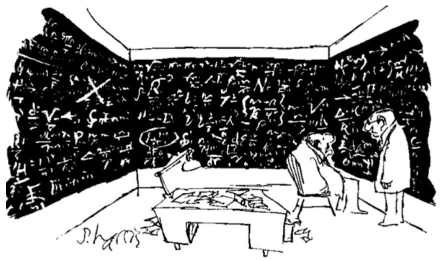
Physically Based Illumination Models

BRDF
Cook-Torrance
Rendering Equation



"Whatever happened to *elegant* solutions?"

Phong Illumination Model

$$I_{\text{total}} = k_a I_{\text{ambient}} + I_{\text{light}}\left[k_d\left(\hat{N}\cdot\hat{L}\right) + k_s\left(\hat{V}\cdot\hat{R}\right)^{n_{\text{shiny}}}\right]$$

Problems with Empirical Models:

- What are k_a, k_s, k_d and n_{shiny}? Are they measurable quantities?
- What are the coefficients for copper?
- How does the incoming light at a point relate to the outgoing light?
- Is energy conserved?
- Just what is light intensity?
- Is my picture accurate?

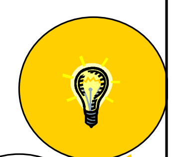
What We Want

- A model that uses physical properties that can be looked up in the CRC Handbook of Chemistry and Physics (indices of refraction, reflectivity, conductivity, etc.)
- Parameters that that have clear physical analogies (how rough or polished a surface is)
- Models that are predictive (the simulation attempts to model the real scene)
- Models that conserve energy
- Complex surface substructures (crystals, amorphous materials, boundary-layer behavior)
- If it was easy... everyone would do it.

Energy and Power of Light

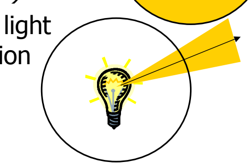
Light energy (Radiant energy): the energy of the photon particles. If we know the number of photon particles emitted, we can sum up the energies of each photon to evaluate the energy of light *(Joules).*

Work: the change in energy. The light does work to emit energy *(Joules).*



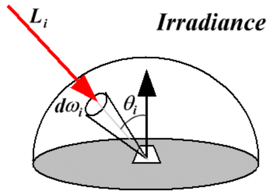
Flux (Radiant power): the rate of work, the rate at which light energy is emitted *(Watt).*

Radiant Intensity: the flux (the rate of light energy change) radiated in a given direction *(W/sr).*



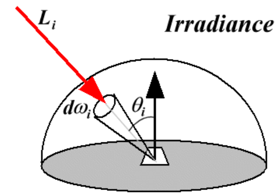
Irradiance

The flux (the rate of radiant energy change) at a surface point per unit surface area *(W/m²)*. In short, flux density. The irradiance function is a two dimensional function describing the incoming light energy impinging on a given point.



$$E_i = \int_\Omega L_i \cos\theta_i \, d\omega_i$$

What does Irradiance look like?



$$E_i = \int_\Omega L_i \cos\theta_i \, d\omega_i$$

What is L_i?

Radiant Intensity?

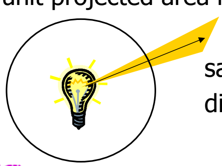
Radiance

The L_i term is not radiant intensity. You can see this by comparing the units:

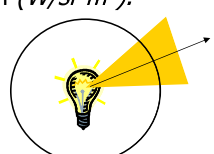
$$E_i\left[\frac{W}{m^2}\right] = \int_\Omega L_i \left[\frac{W}{sr}\right] \cos\theta_i \, d\omega_i \, [sr], \text{ but } \left[\frac{W}{m^2}\right] \neq \left[\frac{W}{sr}\right][sr]$$

Radiant intensity does not account for the size of the surface from the lights perspective: more radiant power (flux) will reach a surface that appears bigger to the light.

Radiance: the angular flux density, the radiant power (flux) per unit projected area in a given direction *(W/sr m²)*.



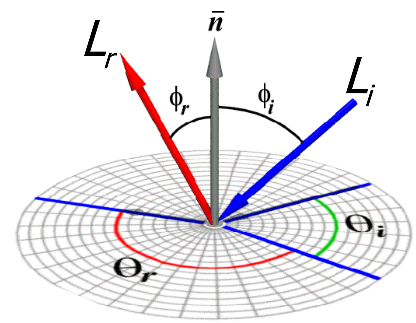
same direction
different radiance



What happens after reflection?

The amount of reflected radiance is proportional to the incident radiance.

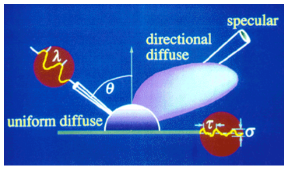
$$L_r = \rho(\theta_r, \phi_r, \theta_i, \phi_i) L_i$$



What does BRDF look like?

Bidirectional Reflectance Distribution Function (BRDF)

$$\rho(\theta_r, \phi_r, \theta_i, \phi_i)$$



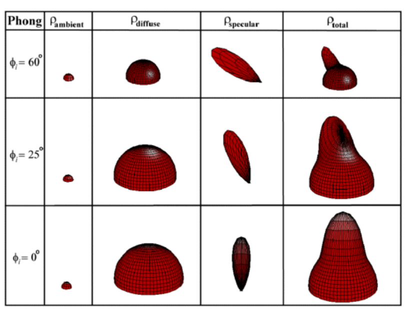
BRDF Approaches

Physically-based models
Measured BRDFs

Local Illumination

$$L_r(\vec{\omega}_r) = \int_{\Omega} \rho(\vec{\omega}_i \mapsto \vec{\omega}_r) L_i(\vec{\omega}_i) \cos\theta_i \, d\omega_i$$

Phong illumination model approximates the BRDF with combination of diffuse and specular components.



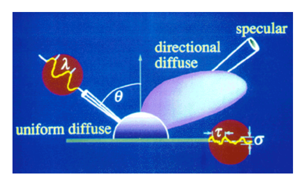
Better Illumination Models

Blinn-Torrance-Sparrow (1977)
- isotropic reflectors with smooth microstructure

Cook-Torrance (1982)
- wavelength dependent Fresnel term

He-Torrance-Sillion-Greenberg (1991)
- adds polarization, statistical microstructure, self-reflectance

Very little of this work has made its way into graphics H/W.



Cook-Torrance Illumination

$$I_\lambda = k_a I_{\lambda,a} + \sum_{i=1}^{\text{lights}} I_{\lambda,i}\left[(1-k_a-k_s)\,\rho_\lambda\left(\hat{l}_i\cdot\hat{n}\right) + k_s \frac{DGF_\lambda(\theta_i)}{\pi(\hat{v}\cdot\hat{n})}\right]$$

$I_{\lambda,a}$ - Ambient light intensity
k_a - Ambient surface reflectance
$I_{\lambda,i}$ - Luminous intensity of light source i
k_s - percentage of light reflected specularly (notice terms sum to one)
ρ_l - Diffuse reflectivity
l_i - vector to light source
n - average surface normal at point
D - microfacet distribution function
G - geometric attenuation Factor
$F_\lambda(\theta_i)$ - Fresnel conductance term
v - vector to viewer

Cook-Torrance BRDF

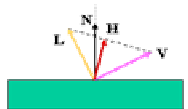
$$\rho_\lambda = \frac{DGF_\lambda(\theta_i)}{\pi\cos\theta_i\cos\theta_r} = \frac{DGF_\lambda(\theta_i)}{\pi\left(\hat{l}\cdot\hat{n}\right)\left(\hat{v}\cdot\hat{n}\right)}$$

Physically based model of a reflecting surface. Assumes a surface is a collection of planar microscopic facets, *microfacets*. Each microfacet is a perfectly smooth reflector. The factor D describes the distribution of microfacet orientations. The factor G describes the masking and shadowing effects between the microfacets. The F term is a Fresnel reflection term related to material's index of refraction.

Microfacet Distribution Function

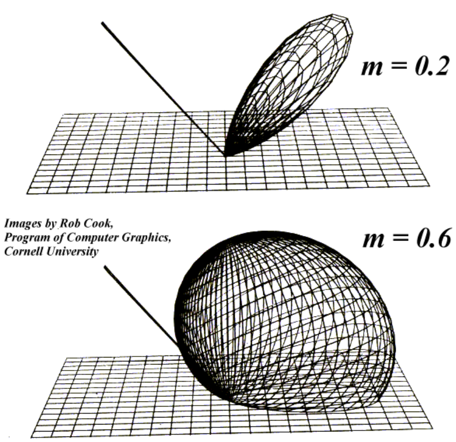
$$D = \frac{e^{-\left(\frac{\tan\beta}{m}\right)^2}}{4m^2\cos^4\beta}$$

$$\hat{H} = \frac{\hat{L}+\hat{V}}{\left|\hat{L}+\hat{V}\right|}$$



- Statistical model of the microfacet variation in the halfway-vector H direction
- Based on a Beckman distribution function
- Consistent with the surface variations of rough surfaces
- β - the angle between N and H
- *m* - the root-mean-square slope of the microfacets
  large *m* indicates steep slopes and the reflections spread out over the surface

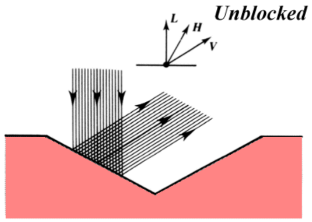
Beckman's Distribution



Images by Rob Cook, Program of Computer Graphics, Cornell University

Geometric Attenuation Factor

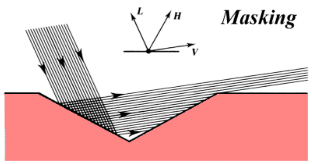
The geometric attenuation factor G accounts for microfacet shadowing. The factor G is in the range from 0 (total shadowing) to 1 (no shadowing). There are many different ways that an incoming beam of light can interact with the surface locally.



The entire beam can simply reflect.

Blocked Reflection

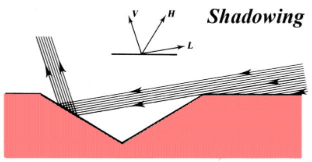
A portion of the out-going beam can be blocked.



This is called *masking*.

Blocked Beam

A portion of the incoming beam can be blocked.



Cook called this *self-shadowing*.

Geometric Attenuation Factor

In each case, the geometric configurations can be analyzed to compute the percentage of light that actually escapes from the surface. The geometric factor, chooses the smallest amount of lost light.

$$G = 1 - \frac{l_{\text{blocked}}}{l_{\text{facet}}}$$

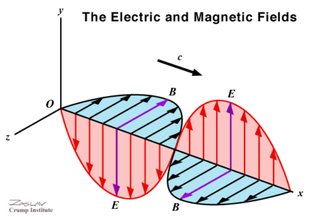
$$G_{\text{masking}} = \frac{2(\vec{n} \cdot \vec{h})(\vec{n} \cdot \vec{v})}{\vec{v} \cdot \vec{h}}$$

$$G_{\text{shadowing}} = \frac{2(\vec{n} \cdot \vec{h})(\vec{n} \cdot \vec{l})}{\vec{v} \cdot \vec{h}}$$

$$G = \min\{1, G_{\text{masking}}, G_{\text{shadowing}}\}$$

Fresnel Reflection

The Fresnel term results from a complete analysis of the reflection process while considering light as an electromagnetic wave. The electric field of light has an associated magnetic field associated with it (hence the name electromagnetic). The magnetic field is always orthogonal to the electric field and the direction of propagation. Over time the orientation of the electric field may rotate. If the electric field is oriented in a particular *constant* direction it is called *polarized*. The behavior of reflection depend on how the incoming electric field is oriented relative to the surface at the point where the field makes contact. This variation in reflectance is called the Fresnel effect.



Fresnel Reflection

The Fresnel effect is wavelength dependent. It behavior is determined by the index-of-refraction of the material (taken as a complex value to allow for attenuation). This effect explains the variation in colors seen in specular regions particular on metals (conductors). It also explains why most surfaces approximate mirror reflectors when the light strikes them at a grazing angle.

$$F_\lambda(\theta_i) = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

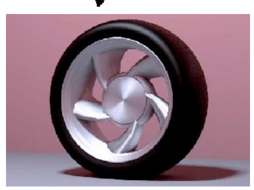
$$c = \cos\theta_i = \vec{l}\cdot\vec{h}$$

$$g = \sqrt{\left(\frac{n_i}{n_f}\right)^2 + c^2 - 1}$$

Remaining Hard Problems

Reflective Diffraction Effects
- thin films
- feathers of a blue jay
- oil on water
- CDs

Anisotropy
- brushed metals
- strands pulled materials
- satin and velvet cloths



Global Illumination

So far, we have looked at local illumination problems, which approximate how the light reflects from a surface under direct illumination. Global illumination computes the more general problem of light transfer between all objects in the scene, including direct and indirect illumination. Rendering equation is the general formulation of the global illumination problem: it describes how the radiance from surface x reflects from the surface x':

$$L(x',\vec{\omega}') = E(x') + \int_S \rho(x')L(x,\vec{\omega})G(x,x')V(x,x')dA$$

- L is the radiance from a point on a surface in a given direction ω
- E is the emitted radiance from a point: E is non-zero only if x' is emissive
- V is the visibility term: 1 when the surfaces are unobstructed along the direction ω, 0 otherwise
- G is the geometry term, which depends on the geometric relationship between the two surfaces x and x'

Next Time

Ray Tracing