Topics in Computer Animation

Kinematics
Dynamics
- Translational
- Rotational

Key Framing
Motion Capture

Different Approaches to Computer Animation

Physically Based Animations -

- All about physics.
- Assign masses to our objects, establish initial and reaction forces, then run simulations.
- Results look real
- Lack of control

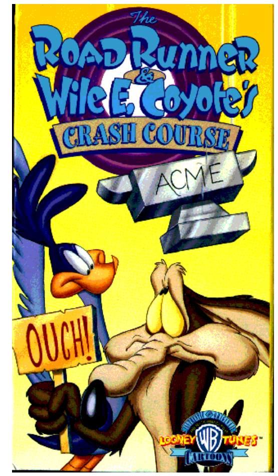
Hand Tweaked Motions -

- Establish the positions and orientations of objects at "key" time steps
- Interpolate the positions of objects in-between
- Very good control
- Making it "look" real is an art, and sometimes we don't want things to look real

Motion Capture -

- Can capture style and nuance
- Looks real
- Good Control
- Hard to edit

Kinematics



Kinematics is that branch of mechanics that describes the motions of bodies without considering the forces required to produce and maintain the motion.

We start with the time varying motions of points as a function of time:

Time varying position: $\dot{x}(t)$

Velocity: $\vec{v}(t) = \frac{d}{dt}\dot{x}(t)$

Acceleration: $\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d^2}{dt^2}\dot{x}(t)$

Consider a point undergoing a constant acceleration ***k***

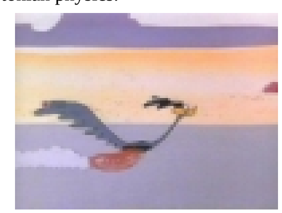
$$\vec{v}(t) = \int k\,dt = kt + \vec{v}(0)$$

$$\dot{x}(t) = \int \vec{v}(t)dt = \int kt + v(0)dt = \frac{k}{2}t^2 + \vec{v}(0)t + \dot{x}(0)$$

Newtonian Physics

Kinematics describes the motion of objects in equilibrium. *Dynamics* (or Kinetics) describes the change in an object's kinematics due to a change in the object's mass or the application of forces. To understand dynamics we'll need to review Newtonian physics.

A moving mass has *momentum*. *Forces* are needed to change the momentum of a mass.



Momentum: $\vec{p} = m\,\vec{v}(t)$

Force: $\vec{F} = m\vec{a}(t) = \frac{d}{dt}\vec{p}$

We can also define aggregate properties of point masses.

Total Mass: $M = \sum_i m_i$

Center of Mass: $\dot{x}_{CM}(t) = \frac{\sum_i m_i \dot{x}_i(t)}{M}$

Total Momentum

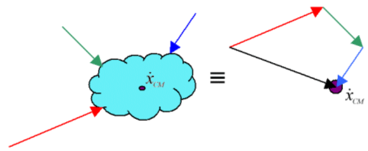
The total momentum of an aggregate object (set of point masses) is given by:

$$\text{Total Momentum:}\ \vec{p}_{Total} = \sum_i m_i \vec{v}_i(t) = \sum_i m_i \tfrac{d}{dt}\dot{x}_i(t) = M \tfrac{d}{dt}\dot{x}_{CM}(t) = M\,\vec{v}_{CM}(t)$$

Since force is the time derivative of momentum. We can also define the total force.

$$\text{Total Force:}\ \vec{F}_{total} = \tfrac{d}{dt}\vec{p}_{Total} = M\tfrac{d}{dt}\vec{v}_{CM}(t) = M\,\vec{a}_{CM}(t)$$

This means that we can treat all forces acting on a given rigid body as if their vector sum was acting on a single point at the body's center of mass with the same mass as the entire body.



Rotational Kinematics

Recall our approach to specifying rotations at the origin around an axis ***a*** by an angle θ from Lecture 10.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \left(\mathbf{Symmetric}\left(\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}\right)(1-\cos\theta) + \mathbf{Skew}\left(\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}\right)\sin\theta + \mathbf{I}\cos\theta \right)\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Assuming that θ is a function of time, we can determine the component of linear velocity that is induced by the rotation by differentiating our rotation expression. We can also orient our frame of reference so that θ = *0* at *t = 0*.

$$\tfrac{d}{dt}\dot{x} = \left(\tfrac{d\theta}{dt}\sin\theta\,(\mathbf{Symmetric}(\vec{a}) - \mathbf{I}) + \tfrac{d\theta}{dt}\cos\theta\,\mathbf{Skew}(\vec{a})\right)\dot{x}$$

$$\theta(0) \to 0 \quad \tfrac{d}{dt}\dot{x} = \tfrac{d\theta}{dt}\mathbf{Skew}(\vec{a})\,\dot{x}$$

The *instantaneous angular velocity vector* is often used to simplify this equation:

$$\vec{\omega} = \tfrac{d\theta}{dt}\vec{a} \quad \tfrac{d}{dt}\dot{x} = Skew(\vec{\omega})\dot{x}$$

Rotating Away from the Origin

Generally, our center of rotation is not at the origin. We can simply fix our equation to handle this case. We introduce the center of rotation, ***o***.

$$\tfrac{d}{dt}(\dot{x} - \dot{o}) = \tfrac{d}{dt}\dot{x} - \tfrac{d}{dt}\dot{o} = \mathbf{Skew}(\vec{\omega})(\dot{x} - \dot{o})$$

$$\tfrac{d}{dt}\dot{x} = \tfrac{d}{dt}\dot{o} + \mathbf{Skew}(\vec{\omega})(\dot{x} - \dot{o})$$

This equation states that the total velocity at a point is the sum of the point's angular velocity and the velocity seen at the center of rotation. The *total acceleration* is given by differentiating once more.

$$\tfrac{d^2}{dt^2}\dot{x} = \tfrac{d^2}{dt^2}\dot{o} + \tfrac{d}{dt}\mathbf{Skew}(\vec{\omega})(\dot{x} - \dot{o}) + \mathbf{Skew}(\vec{\omega})\tfrac{d}{dt}(\dot{x} - \dot{o})$$

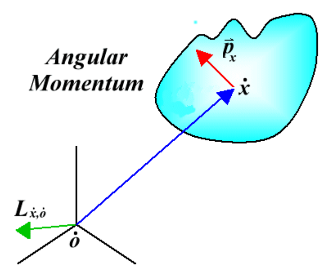
$$\tfrac{d^2}{dt^2}\dot{x} = \tfrac{d^2}{dt^2}\dot{o} + \tfrac{d}{dt}\mathbf{Skew}(\vec{\omega})(\dot{x} - \dot{o}) + \mathbf{Skew}(\vec{\omega})(\mathbf{Skew}(\vec{\omega})(\dot{x} - \dot{o}))$$

Often, you will see the angular velocity vector defined as:

$$\vec{\alpha} = \tfrac{d}{dt}\vec{\omega} = \tfrac{d^2}{dt^2}\theta\,\vec{a} \qquad \tfrac{d^2}{dt^2}\dot{x} = \tfrac{d^2}{dt^2}\dot{o} + \mathbf{Skew}(\vec{\alpha})(\dot{x} - \dot{o}) + \mathbf{Skew}(\vec{\omega})\mathbf{Skew}(\vec{\omega})(\dot{x} - \dot{o})$$

What's going on with that third term in the total acceleration expression?

Angular Dynamics



Angular momentum is the component of momentum due to rotation. Angular momentum must be specified relative to a center of rotation.

$$L_{\dot{x},\dot{o}} = \mathbf{Skew}(\dot{x} - \dot{o})\vec{p}_{\dot{x}} = \mathbf{Skew}(\dot{x} - \dot{o})m_{\dot{x}}\vec{v}_{\dot{x}}$$

Angular momentum describes the rotational motion of the vector from **o** to **x** due to the motion at **x** (**p**). In other words, the fraction of **x**'s momentum rotating around **o**. This rotational motion will be about an axis perpendicular to both the vector and **p**. The angular momentum captures this axis of rotation (centered at **o**).

The time derivative of angular momentum is called *torque*.

$$\tau_{\dot{x},\dot{o}} = \tfrac{d}{dt}L_{\dot{x},\dot{o}} = \mathbf{Skew}(\dot{x} - \dot{o})\tfrac{d}{dt}\vec{p}_{\dot{x}} = \mathbf{Skew}(\dot{x} - \dot{o})\vec{F}_{\dot{x}}$$

Total Angular Momentum

We can compute the total angular momentum of a body relative to the point **o** as follows:

$$L_{T,\dot{o}} = \sum_i \mathbf{Skew}(\dot{x}_i - \dot{o}) m_i \vec{v}_i = \sum_i \mathbf{Skew}(\dot{x}_i - \dot{o}) m_i \tfrac{d}{dt}\dot{x}_i$$

Substituting in the angular velocity term from two slides ago gives:

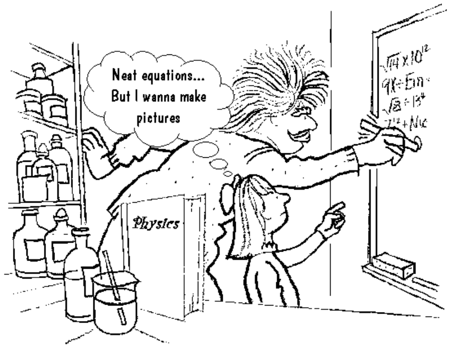
$$L_{T,\dot{o}} = \sum_i m_i \mathbf{Skew}(\dot{x}_i - \dot{o})\ \mathbf{Skew}(\vec{\omega})(\dot{x}_i - \dot{o}) = \sum_i -m_i \mathbf{Skew}(\dot{x}_i - \dot{o})\ \mathbf{Skew}(\dot{x}_i - \dot{o})\vec{\omega}$$

Now the only term that we have that varies with time is the angular velocity vector. We can factor it out of the summation and the resulting summation is fixed for a given **o**. We call this term the *Inertial tensor* (it's just a 3 by 3 matrix, however).

$$L_{T,\dot{o}} = \left(\sum_i - m_i \mathbf{Skew}(\dot{x}_i - \dot{o})\ \mathbf{Skew}(\dot{x}_i - \dot{o})\right)\vec{\omega} = I_{\dot{o}}\vec{\omega}$$

For rigid bodies we will find it convenient to specify the Inertial tensor relative to the object's center of mass.

What Now?



Generally, we will be given rigid bodies with forces applied to them. Those forces cause the objects to move (animate). We now know everything we need to simulate this.

The Key to Dynamics

The motion of an object is changed by the application of Forces. How the object's translation changes can be determined by summing up all of the Forces applied to the object. The object's rotation is changed according to where the forces are applied relative to the center of mass.

We only need to know a few more things. We'll need a method for solving first-order differential equations. Here we'll use the simplest solution technique, *Euler Integration*.

$$\vec{x}(t+h) \approx \vec{x}(t) + h\frac{d}{dt}\vec{x}(t)$$

The Simulation Loop

Determine rigid body constants : I_{CM}^{-1}, M

Initial conditions : $x_{CM}(0), \vec{v}(0), R(0), L_{CM}(0)$

Compute Auxillary information : $I_{RCM}^{-1}(0) = R(0) I_{CM}^{-1} R^T(0) \qquad \vec{\omega}(0) = I_{CM}^{-1} L_{CM}(0)$

Find individual forces and their points of application : F_i, x_i

Compute total forces and torques :

$$F_T(t) = \sum_i F_i$$

$$\tau_T(t) = \sum_i \mathbf{Skew}(x_i - x_{CM}) F_i$$

Integrate over interval h :

$$x_{CM}(t+h) = x_{CM}(t) + h\vec{v}_{CM}(t)$$

$$\vec{v}_{CM}(t+h) = \vec{v}_{CM}(t) + h\frac{F_T(t)}{M}$$

$$R(t+h) = R(t) + h\ \mathbf{Skew}(\vec{\omega})R(t) \quad \text{// we'll need to reorthogonalize } R(t+h)$$

$$L_{CM}(t+h) = L_{CM}(t) + h\ \tau_T(t)$$

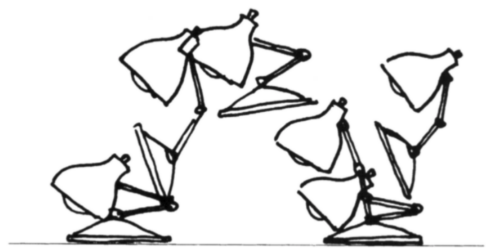
Update Auxillary information :

$$I_{RCM}^{-1}(t+h) = R(t+h) I_{CM}^{-1} R^T(t+h)$$

$$\vec{\omega}(t+h) = I_{CM}^{-1} L_{CM}(t+h)$$

Key Framing

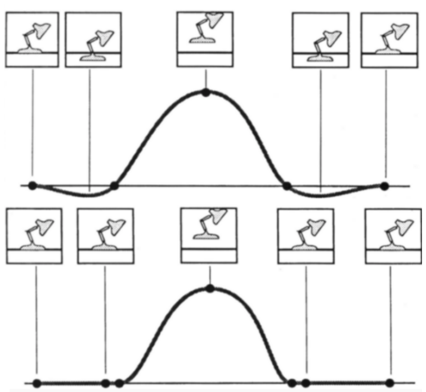
An alternative approach to simulation is "key framing". This is the animation approach used in traditional cel animation. The animator starts by specifying the positions and orientations at various key points in time. In-betweeners generate the images in-between the key frames. This was once a job for apprentice animators. These days it is largely done by computer.



The images in this section are taken from a classical paper on animation by John Lasseter from Pixar. *"Principles of Traditional Animation Applied to 3D Computer Graphics,"* **SIGGRAPH'87**, pp. 35-44.

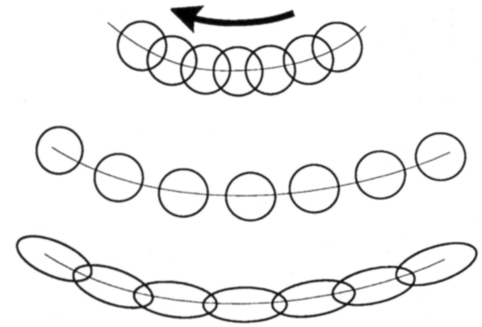
Interpolating Key Frames

Largely, splines are used to interpolate the positions on objects between key frames. This is actually very similar to the spline techniques that we used when discussing image reconstruction earlier.

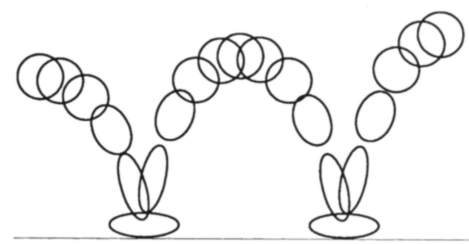


Cartoon Physics

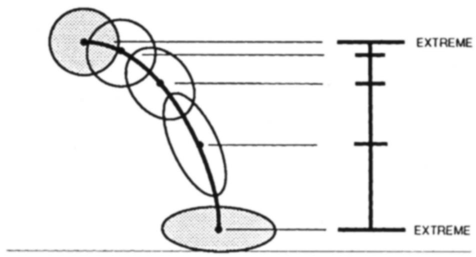
Skilled animators also play with physics to exaggerate and overcome limitations of the display medium.

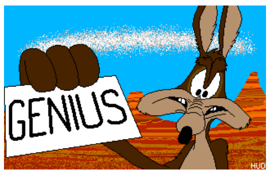


More Examples



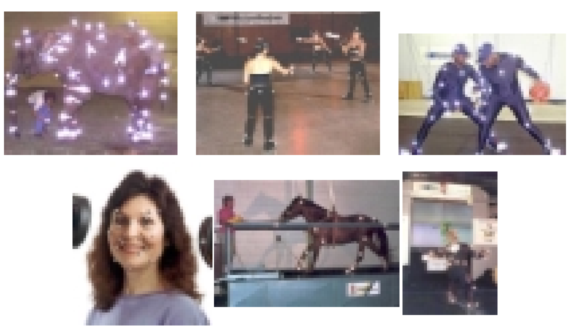
But They Need to Know Physics





Motion Capture

Motion capture is becoming an increasingly common technique for animating computer generated characters. These techniques rely on computer vision and tracking technologies to acquire motion that is retargeted to the animated character.



The system shown above is by MotionAnalysis Inc.

Next Time

Animation is a wide open area in computer graphics. There are still lot's of things to do.