Surface Modeling

Types:
- Polygon surfaces
- Curved surfaces
- Volumes

Generating models:
- Interactive
- Procedural

Polygon Surfaces

Set of surface polygons that enclose an object interior

Polygon Tables

We specify a polygon surface with a set of vertex coordinates and associated attribute parameters.

Curved Surfaces

- Implicit
  Curve defined in terms of an implicit function:
  \[ f(x, y, z) = 0 \]

- Parametric
  Parametrically defined curve in three dimensions is given by three univariate functions:
  \[ Q(u) = (X(u), Y(u), Z(u)) \]

\[ Q(u) = (\cos u, \sin u) \quad f(x, y) = x^2 + y^2 - 1 = 0 \]
Implicit vs. Parametric

Surface Display
✓ Parametric

Surface Intersections
✓ Implicit

Changing Topology
✓ Implicit

Parametric Example: Beziér Curves

A Beziér curve can be defined in terms of a set of control points denoted in red. Consider, for example, a cubic, or curve of degree 3:

\[ Q(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2 (1-u) p_2 + u^3 p_3 \]

We can generate points on the curve by repeated linear interpolation. Starting with the control polygon (in red), the edges are subdivided (as noted in blue). These points are then connected in order and the resulting edges subdivided. The recursion stops when only one edge remains. This allows us to approximate the curve at multiple resolutions.

Beziér Patches

Control polyhedron with 16 points and the resulting bicubic patch:

Example: The Utah Teapot

32 patches

single shaded patch

wireframe of the control points

Patch edges
Subdivision of Beziér Surfaces

We can now apply the same basic idea to a surface, to yield increasingly accurate polygonal representations.

Deforming a Patch

The net of control points forms a polyhedron in cartesian space, and the positions of the points in this space control the shape of the surface.

The effect of lifting one of the control points is shown on the right.

Patch Representation vs. Polygon Mesh

It’s fair to say that a polygon is a simple and flexible building block. However, a parametric representation of an object has certain key advantages:

- Conciseness
  - A parametric representation is exact and economic since it is analytical. With a polygonal object, exactness can only be approximated at the expense of extra processing and database costs.

- Deformation and shape change
  - Deformations of parametric surfaces is no less well defined than its undeformed counterpart, so the deformations appear smooth. This is not generally the case with a polygonal object.

Sweep Representations

Solid modeling packages often provide a number of construction techniques. A good example is a sweep, which involves specifying a 2D shape and a “sweep” that moves the shape through a region of space.
Constructive Solid-Geometry Methods (CSG)

Another modeling technique is to combine the volumes occupied by overlapping 3D shapes using set operations. This creates a new volume by applying the union, intersection, or difference operation to two volumes.

intersection  difference  union

A CSG Tree Representation

Example Modeling Package: Alias Studio

Volume Modeling
Marching Cubes Algorithm

Extracting a surface from voxel data:

1. Select a cell
2. Calculate the inside/outside state of each vertex of the cell
3. Create an index
4. Use the index to look up the state of the cell in the case table (see next slide)
5. Calculate the contour location (via interpolation) for each edge in the case table

Marching Cube Cases

Extracted Polygonal Mesh

Procedural Techniques: Fractals

Apply algorithmic rules to generate shapes
Terrains

Midpoint Displacement

Example:
\[ d = \text{edge size} \cdot \text{random}(-1,1) \]

Example: L-systems

Biologically-motivated approach to modeling botanical structures

Example of a complex L-system model

Next Time