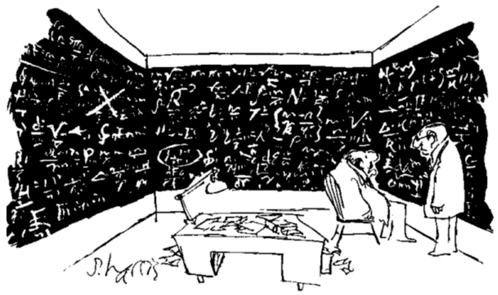
Physically-Based Illumination Models

- What and Why?
- Review of Electromagnetics
- Cook-Torrance
- Energy Conservation



"Whatever happened to *elegant* solutions?"

Revisiting Phong's Illumination Model

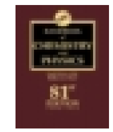
$$I_{total} = k_a I_{ambient} + \sum_{i=1}^{lights} I_i \left(k_d \left(\hat{N} \cdot \hat{L} \right) + k_s \left(\hat{V} \cdot \hat{R} \right)^{n_{shiney}} \right)$$

Problems with Empircal Models:

- What are the coefficients for copper?
- What are k_a, k_s, and n_{shiney}?
  Are they measurable quanities?
- How does the incoming light at a point relate to the outgoing light?
  Is energy conserved?
- Just what is light intensity?
- Is my picture accurate?

Desiderata

- A model that uses physical properties that can be looked up in the CRC Handbook of Chemistry and Physics (indices of refraction, reflectivity, conductivity, etc.)

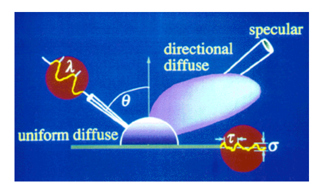


- Parameters that that have clear physical analogies (how rough or polished a surface is)
- Models that are predictive (the simulation attempts to model the real scene)
- Models that conserve energy
- Complex surface substructures
  (crystals, amorphous materials, boundary-layer behavior)
- If it was easy... everyone would do it.

Better Illuminance Models

- Blinn-Torrance-Sparrow (1977)
  - isotropic reflectors with smooth microstructure
- Cook-Torrance (1982)
  - wavelength dependent Fresnel term
- He-Torrance-Sillion-Greenberg (1991)
  - adds polarization, statistical microstructure, self-reflectance

Very little of this work has made its way into graphics H/W. Why?



Cook-Torrance Illumination

$$I_{\lambda,r} = I_{\lambda,a}k_a + \sum_{i=1}^{lights} I_{\lambda,i}\left((1-k_a-k_s)\rho_\lambda(\bar{l}_i\cdot\bar{n}) + k_s\frac{DGF_\lambda(\theta_i)}{\pi(\bar{v}\cdot\bar{n})}\right)$$

Definitions:

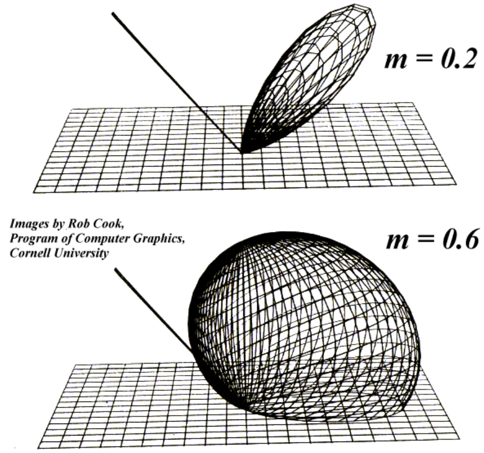
- $I_{\lambda,a}$ - Ambient light intensity (same old hack)
- k_a - Ambient surface reflectance (hacks beget hacks)
- $I_{\lambda,i}$ - Luminous intensity of light source i
- k_s - percentage of light reflected specularly (notice terms sum to one)
- ρ_λ - Diffuse reflectivity
- l_i - vector to light source
- n - average surface normal at point
- D - microfacet distribution function
- G - geometric attenuation Factor
- $F_\lambda(\theta_i)$ - Fresnel conductance term
- v - vector to viewer

Microfacet Distribution Function

$$D = \frac{e^{-\left(\frac{\tan\beta}{m}\right)^2}}{4m^2\cos^4\beta}$$

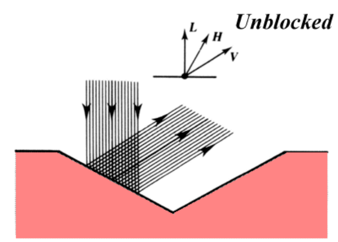
- Statistical model of the variation in normal direction
- Based on a Beckman distribution function
- Consistent with the surface variations of rough surfaces
- B - the angle between *n* and *h*
- *m* - the root-mean-square slope of the microfacets
  large *m* indicates steep slopes and the reflections spread out over the surface

Beckman's Distribution



Geometric Attenuation Factor

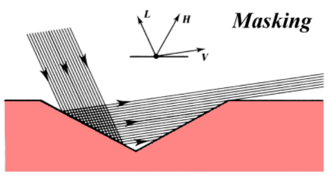
There are many different ways that an incoming beam of light can interact with the surface locally.



The entire beam can simply reflect.

Blocked Reflection

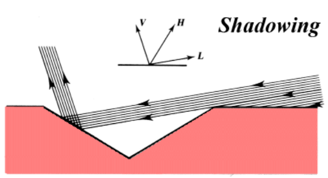
A portion of the out-going beam can be blocked.



This is called *masking*.

Blocked Beam

A portion of the incoming beam can be blocked.



Cook called this *self-shadowing*.

Geometric Attenuation Factor

In each case, the geometric configurations can be analyzed to compute the percentage of light that actually escapes from the surface. Blinn first did this analysis. The results are:

$$G = 1 - \frac{l_{blocked}}{l_{facet}}$$

$$G_{masking} = \frac{2(\bar{n}\cdot\bar{h})(\bar{n}\cdot\bar{v})}{\bar{v}\cdot\bar{h}}$$

$$G_{shadowing} = \frac{2(\bar{n}\cdot\bar{h})(\bar{n}\cdot\bar{l})}{\bar{v}\cdot\bar{h}}$$

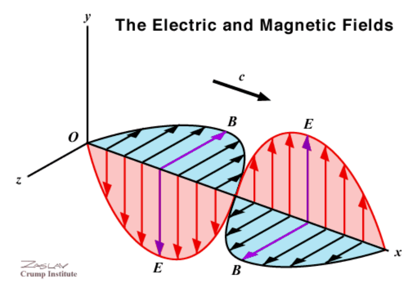
$$G = \min\{1, G_{masking}, G_{shadowing}\}$$

The geometric factor chooses the smallest amount of light that is lost as the local self-shadowing model.

Fresnel Reflection

The Fresnel term results from a complete analysis of the reflection process while considering light as an electromagnetic wave.

The electric field of light has a magnetic field associated with it (hence the name electromagnetic). The magnetic field is always orthogonal to the electric field and the direction of propagation. Over time the orientation of the electric field may rotate. If the electric field is oriented in a particular *constant* direction it is called *polarized*.



The behavior of reflection depend on how the incoming electric field is oriented relative to the surface at the point where the field makes contact. This variation in reflectance is called the Fresnel effect.

Fresnel Reflection

The Fresnel effect is wavelength dependent. It behavior is determined by the index-of-refraction of the material (taken as a complex value to allow for attenuation). This effect explains the variation in colors seen in specular regions particular on metals (conductors). It also explains why most surfaces approximate mirror reflectors when the light strikes them at a grazing angle.

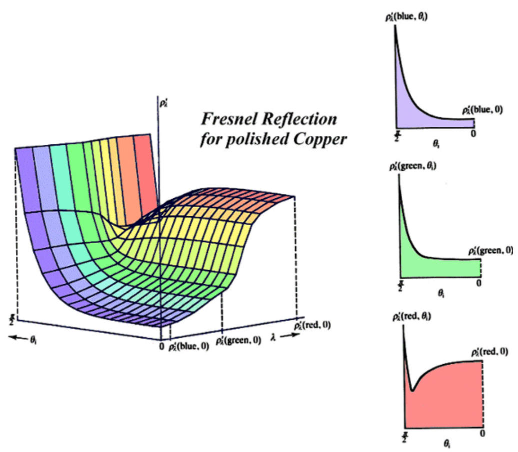
$$F_\lambda(\theta_i) = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1+\frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right)$$

$$c = \cos\theta_i = \bar{l}\cdot\bar{h}$$

$$g = \sqrt{\left(\tfrac{n_i}{n_t}\right)^2 + c^2 - 1}$$

This version of the equation ignores the polarization of the incoming and reflected rays.

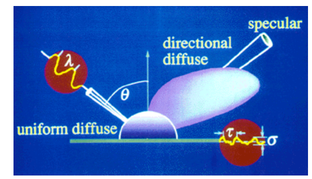
A Plot of the Fresnel Factor



Energy Conserving Approaches

There are still noticable flaws in physically based models.

$$Light_{out} = Light_{emitted} + Light_{in} + Light_{absorbed}$$

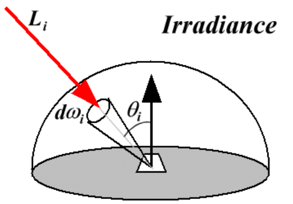


Definitions

- *Radiant flux (W)* - the rate at which light energy is emitted
- *Steradian (sr)* - a unit of solid (3D) angle (there are 4 π steradians in a sphere)
- *Radiant Intensity (W/sr)* - the rate that light energy is radiated through a given solid angle
- *Radiance (W/(sr m^2))* - the rate of energy radiated through a given solid angle as seen reflected from a surface (i.e. the hemisphere is projected onto the surface)
- *Irradiance (W/m^2)* - the rate of incident or incoming energy at a surface point per unit surface area.

Irradiance

The irradiance function is a two dimensional function describing the incoming light energy impinging on a given point.



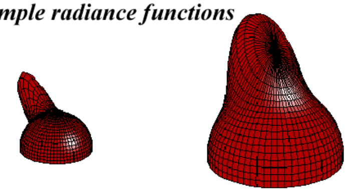
$$E_i = \int_{\Omega_i} L_i \cos\theta_i d\omega_i$$

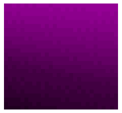
What does Irradiance look like?

Radiance

Radiance is a two dimensional function representing the light reflected from a surface.

Simple radiance functions

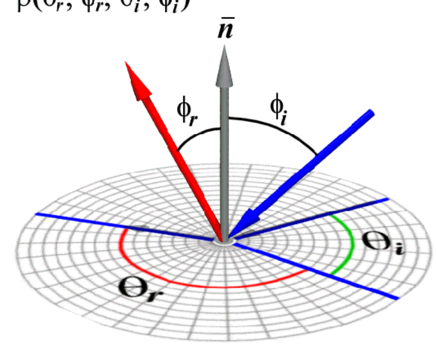




Combining these Functions

Bidirectional Reflectance Distribution Function (BRDF)

$\rho(\theta_r, \phi_r, \theta_i, \phi_i)$



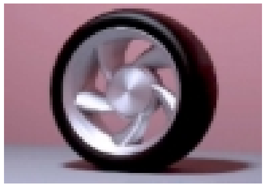
BRDF Approaches

1. Physically-based models
2. Measured BRDFs

Remaining Hard Problems

- Reflective Diffraction Effects
  - thin films
  - feathers of a blue jay
  - oil on water
  - CDs

- Anisotropy
  - brushed metals
  - strands pulled materials
  - Satin and velvet cloths



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