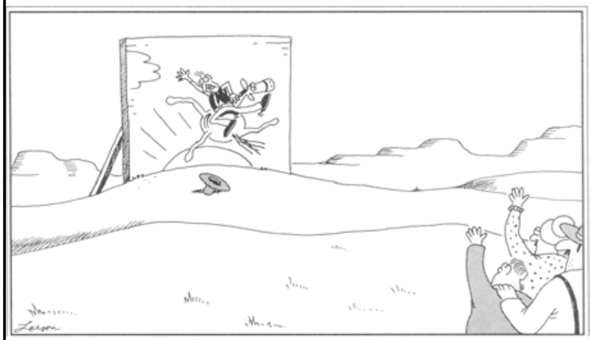
# MIT 6.837 - Ray Tracing



The embarrassment of riding off into a fake sunset



# Ray Tracing

MIT EECS 6.837

Frédo Durand and Barb Cutler

Some slides courtesy of Leonard McMillan

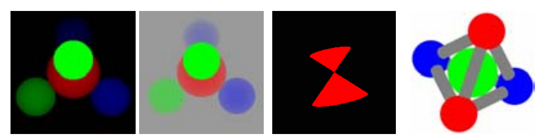


# Administrative

- Assignment 2
  - Due tomorrow at 11:59pm
- Assignment 3
  - Online this evening
  - Due Wednesday October 1



# Cool assignment 1 results



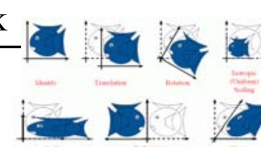
emmav seantek dym koi



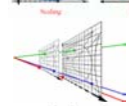
# Review of last week?



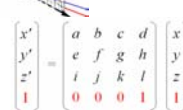
# Review of last week



- Linear, affine and projective transforms



- Homogeneous coordinates
- Matrix notation
- Transformation composition is not commutative
- Orthonormal basis change

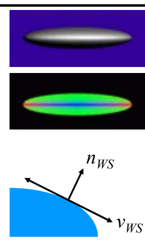


# Review of last week

- Transformation for ray tracing
  - Transforming the ray
    - For the direction, linear part of the transform only
  - Transforming t or not
  - Normal transformation

$$n_{WS}^{\mathbf{T}} = n_{OS}\ (\mathbf{M}^{-1})$$

- Constructive Solid Geometry (CSG)

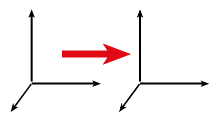





# Fun with transformations: Relativity

Digression

- Special relativity: Lorentz transformation
  - 4 vector (t, x, y, z)
    - 4th coordinate can be ct or t
  - Lorentz transformation depends on object speed v



$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

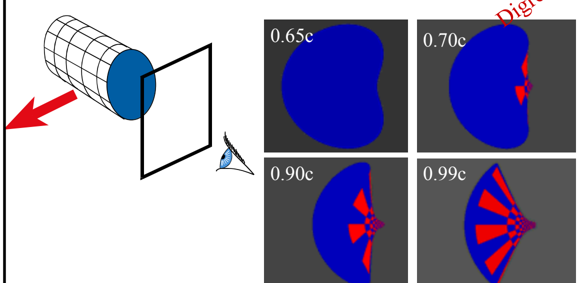
http://casa.colorado.edu/~ajsh/sr/sr.shtml



# Relativity

Digression

- Transform ray by Lorentz transformation




See also http://www.cs.mu.oz.au/~andrbh/raytrace/raytrace.html



# Today: Ray Tracing

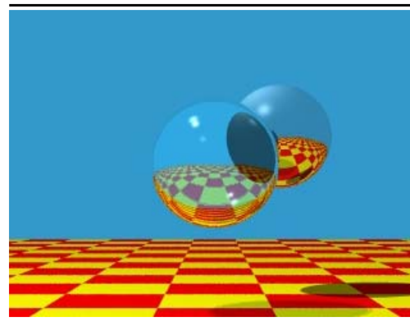
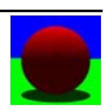


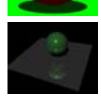
Image by Turner Whitted



# Overview of today

- Shadows
- Reflection
- Refraction
- Recursive Ray Tracing



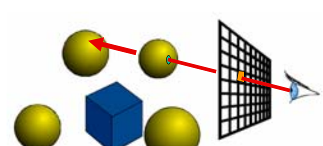




# Ray Casting (a.k.a. Ray Shooting)

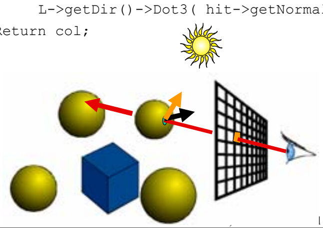
```
For every pixel (x,y)
  Construct a ray from the eye
  color[x,y]=castRay(ray)
```

- Complexity?
  - O(n * m)
  - n: number of objects, m: number of pixels



## Ray Casting with diffuse shading

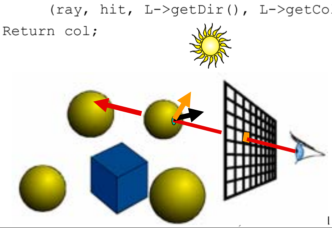
```
Color castRay(ray)
   Hit hit();
   For every object ob
        ob->intersect(ray, hit, tmin);
    Color col=ambient*hit->getColor();
    For every light L
        col=col+hit->getColorL()*L->getColor*
          L->getDir()->Dot3( hit->getNormal() );
    Return col;
```





## Encapsulating shading

```
Color castRay(ray)
   Hit hit();
   For every object ob
        ob->intersect(ray, hit, tmin);
    Color col=ambient*hit->getMaterial()->getDiffuse();
    For every light L
        col=col+hit->getMaterial()->shade
          (ray, hit, L->getDir(), L->getColor());
    Return col;
```





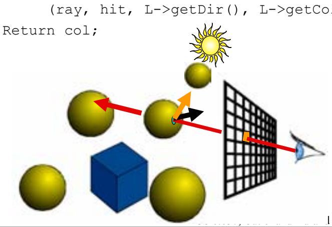
## Questions?

- Image computed using the RADIANCE system by Greg Ward



## How can we add shadows?

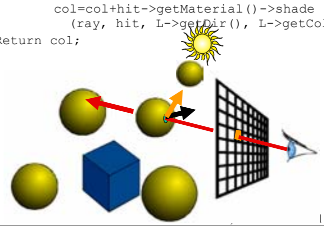
```
Color castRay(ray)
   Hit hit();
   For every object ob
        ob->intersect(ray, hit, tmin);
    Color col=ambient*hit->getMaterial()->getDiffuse();
    For every light L
        col=col+hit->getMaterial()->shade
          (ray, hit, L->getDir(), L->getColor());
    Return col;
```

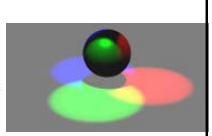




## Shadows

```
Color castRay(ray)
   Hit hit();
   For every object ob
        ob->intersect(ray, hit, tmin);
    Color col=ambient*hit->getMaterial()->getDiffuse();
    For every light L
        Ray ray2(hitPoint, L->getDir()); Hit hit2(L->getDist(),,)
        For every object ob
            ob->intersect(ray2, hit2, 0);
        If (hit->getT> L->getDist())
            col=col+hit->getMaterial()->shade
              (ray, hit, L->getDir(), L->getColor());
    Return col;
```

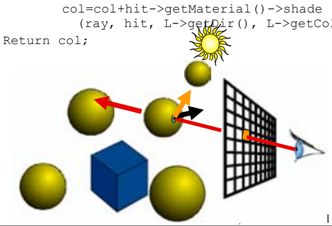


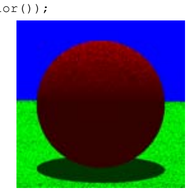




## Shadows – problem?

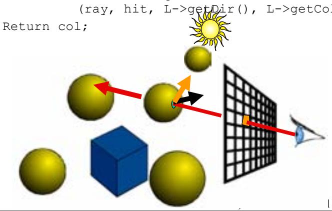
```
Color castRay(ray)
   Hit hit();
   For every object ob
        ob->intersect(ray, hit, tmin);
    Color col=ambient*hit->getMaterial()->getDiffuse();
    For every light L
        Ray ray2(hitPoint, L->getDir()); Hit hit2(L->getDist(),,)
        For every object ob
            ob->intersect(ray2, hit2, 0);
        If (hit->getT> L->getDist())
            col=col+hit->getMaterial()->shade
              (ray, hit, L->getDir(), L->getColor());
    Return col;
```

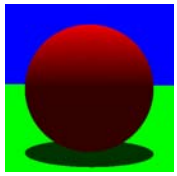




# Avoiding self shadowing

```
Color castRay(ray)
    Hit hit();
    For every object ob
        ob->intersect(ray, hit, tmin);
    Color col=ambient*hit->getMaterial()->getDiffuse();
    For every light L
        Ray ray2(hitPoint, L->getDir()); Hit hit2(L->getDist(),,)
        For every object ob
            ob->intersect(ray2, hit2, epsilon);
        If (hit->getT> L->getDist())
            col=col+hit->getMaterial()->shade
              (ray, hit, L->getDir(), L->getColor());
    Return col;
```

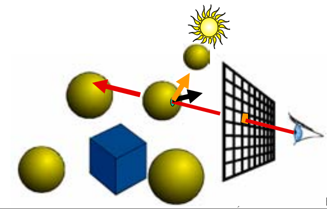






# Shadow optimization

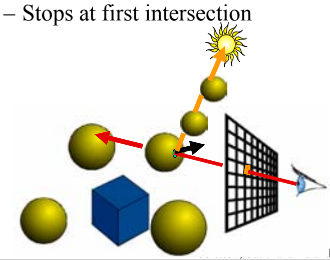
- Shadow rays are special
- How can we accelerate our code?





# Shadow optimization

- We only want to know whether there is an intersection, not which one is closest
- Special routine `Object3D::intersectShadowRay()`
  - Stops at first intersection





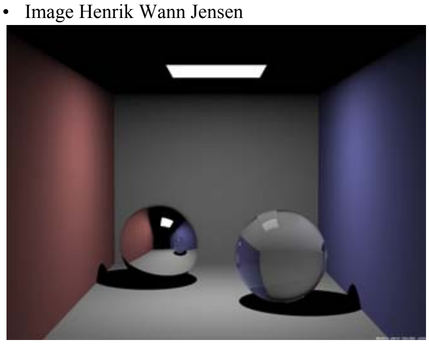
# Shadow ray casting history

- Due to Appel [1968]
- First shadow method in graphics
- Not really used until the 80s



# Questions?

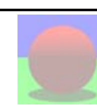
- Image Henrik Wann Jensen

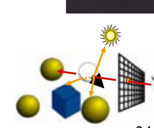




# Overview of today

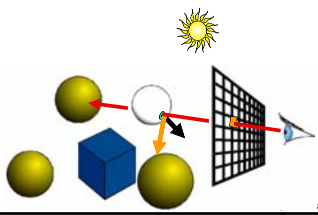
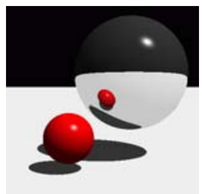
- Shadows
- Reflection
- Refraction
- Recursive Ray Tracing





## Mirror Reflection

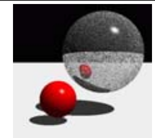
- Compute mirror contribution
- Cast ray
  - In direction symmetric wrt normal
- Multiply by reflection coefficient (color)

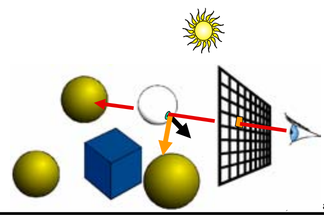
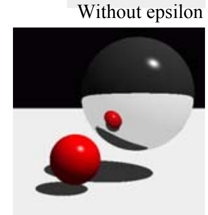


## Mirror Reflection

- Cast ray
  - In direction symmetric wrt normal
- Don't forget to add epsilon to the ray



Without epsilon

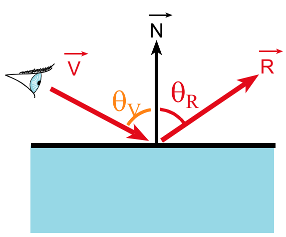



With epsilon



## Reflection

- Reflection angle = view angle

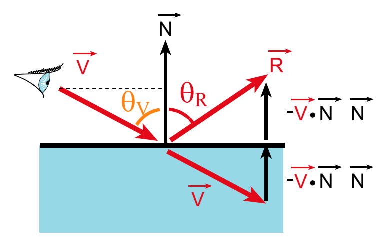





## Reflection

- Reflection angle = view angle

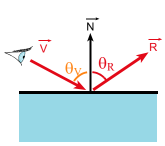
$$\vec{R} = \vec{V} - 2(\vec{V} \bullet \vec{N})\vec{N}$$






## Amount of Reflection

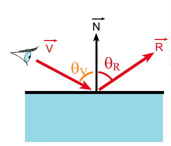
- Traditional (hacky) ray tracing
  - Constant coefficient `reflectionColor`
  - Component per component multiplication

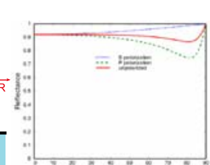




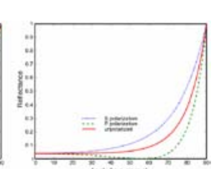

## Amount of Reflection

- More realistic:
  - Fresnel reflection term
  - More reflection at grazing angle
  - Schlick's approximation: $R(\theta)=R_0+(1-R_0)(1-\cos\theta)^5$



metal



Dielectric (glass)

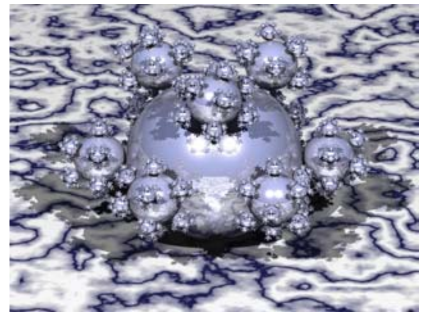
# Fresnel reflectance demo

- Lafortune et al., Siggraph 1997



# Questions?

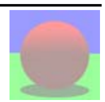
- Image by Henrik Wann Jensen



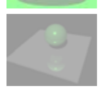


# Overview of today

- Shadows

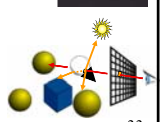


- Reflection



- Refraction

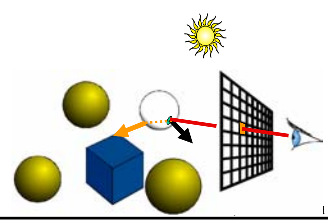
- Recursive Ray Tracing





# Transparency

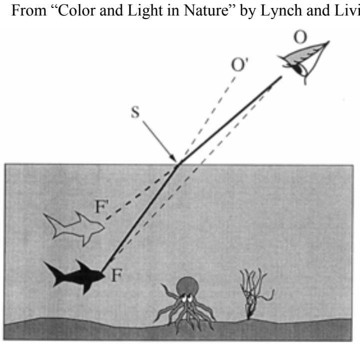
- Compute transmitted contribution
- Cast ray
  - In refracted direction
- Multiply by transparency coefficient (color)





# Qualitative refraction

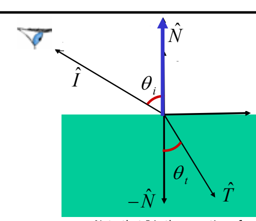
- From “Color and Light in Nature” by Lynch and Livingston






# Refraction

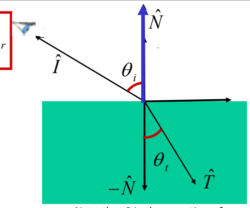
Snell-Descartes Law




Note that I is the negative of the incoming ray

# Refraction

Snell-Descartes Law $\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i} = \eta_r$




Note that I is the negative of the incoming ray

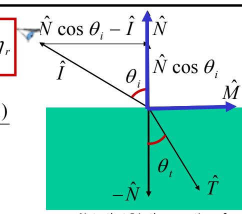


# Refraction

Snell-Descartes Law $\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i} = \eta_r$

$$\hat T = \sin\theta_t \hat M - \cos\theta_t \hat N$$

$$\hat M = \frac{(\hat N \cos\theta_i - \hat I)}{\sin\theta_i}$$




Note that I is the negative of the incoming ray



# Refraction

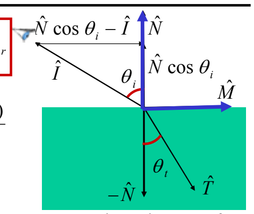
Snell-Descartes Law $\frac{\sin\theta_t}{\sin\theta_i} = \frac{\eta_i}{\eta t} = \eta_r$

$$\hat T = \sin\theta_t \hat M - \cos\theta_t \hat N$$

$$\hat M = \frac{(\hat N \cos\theta_i - \hat I)}{\sin\theta_i}$$

$$\hat T = \frac{\sin\theta_t}{\sin\theta_i}(\hat N \cos\theta_i - \hat I) - \cos\theta_t \hat N$$

$$\hat T = (\eta_r \cos\theta_i - \cos\theta_t)\hat N - \eta_r \hat I$$




Note that I is the negative of the incoming ray



# Refraction

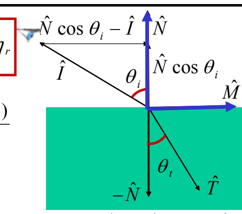
Snell-Descartes Law $\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i} = \eta_r$

$$\hat T = \sin\theta_t \hat M - \cos\theta_t \hat N$$

$$\hat M = \frac{(\hat N \cos\theta_i - \hat I)}{\sin\theta_i}$$

$$\hat T = \frac{\sin\theta_t}{\sin\theta_i}(\hat N \cos\theta_i - \hat I) - \cos\theta_t \hat N$$

$$\hat T = (\eta_r \cos\theta_i - \cos\theta_t)\hat N - \eta_r \hat I$$




Note that I is the negative of the incoming ray

$$\cos\theta_i = \hat N \cdot \hat I$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \eta_r^2 \sin^2\theta_i} = \sqrt{1 - \eta_r^2(1 - (\hat N \cdot \hat I)^2)}$$



# Refraction

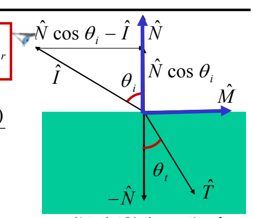
Snell-Descartes Law $\frac{\sin\theta_t}{\sin\theta_i} = \frac{\eta_i}{\eta_t} = \eta_r$

$$\hat T = \sin\theta_t \hat M - \cos\theta_t \hat N$$

$$\hat M = \frac{(\hat N \cos\theta_i - \hat I)}{\sin\theta_i}$$

$$\hat T = \frac{\sin\theta_t}{\sin\theta_i}(\hat N \cos\theta_i - \hat I) - \cos\theta_t \hat N$$

$$\hat T = (\eta_r \cos\theta_i - \cos\theta_t)\hat N - \eta_r \hat I$$




Note that I is the negative of the incoming ray

$$\cos\theta_i = \hat N \cdot \hat I$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \eta_r^2 \sin^2\theta_i} = \sqrt{1 - \eta_r^2(1 - (\hat N \cdot \hat I)^2)}$$

$$\hat T = \left(\eta_r(\hat N \cdot \hat I) - \sqrt{1 - \eta_r^2(1 - (\hat N \cdot \hat I)^2)}\right)\hat N - \eta_r \hat I$$

Don't forget to normalize

Total internal reflection when the square root is imaginary



# Total internal reflection

- From "Color and Light in Nature" by Lynch and Livingstone

Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air. Section 1.9. (Photo by D. Granger)

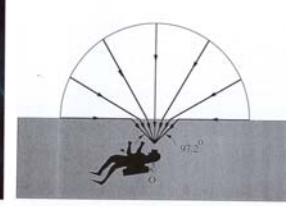



Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

## Cool refraction demo

- Enright, D., Marschner, S. and Fedkiw, R.,



## Cool refraction demo

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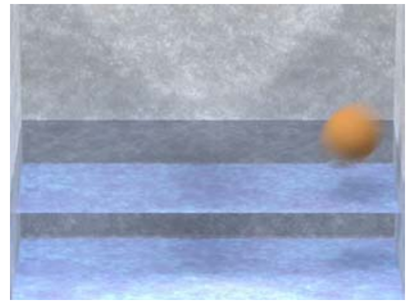
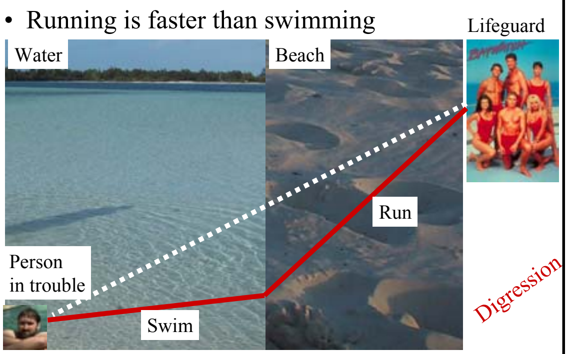




## Refraction and the lifeguard problem

- Running is faster than swimming




Digression



## Wavelength

- Refraction is wavelength-dependent
- Newton's experiment
- Usually ignored in graphics

Pink Floyd, *The Dark Side of the Moon*

Pittoni, 1725, Allegory to Newton



## Rainbow

- From "Color and Light in Nature" by Lynch and Livingstone

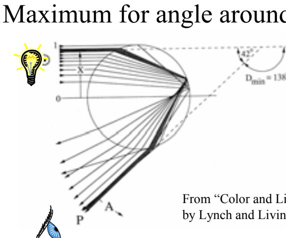
Digression



## Rainbow

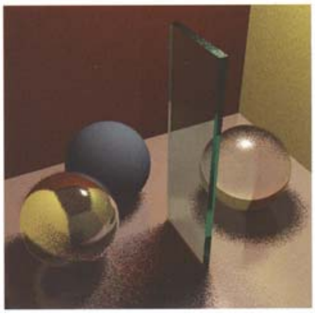
- Refraction depends on wavelength
- Rainbow is caused by refraction+internal reflection+refraction
- Maximum for angle around 42 degrees

Digression




From "Color and Light in Nature" by Lynch and Livingstone

# Questions?

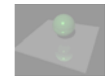




# Overview of today

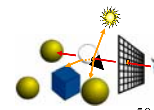
- Shadows

- Reflection



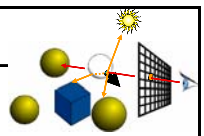
- Refraction

- Recursive Ray Tracing





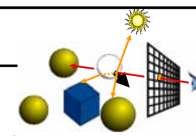
# Recap: Ray Tracing



```
traceRay
   Intersect all objects
   Ambient shading
   For every light
      Shadow ray
      shading
   If mirror
      Trace reflected ray
   If transparent
      Trace transmitted ray
```



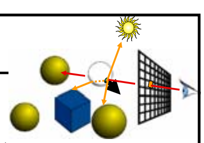
# Recap: Ray Tracing



```
Color traceRay(ray)
   For every object ob
       ob->intersect(ray, hit, tmin);
   Color col=ambient*hit->getMaterial()->getDiffuse();
   For every light L
       If ( not castShadowRay( hit->getPoint(), L->getDir())
           col=col+hit->getMaterial()->shade
             (ray, hit, L->getDir(), L->getColor());
   If (hit->getMaterial()->isMirror())
       Ray rayMirror (hit->getPoint(),
         getMirrorDir(ray->getDirection(), hit->getNormal());
       Col=col+hit->getMaterial->getMirrorColor()
         *traceRay(rayMirror, hit2);
   If (hit->getMaterial()->isTransparent()
       Ray rayTransmitted(hit->getPoint(),
         getRefracDir(ray, hit->getNormal(), curentRefractionIndex,
         hit->Material->getRefractionIndex());
       Col=col+hit->getMaterial->getTransmittedColor()
         *traceRay(rayTransmitted, hit3);
   Return col;
```



# Does it end?



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       Col=col+hit->getMaterial->getTransmittedColor()
         *traceRay(rayTransmitted, hit3);
   Return col;
```



# Avoiding infinite recursion

Stopping criteria:

- Recursion depth
  - Stop after a number of bounces
- Ray contribution
  - Stop if transparency/transmitted attenuation becomes too small

Usually do both


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       ob->intersect(ray, hit, tmin);
   Color col=ambient*hit->getMaterial()->getDiffuse();
   For every light L
       If ( not castShadowRay( hit->getPoint(), L->getDir())
           col=col+hit->getMaterial()->shade
             (ray, hit, L->getDir(), L->getColor());
   If (hit->getMaterial()->isMirror())
       Ray rayMirror (hit->getPoint(),
         getMirrorDir(ray->getDirection(), hit->getNormal());
       Col=col+hit->getMaterial->getMirrorColor()
         *traceRay(rayMirror);
   If (hit->getMaterial()->isTransparent()
       Ray rayTransmitted(hit->getPoint(),
         getRefracDir(ray, hit->getNormal(),
         curentRefractionIndex, hit->Material-
         >getRefractionIndex());
       Col=col+hit->getMaterial->getTransmittedColor()
         *traceRay(rayTransmitted);
   Return col;
```
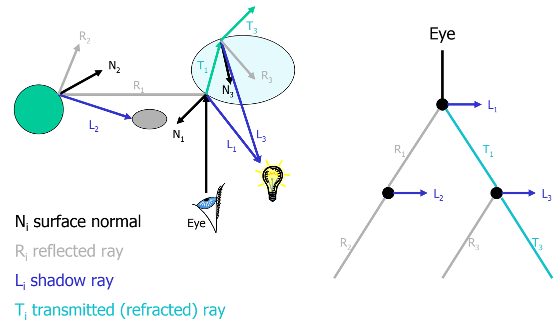

# Recursion for reflection

0 recursion | 1 recursion | 2 recursions



# The Ray Tree

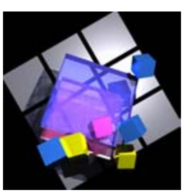





# Kewl visualization

- Ben Garlick's SGI demo flyray
- On an Athena SGI O2:

```
add 6.837
cd /mit/6.837/demos/flyray/data
../flyray
```





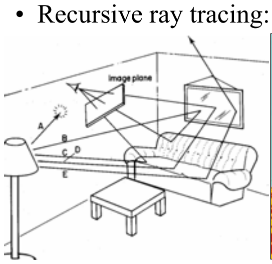
# Real-time ray tracing

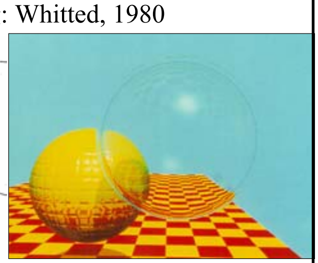
- Steve Parker et al. (U. of Utah)




# Ray Tracing History

- Ray Casting: Appel, 1968
- CSG and quadrics: Goldstein & Nagel 1971
- Recursive ray tracing: Whitted, 1980

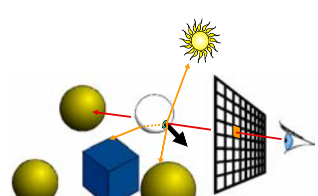




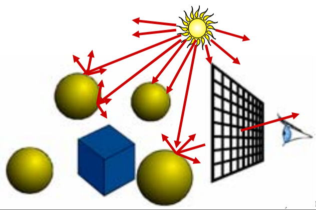
# Does Ray Tracing simulate physics?

- Photons go from the light to the eye, not the other way
- What we do is backward ray tracing



## Forward ray tracing

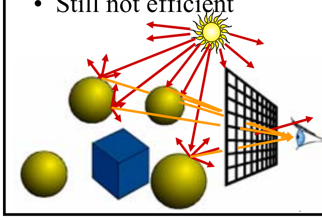
- Start from the light source
- But low probability to reach the eye
  - What can we do about it?





## Forward ray tracing

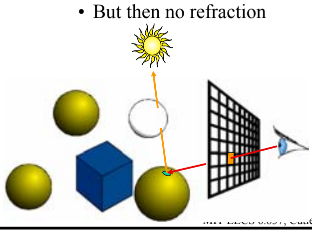
- Start from the light source
- But low probability to reach the eye
  - What can we do about it?
  - Always send a ray to the eye
- Still not efficient





## Does Ray Tracing simulate physics?

- Ray Tracing is full of dirty tricks
- e.g. shadows of transparent objects
  - Dirtiest: opaque
  - Still dirty: multiply by transparency color
    - But then no refraction





## Correct transparent shadow

Digression

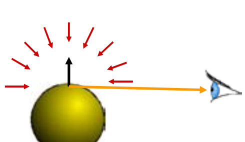
Animation by Henrik Wann Jensen

Using advanced refraction technique
(refraction for illumination is usually not handled that well)




## The Rendering equation

- Clean mathematical framework for light-transport simulation
- We'll see that in November
- At each point, outgoing light in one direction is the integral of incoming light in all directions multiplied by reflectance property





## Thursday

- Reflectance properties, shading and BRDF
- Guest lecture by Wojciech Matusik