{n} - \mathbf{l}$$

$$L(\omega_r) = k_s(\mathbf{v}\cdot\mathbf{r})^q \frac{\Phi_s}{4\pi\, d^2} =$$

$$= k_s(\mathbf{v}\cdot(2(\mathbf{n}\cdot\mathbf{l})\mathbf{n} - \mathbf{l}))^q \frac{\Phi_s}{4\pi\, d^2}$$

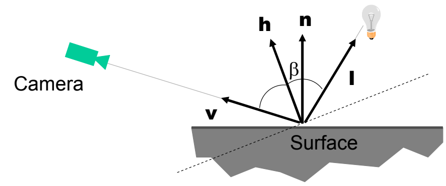


Blinn-Torrance Variation

- Uses the halfway vector **h** between **l** and **v**.

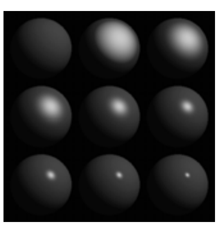
$$\mathbf{h} = \frac{\mathbf{l} + \mathbf{v}}{\|\mathbf{l} + \mathbf{v}\|}$$

$$L(\omega_r) = k_s(\cos\beta)^q \frac{\Phi_s}{4\pi\, d^2} = k_s(\mathbf{n}\cdot\mathbf{h})^q \frac{\Phi_s}{4\pi\, d^2}$$

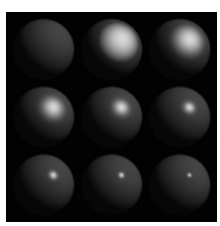


Phong Examples

- The following spheres illustrate specular reflections as the direction of the light source and the coefficient of shininess is varied.



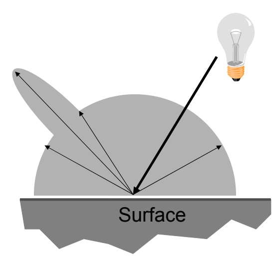
Phong



Blinn-Torrance

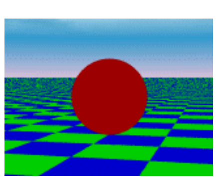
The Phong Model

- Sum of three components:
  diffuse reflection +
  specular reflection +
  "ambient".



Ambient Illumination

- Represents the reflection of all indirect illumination.
- This is a total hack!
- Avoids the complexity of global illumination.

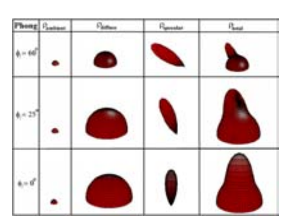


$$L(\omega_r) = k_a$$

Putting it all together

- Phong Illumination Model

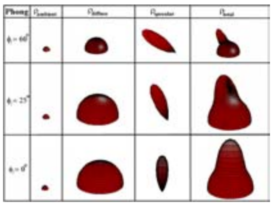
$$L(\omega_r) = k_a + \left(k_d(\mathbf{n}\cdot\mathbf{l}) + k_s(\mathbf{v}\cdot\mathbf{r})^q\right)\frac{\Phi_s}{4\pi\, d^2}$$



For Assignment 3

- Variation on Phong Illumination Model

$$L(\omega_r) = k_d L_a + \left(k_d(\mathbf{n}\cdot\mathbf{l}) + k_s(\mathbf{v}\cdot\mathbf{r})^q\right)L_i$$



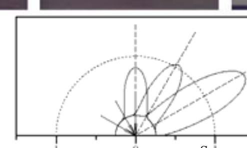
Adding color

- Diffuse coefficients:
  - $k_{d\text{-}red}$, $k_{d\text{-}green}$, $k_{d\text{-}blue}$
- Specular coefficients:
  - $k_{s\text{-}red}$, $k_{s\text{-}green}$, $k_{s\text{-}blue}$
- Specular exponent:
  q

Phong Demo

Fresnel Reflection

- Increasing specularity near grazing angles.



Source: Lafortune et al. 97

Off-specular & Retro-reflection

- Off-specular reflection
  - Peak is not centered at the reflection direction
- Retro-reflection:
  - Reflection in the direction of incident illumination
  - Examples: Moon, road markings

The Phong Model

- Is it non-negative?
- Is it energy-conserving?
- Is it reciprocal?
- Is it isotropic?

Shaders (`Material` class)

- Functions executed when light interacts with a surface
- Constructor:
  - set shader parameters
- Inputs:
  - Incident radiance
  - Incident & reflected light directions
  - surface tangent (anisotropic shaders only)
- Output:
  - Reflected radiance

Questions?

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