Ray Casting II

MIT EECS 6.837
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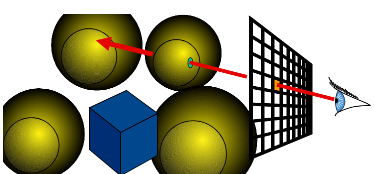
Some slides courtesy of Leonard McMillan

Review of Ray Casting

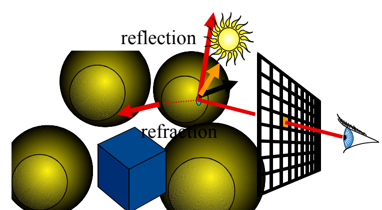
Ray Casting

```
For every pixel
  Construct a ray from the eye
  For every object in the scene
      Find intersection with the ray
      Keep if closest
```



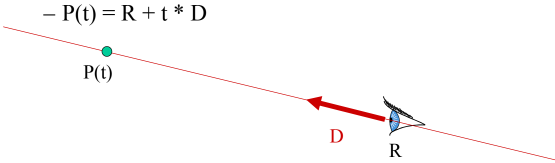
Ray Tracing

- Secondary rays (shadows, reflection, refraction)
- In a couple of weeks



Ray representation

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line
  - P(t) = R + t * D



Explicit vs. implicit

- Implicit
  - Solution of an equation
  - Does not tell us how to generate a point on the plane
  - Tells us how to check that a point is on the plane
- Explicit
  - Parametric
  - How to generate points
  - Harder to verify that a point is on the ray

Durer's Ray casting machine

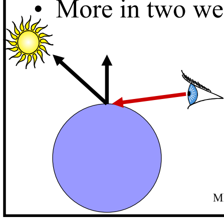
- Albrecht Durer, 16th century


Durer's Ray casting machine

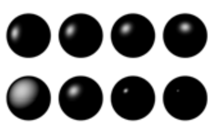
- Albrecht Durer, 16th century

A note on shading

- Normal direction, direction to light
- Diffuse component: dot product
- Specular component for shiny materials
  - Depends on viewpoint
- More in two weeks



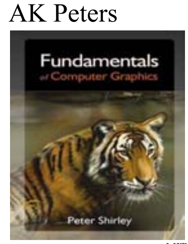

Diffuse sphere



shiny spheres

Textbook

- Recommended, not required
- Peter Shirley
  Fundamentals of Computer Graphics
  AK Peters



References for ray casting/tracing

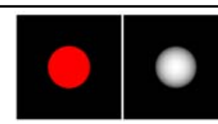
- Shirley Chapter 9
- Specialized books:

- Online resources

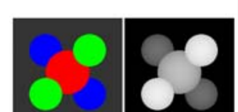
http://www.irtc.org/
http://www.acm.org/tog/resources/RTNews/html/
http://www.povray.org/
http://www.siggraph.org/education/materials/HyperGraph/raytrace/rtrace0.htm
http://www.siggraph.org/education/materials/HyperGraph/raytrace/rt_java/raytrace.html

Assignment 1

- Write a basic ray caster
  - Orthographic camera
  - Spheres
  - Display: constant color and distance

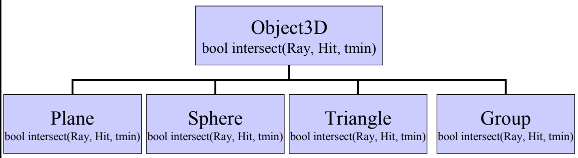


- We provide
  - Ray
  - Hit
  - Parsing
  - And linear algebra, image



Object-oriented design

- We want to be able to add primitives easily
  - Inheritance and virtual methods
- Even the scene is derived from Object3D!



Ray

```
class Ray {

public:

  // CONSTRUCTOR & DESTRUCTOR
  Ray () {}
  Ray (const Vec3f &dir, const Vec3f &orig) {
    direction = dir;
    origin = orig; }
  Ray (const Ray& r) {*this=r;}

  // ACCESSORS
  const Vec3f& getOrigin() const { return origin; }
  const Vec3f& getDirection() const { return direction; }
  Vec3f pointAtParameter(float t) const {
    return origin+direction*t; }

private:

  // REPRESENTATION
  Vec3f direction;
  Vec3f origin;

};
```

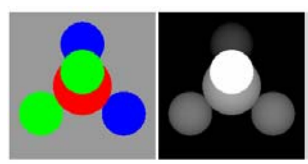

Hit

- Store intersection point & various information

```
class Hit {
public:
    // CONSTRUCTOR & DESTRUCTOR
    Hit(float _t, Vec3f c) { t = _t; color = c; }
    ~Hit() {}
    // ACCESSORS
    float getT() const { return t; }
    Vec3f getColor() const { return color; }
    // MODIFIER
    void set(float _t, Vec3f c) { t = _t; color = c; }
private:
    // REPRESENTATION
    float t;
    Vec3f color;
    //Material *material;
    //Vec3f normal;
};
```


Tasks

- Abstract Object3D
- Sphere and intersection
- Group class
- Abstract camera and derive Orthographic
- Main function

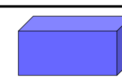


Questions?

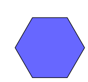
Image by Henrik Wann Jensen

Overview of today

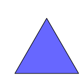
- Ray-box intersection



- Ray-polygon intersection

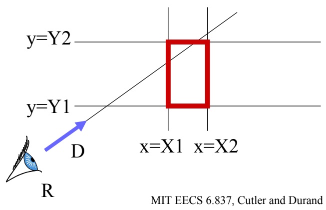


- Ray-triangle intersection



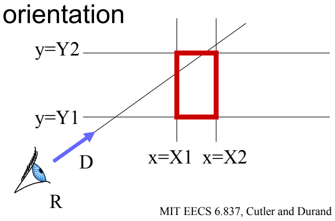
Ray-Parallelepiped Intersection

- Axis-aligned
- From (X1, Y1, Z1) to (X2, Y2, Z2)
- Ray P(t)=R+Dt



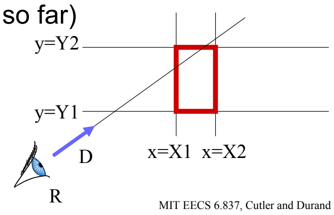
Naïve ray-box Intersection

- Use 6 plane equations
- Compute all 6 intersection
- Check that points are inside box Ax+by+Cz+D<0 with proper normal orientation



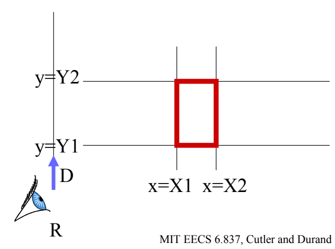
Factoring out computation

- Pairs of planes have the same normal
- Normals have only one non-0 component
- Do computations one dimension at a time
- Maintain tnear and tfar (closest and farthest so far)



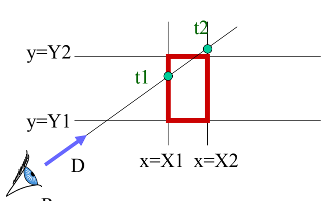
Test if parallel

- If Dx=0, then ray is parallel
  - If Rx<X1 or Rx>x2 return false



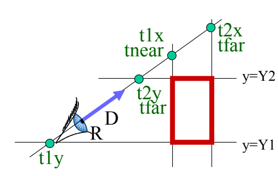
If not parallel

- Calculate intersection distance t1 and t2
  - t1=(X1-Rx)/Dx
  - t2=(X2-Rx)/Dx



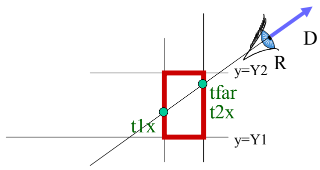
Test 1

- Maintain tnear and tfar
  - If t1>t2, swap
  - if t1>tnear, tnear =t1 if t2<tfar, tfar=t2
- If tnear>tfar, box is missed



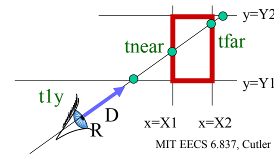
Test 2

- If tfar<0, box is behind



Algorithm recap

- Do for all 3 axis
  - Calculate intersection distance t1 and t2
  - Maintain tnear and tfar
  - If tnear>tfar, box is missed
  - If tfar<0, box is behind
- If box survived tests, report intersection at tnear



Efficiency issues

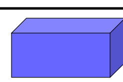
- Do for all 3 axis
  - Calculate intersection distance t1 and t2
  - Maintain tnear and tfar
  - If tnear>tfar, box is missed
  - If tfar<0, box is behind
- If box survived tests, report intersection at tnear
- 1/Dx, 1/Dy and 1/Dz can be precomputed and shared for many boxes
- Unroll the loop
  - Loops are costly (because of termination if)
  - Avoids the tnear tfar for X dimension

Questions?

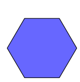
Image by Henrik Wann Jensen

Overview of today

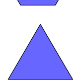
- Ray-box intersection



- Ray-polygon intersection

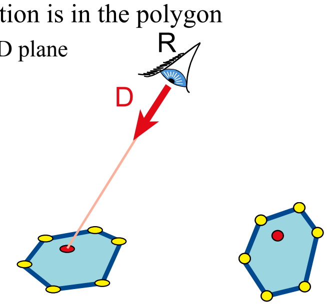


- Ray-triangle intersection



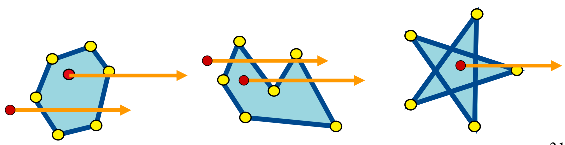
Ray-polygon intersection

- Ray-plane intersection
- Test if intersection is in the polygon
  - Solve in the 2D plane



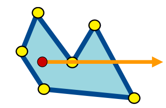
Point inside/outside polygon

- Ray intersection definition:
  - Cast a ray in any direction
    - (axis-aligned is smarter)
  - Count intersection
  - If odd number, point is inside
- Works for concave and star-shaped



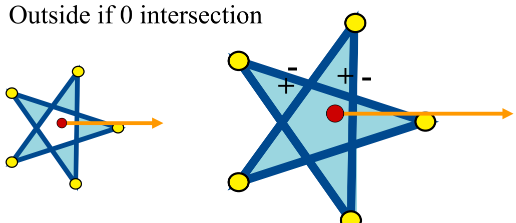
Precision issue

- What if we intersect a vertex?
  - We might wrongly count an intersection for each adjacent edge
- Decide that the vertex is always above the ray



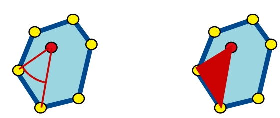
Winding number

- To solve problem with star pentagon
- Oriented edges
- Signed number of intersection
- Outside if 0 intersection



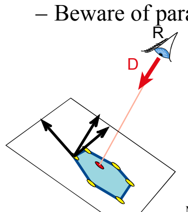
Alternative definitions

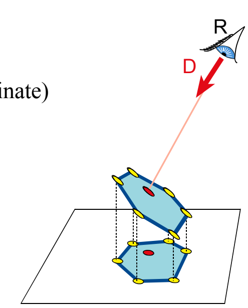
- Sum of the signed angles from point to vertices
  - 360 if inside, 0 if outside
- Sum of the signed areas of point-edge triangles
  - Area of polygon if inside, 0 if outside



How do we project into 2D?

- Along normal
  - Costly
- Along axis
  - Smarter (just drop 1 coordinate)
  - Beware of parallel plane





Questions?

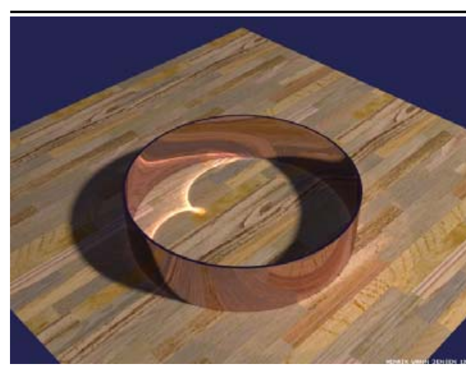
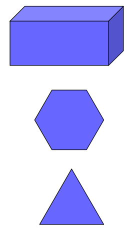


Image by Henrik Wann Jensen

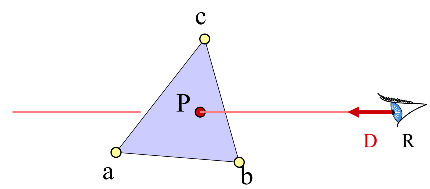
Overview of today

- Ray-box intersection
- Ray-polygon intersection
- Ray-triangle intersection



Ray triangle intersection

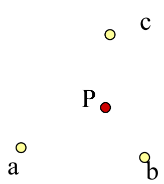
- Use ray-polygon
- Or try to be smarter
  - Use barycentric coordinates



Barycentric definition of a plane

[Möbius, 1827]

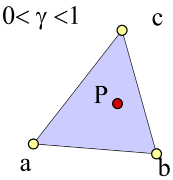
- P(α, β, γ)=αa+βb+γc
  with $\alpha + \beta + \gamma = 1$
- Is it explicit or implicit?



Barycentric definition of a triangle

- P(α, β, γ)=αa+βb+γc
  with $\alpha + \beta + \gamma = 1$

  $0< \alpha <1$
  $0< \beta <1$
  $0< \gamma <1$

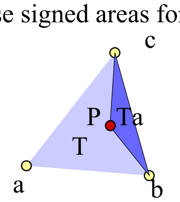


Given P, how can we compute α, β, γ ?

- Compute the areas of the opposite subtriangle
  - Ratio with complete area

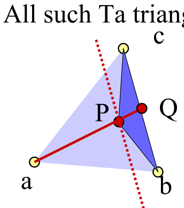
  $\alpha=A_a/A$, $\beta=A_b/A$ $\gamma=A_c/A$

Use signed areas for points outside the triangle



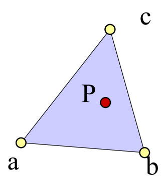
Intuition behind area formula

- P is barycenter of a and Q
- A is the interpolation coefficient on aQ
- All points on line parallel to bc have the same α
- All such Ta triangles have same height/area



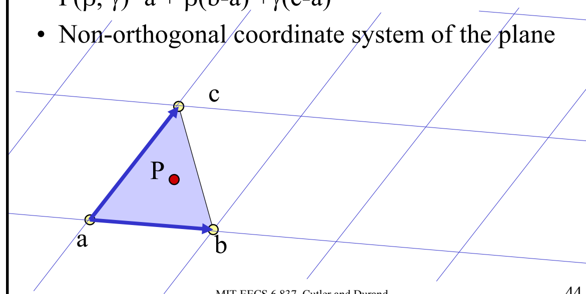
Simplify

- Since $\alpha + \beta + \gamma = 1$
  we can write $\alpha = 1 - \beta - \gamma$
- $P(\beta, \gamma) = (1-\beta-\gamma)\, a + \beta b + \gamma c$



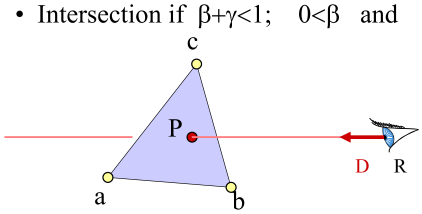
Simplify

- $P(\beta, \gamma) = (1-\beta-\gamma)\, a + \beta b + \gamma c$
- $P(\beta, \gamma) = a + \beta(b-a) + \gamma(c-a)$
- Non-orthogonal coordinate system of the plane



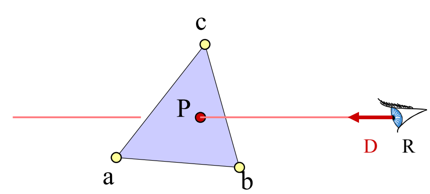
How do we use it for intersection?

- Insert ray equation into barycentric expression of triangle
- $P(t) = a + \beta\,(b-a) + \gamma\,(c-a)$
- Intersection if $\beta+\gamma<1$; $0<\beta$ and $0<\gamma$



Intersection

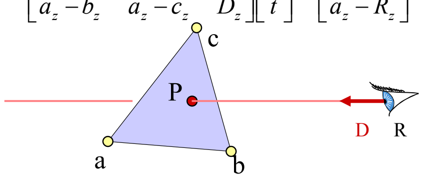
- $R_x + tD_x = a_x + \beta\,(b_x - a_x) + \gamma\,(c_x - a_x)$
- $R_y + tD_y = a_y + \beta\,(b_y - a_y) + \gamma\,(c_y - a_y)$
- $R_z + tD_z = a_z + \beta\,(b_z - a_z) + \gamma\,(c_z - a_z)$



Matrix form

- $R_x + tD_x = a_x + \beta\,(b_x - a_x) + \gamma\,(c_x - a_x)$
- $R_y + tD_y = a_y + \beta\,(b_y - a_y) + \gamma\,(c_y - a_y)$
- $R_z + tD_z = a_z + \beta\,(b_z - a_z) + \gamma\,(c_z - a_z)$

$$\begin{bmatrix} a_x - b_x & a_x - c_x & D_x \\ a_y - b_y & a_y - c_y & D_y \\ a_z - b_z & a_z - c_z & D_z \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} a_x - R_x \\ a_y - R_y \\ a_z - R_z \end{bmatrix}$$

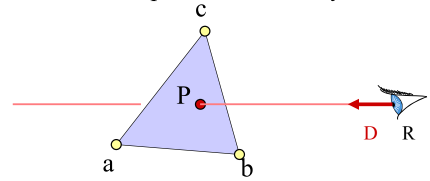


Cramer's rule

- | | denotes the determinant

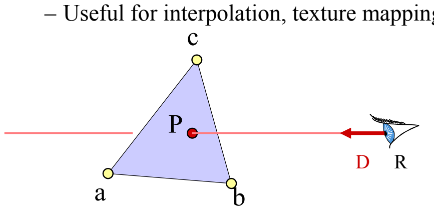
$$\beta = \frac{\begin{vmatrix} a_x - R_x & a_x - c_x & D_x \\ a_y - R_y & a_y - c_y & D_y \\ a_z - R_z & a_z - c_z & D_z \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_x & D_x \\ a_y - b_y & a_y - R_y & D_y \\ a_z - b_z & a_z - R_z & D_z \end{vmatrix}}{|A|} \qquad t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_x \\ a_y - b_y & a_y - c_y & a_y - R_y \\ a_z - b_z & a_z - c_z & a_z - R_z \end{vmatrix}}{|A|}$$

- Can be copied mechanically in the code



Advantage

- Efficient
- Store no plane equation
- Get the barycentric coordinates for free
  - Useful for interpolation, texture mapping

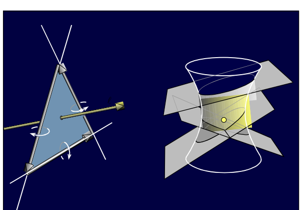


Questions?

- Image computed using the RADIANCE system by Greg Ward

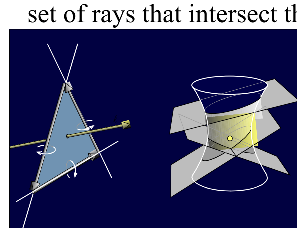
Plucker computation

- Plucker space:
  6 or 5 dimensional space describing 3D lines
- A line is a point in Plucker space



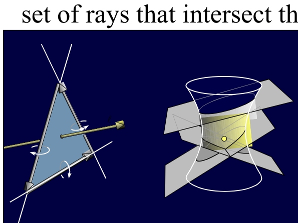
Plucker computation

- The rays intersecting a line are a hyperplane
- A triangle defines 3 hyperplanes
- The polytope defined by the hyperplanes is the set of rays that intersect the triangle



Plucker computation

- The rays intersecting a line are a hyperplane
- A triangle defines 3 hyperplanes
- The polytope defined by the hyperplanes is the set of rays that intersect the triangle



- Ray-triangle intersection becomes a polytope inclusion
- Couple of additional issues

Next week: Transformations

- Permits 3D IFS ;-)