



Clipping

MIT EECS 6.837
Frédo Durand and Seth Teller
Some slides and images courtesy of Leonard McMillan

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Administrative

- Assignment 2
 - Due Friday 27 at 5pm
 - Model a scene using iv files from assignt 1
 - Lighting: at least one spot and one other light

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Assignment 1 Gallery

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Overview of graphics pipeline?

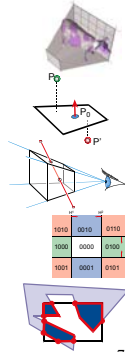
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Questions about previous lectures?

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Today's lecture: Clipping

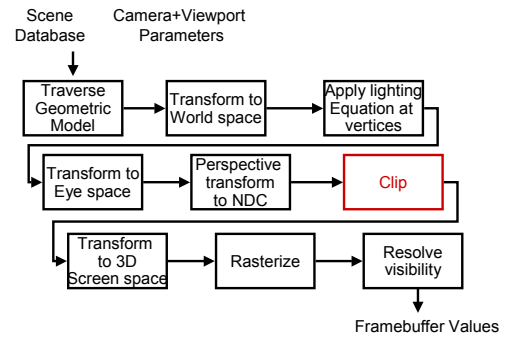
- Overview and motivation
- Plane equation & point clipping
- Line & segment clipping
- Acceleration using outcodes
- Polygon clipping



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Overview of graphics pipeline?

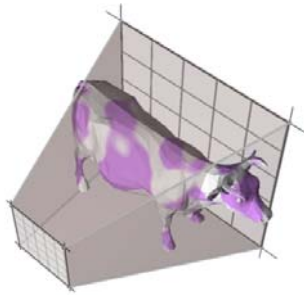


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Clipping

- Eliminate parts of primitives outside the viewing frustum
- Shirley p 210
- *Jim Blinn's Corner, A Trip Down The Graphics Pipeline*, Morgan Kaufman, chapter 13

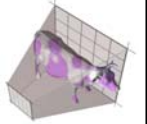


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Why clipping?

- Avoid degeneracy
 - Don't draw outside image
 - Avoid division by 0 and overflow
- Efficiency
 - Don't waste time on object outside the screen
- Other graphics methods (usually more complex cases, non convex)
 - Hidden-surface removal
 - Shadows
 - Picking
 - Binning
 - CSG (Boolean) operations (2D & 3D)

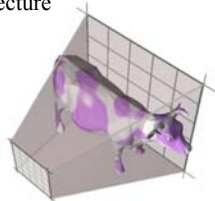


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Clipping strategies

- Don't clip (and hope for the best)
- Clip on –the-fly during rasterization
- Analytical clipping: alter input geometry
 - This is the subject of today's lecture

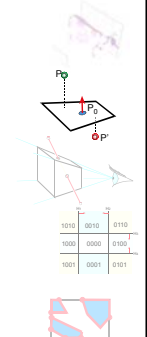


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Plan

- Overview and motivation
- Plane equation & point clipping
- Line & segment clipping
- Acceleration using outcodes
- Polygon clipping

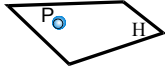


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3D plane equation

- Implicit plane equation
 $H(p) = Ax + By + Cz + D = 0$
- Gradient of H?

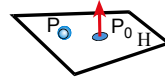


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3D plane equation

- Implicit plane equation
 $H(p) = Ax + By + Cz + D = 0$
- Gradient of H?
- Plane defined by
 - $P_0(x, y, z, 1)$
 - $n(A, B, C, 1)$

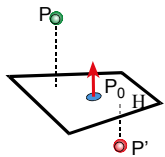


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Plane-point distance

- Plane $H_p=0$
- If n is normalized
 $d=HP$
- Signed distance!

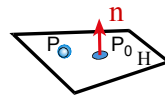


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Homogeneous plane equation

- $H=(A,B,C,D)$ $p=(x,y,z,1)$
- Dot product in homogeneous coordinates
 $H \cdot p = 0$ ($H^T p = 0$)
- Duality between points and hyperplanes...

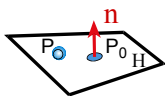


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Homogeneous plane equation

- $H=(A,B,C,D)$ $p=(x,y,z,1)$
- Infinite number of equivalent plane expressions
 $sAx + sBy + sCz + sD = 0$
- Infinite number of homogeneous coordinate
 (sx, sy, sz, sw)

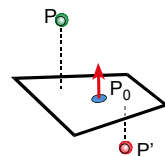


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Clipping wrt 1 plane

- Pass through if $H_p \geq 0$
- Clip (or cull or reject) if $H_p < 0$

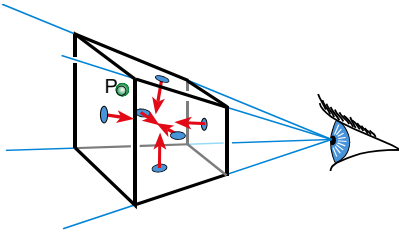


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Point clipping

- Test against each of the 6 planes
 - Normals oriented towards the interior
- Clip (or cull or reject) if $H_p < 0$

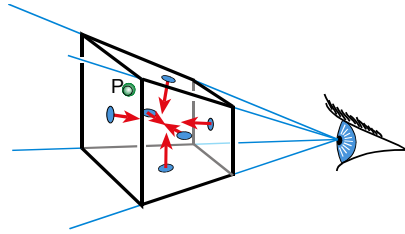


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Point clipping & transformation

- Transform M (e.g. from world space to NDC)
- The plane equation is transformed with $(M^{-1})^T$

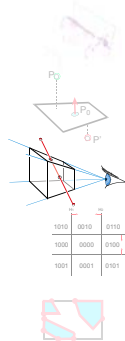


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Plan

- Overview and motivation
- Plane equation & point clipping
- **Line & segment clipping**
- Acceleration using outcodes
- Polygon clipping

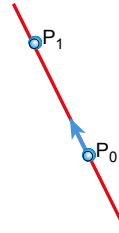


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Parametric expression of line

- ?

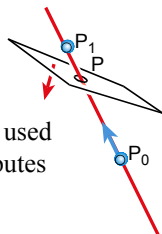


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Line-plane intersection

- Insert explicit equation of line into implicit equation of plane



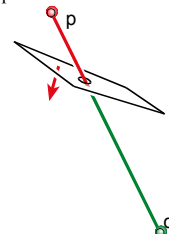
- Parameter t can be used to interpolate attributes (color, etc.)

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Segment clipping

- If $H_q < 0$ and $H_p > 0$, "clip q to plane"
- If $H_p < 0$ and $H_q > 0$, "clip p to plane"
- If $H_p > 0$ and $H_q > 0$, "pass through"
- If $H_p < 0$ and $H_q < 0$, "clipped out"



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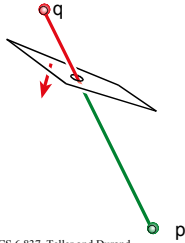
Segment clipping

If $H_q < 0$ and $H_p > 0$, "clip q to plane"

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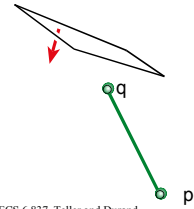
Segment clipping

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If $H_p > 0$ and $H_q > 0$, "pass through"

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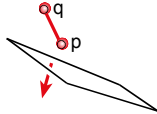
Segment clipping

If $H_q < 0$ and $H_p > 0$, "clip q to plane"

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If $H_p > 0$ and $H_q > 0$, "pass through"

If $H_p < 0$ and $H_q < 0$, "clipped out"



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Segment clipping

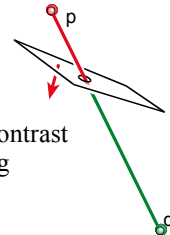
If $H_q < 0$ and $H_p > 0$, "clip q to plane"

If $H_p < 0$ and $H_q > 0$, "clip p to plane"

If $H_p > 0$ and $H_q > 0$, "pass through"

If $H_p < 0$ and $H_q < 0$, "clipped out"

- Note qualitative contrast with point clipping
 - "Triage"



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Clipping against the frustum

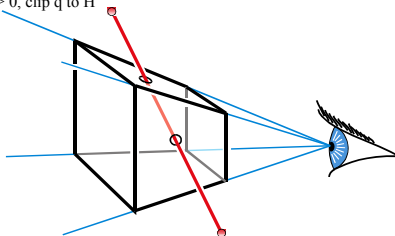
For each frustum plane H

If $H_p < 0$ and $H_q < 0$, clipped out; break

If $H_p > 0$ and $H_q > 0$, pass through

If $H_p < 0$ and $H_q > 0$, clip p to H

If $H_q < 0$ and $H_p > 0$, clip q to H

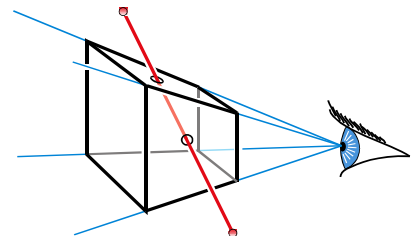


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Segment clipping

- Note: must interpolate associated attributes
 - (color, normal, texture, etc.)

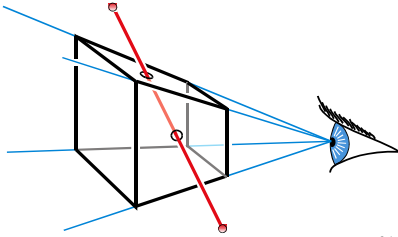


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Line clipping

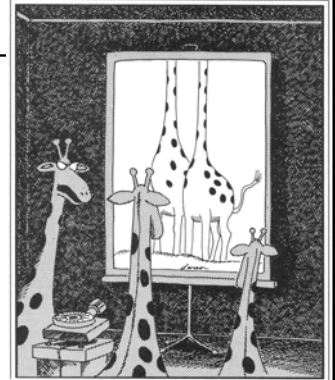
- Result is a single segment
- Why?



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Questions?



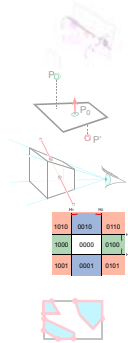
"Oh, lovely — just the hundredth time you've managed to cut everyone's head off."

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Plan

- Overview and motivation
- Plane equation & point clipping
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- Acceleration using outcodes
- Polygon clipping



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Acceleration using outcodes

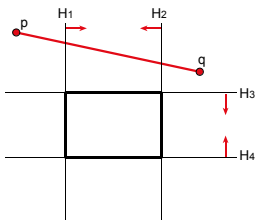
- Avoid many geometric calculations
- Eliminate obvious rejects
- We'll show it in 2D
- Same principle in n-D

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Is it efficient?

- For each frustum plane H
- If $H_p < 0$ and $H_q < 0$, clipped out; break
 - If $H_p > 0$ and $H_q > 0$, pass through
 - If $H_p < 0$ and $H_q > 0$, clip p to H
 - If $H_q < 0$ and $H_p > 0$, clip q to H

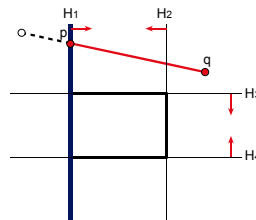


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Is it efficient?

- For each frustum plane H
- If $H_p < 0$ and $H_q < 0$, clipped out; break
 - If $H_p > 0$ and $H_q > 0$, pass through
 - If $H_p < 0$ and $H_q > 0$, clip p to H
 - If $H_q < 0$ and $H_p > 0$, clip q to H



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Is it efficient?

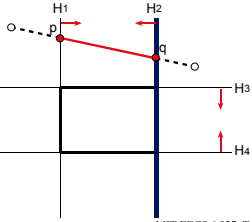
For each frustum plane H

If $H_p < 0$ and $H_q < 0$, clipped out; break

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If $H_p < 0$ and $H_q > 0$, clip p to H

If $H_q < 0$ and $H_p > 0$, clip q to H



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Is it efficient?

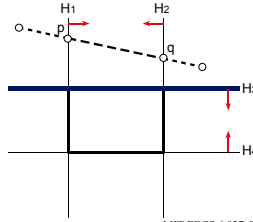
For each frustum plane H

If $H_p < 0$ and $H_q < 0$, clipped out; break

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If $H_p < 0$ and $H_q > 0$, clip p to H

If $H_q < 0$ and $H_p > 0$, clip q to H



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What is the problem?

Is it efficient?

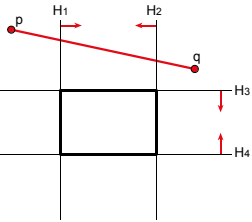
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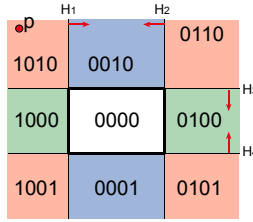
What is the problem?

The computation of the intersections were unnecessary

How can we detect this earlier?

Outcodes

- Compute the n plane equations for point p
- Combine into an **outcode** (binary)
- 0 means : is valid wrt this plane



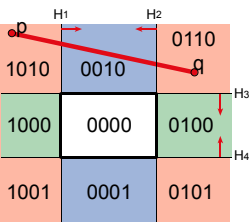
Outcode of p : 1010

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Outcodes

- Compute the n plane equations for point p and q
- Combine into an **outcode** (binary)
- Logical AND



Outcode of p : 1010

Outcode of q : 0110

Outcode of [pq] : 0010

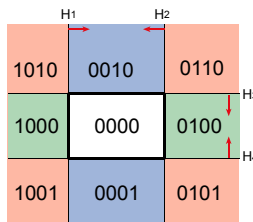
Rejected because there is a 1

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Outcodes

- When do we fail to save computation?

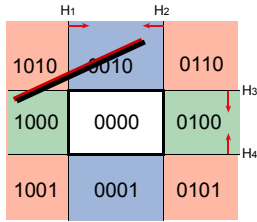


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Outcodes

- When do we fail to save computation?
- Conservative rejection
- And we have the info on what to intersect



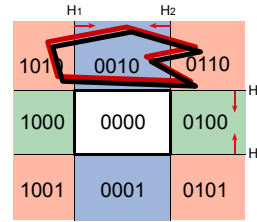
Outcode of p : 1000
 Outcode of q : 0010
 Outcode of [pq] : 0000
 Not rejected

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Outcodes

- It works for arbitrary primitives



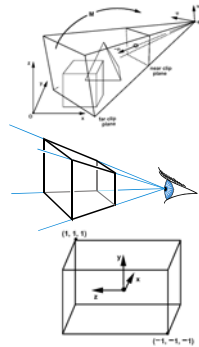
Outcode of p : 1010
 Outcode of q : 0110
 Outcode of r : 0110
 Outcode of s : 0010
 Outcode of t : 0110
 Outcode of u : 0010
 Outcode : 0010
 Rejected

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When to clip?

- Before perspective transform in 3D space
 - Use the equation of 6 planes
 - Advantages? Problems?
- In homogeneous coordinates after perspective transform (NDC)
 - Before perspective divide
 - Advantages? Problems
- In the transformed 3D screen space after perspective division
 - Advantages? Problems?

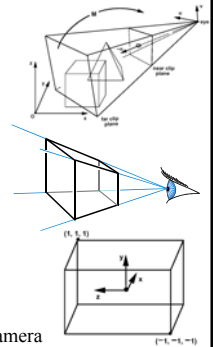


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When to clip?

- Before perspective transform in 3D space
 - Use the equation of 6 planes
 - Natural, not too degenerate
- In homogeneous coordinates after perspective transform (Clip space)
 - Before perspective divide (4D space, weird w values)
 - Canonical, independent of camera
 - The simplest to implement in fact
- In the transformed 3D screen space after perspective division
 - Problem: objects in the plane of the camera

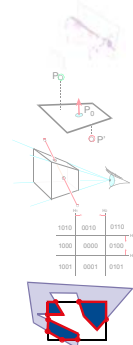


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Plan

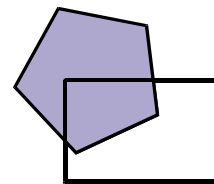
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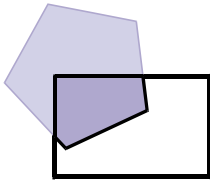
Polygon clipping



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Polygon clipping

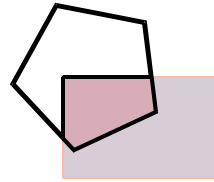


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Polygon clipping

- Clipping is symmetric

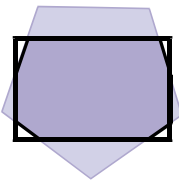


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Polygon clipping is complex

- Even when the polygons are convex

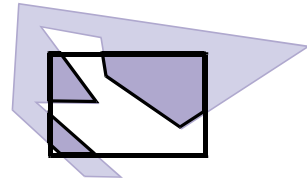


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Polygon clipping is nasty

- When the polygons are concave

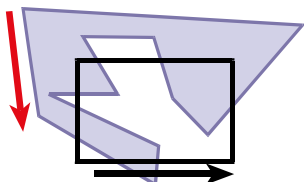


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Weiler-Atherton Clipping

- Strategy: "Walk" polygon/window boundary
- Polygons are oriented (CCW)

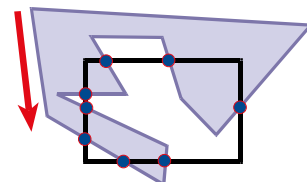


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Weiler-Atherton Clipping

- Compute intersection points

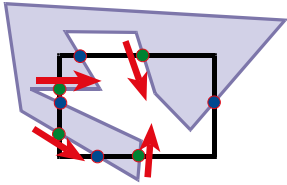


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Weiler-Atherton Clipping

- Compute intersection points
- Mark points where polygons enters clipping window (green here)

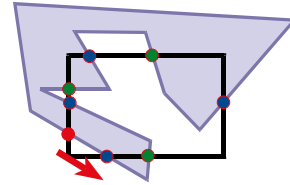


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Clipping

- While there is still an unprocessed entering intersection
- Walk" polygon/window boundary

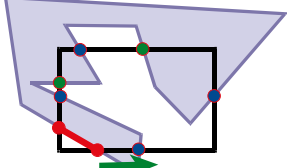


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Walking rules

- Out-to-in pair:
 - Record clipped point
 - Follow polygon boundary (ccw)
- In-to-out pair:
 - Record clipped point
 - Follow window boundary (ccw)

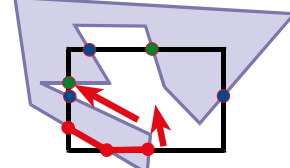


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Walking rules

- Out-to-in pair:
 - Record clipped point
 - Follow polygon boundary (ccw)
- In-to-out pair:
 - Record clipped point
 - Follow window boundary (ccw)

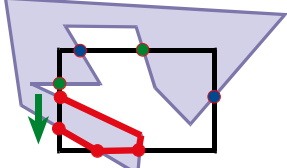


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Walking rules

- Out-to-in pair:
 - Record clipped point
 - Follow polygon boundary (ccw)
- In-to-out pair:
 - Record clipped point
 - Follow window boundary (ccw)

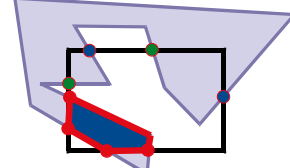


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Walking rules

- Out-to-in pair:
 - Record clipped point
 - Follow polygon boundary (ccw)
- In-to-out pair:
 - Record clipped point
 - Follow window boundary (ccw)

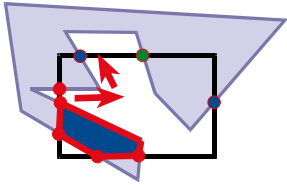


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Walking rules

While there is still an unprocessed entering intersection
Walk" polygon/window boundary

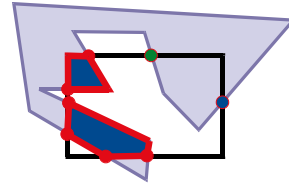


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Walking rules

While there is still an unprocessed entering intersection
Walk" polygon/window boundary

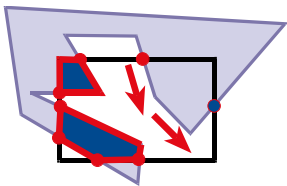


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Walking rules

While there is still an unprocessed entering intersection
Walk" polygon/window boundary

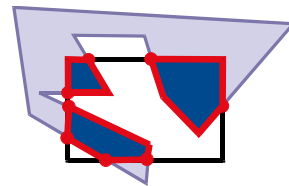


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Walking rules

While there is still an unprocessed entering intersection
Walk" polygon/window boundary

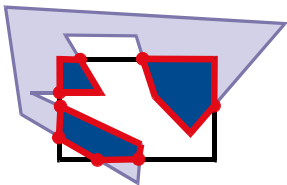


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Weiler-Atherton Clipping

- Importance of good adjacency data structure (here simply list of oriented edges)

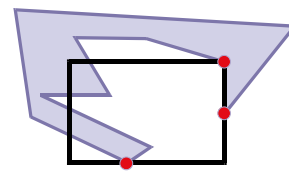


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Robustness, precision, degeneracies

- What if a vertex is on the boundary?
- What happens if it is "almost" on the boundary?
 - Problem with floating point precision
- Welcome to the real world of 3D geometry!



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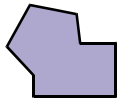
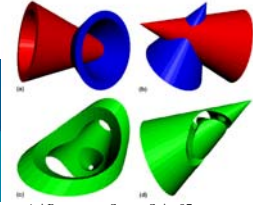
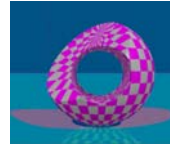
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Clipping

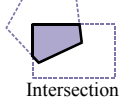
- Many other clipping algorithms:
- Parametric, general windows, region-region, One-Plane-at-a-Time Clipping, etc.

Constructive Solid Geometry (CSG)

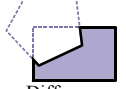
- Sort of generalized clipping
- Boolean operations
- Very popular in CAD/CAM
- CSG tree



Union



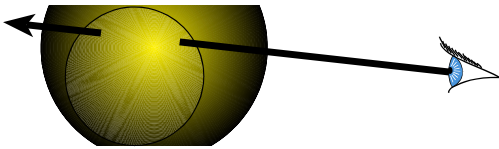
Intersection



Difference

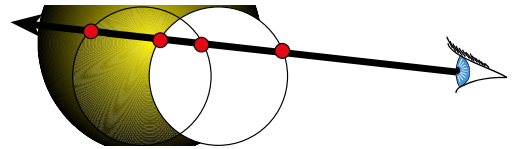
Constructive Solid Geometry (CSG)

- Simple with some rendering algorithms (Ray Tracing)



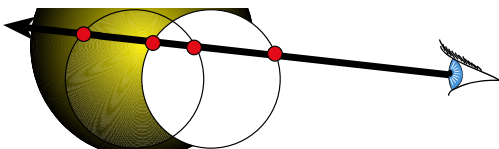
Constructive Solid Geometry (CSG)

- Simple with some rendering algorithms (Ray Tracing)



Constructive Solid Geometry (CSG)

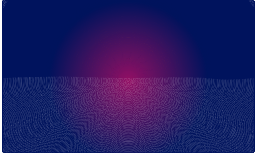
- Simple with some rendering algorithms (Ray Tracing)
- Very hard with polygonal models
- Can also be done with hardware tricks



Questions?

Playtime: Sunset

- We have seen why the sunset is red



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Playtime: The Green Flash

- Novel by Jules Verne (19th century)
 - Movie by Rohmer, *Summer*
- Visual phenomenon at sunset



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Playtime: The Green Flash



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Green Flash

- <http://mintaka.sdsu.edu/GF/>
- http://www.mtwilson.edu/Tour/Lot/Green_Flash/
- <http://www.intersoft.it/galaxlux/GreenFlashGallery.htm>
- <http://www.isc.tamu.edu/~astro/research/sandiego.html>
- <http://www.meteores.net/rv.html>
- <http://flzhgn.home.mindspring.com/gnray.htm>



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Green flash



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Green flash



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