Reading Suggestions: Surfaces and Curvature

REMINDER: The goal of this assignment is to explore how shape analysis can be used in a discipline *you* find interesting. We'll provide a few suggestions for further reading, but do not let these stop you from exploring! The papers we link are not necessarily the best in their respective fields, just places to get you started.

Smooth Theory

The theory of smooth surfaces is incredibly rich, and typically covered in semester-long courses. For a brief introduction see any of these texts:

- Chapter 2 of **Differential Geometry of Curves and Surfaces**, do Carmo, Courier Dover Publications, 2016
- Chapter 2 of **Curves and Surfaces**, Montiel and Ros, AMS, 2009 (second edition)
- Chapter 2 of **A Quick and Dirty Introduction to Discrete Differential Geometry**, Crane
- Chapters 3 and 5 of **Discrete Differential Geometry: An Applied Introduction**, Grinspun et al., SIGGRAPH 2006
- *Historical*: **General Investigations of Curved Surfaces**, Gauss, Typis Dieterichianis, 1827.

Data Structures

In class, we discussed a few data structures for representing manifold meshes. This was a very active field of research in the early days of computer graphics and computer-aided geometric design, and many of these data structures are still in use today in CAD software and packages such as CGAL. Here are a few representative papers:

- *Winged Edge*: **Winged Edge Polyhedron Representation**, Baumgart, 1972
- *Half Edge*: **Finding the Intersection of Two Convex Polyhedra**, Muller and Preparata, Theoretical Computer Science, 1977
- *Quad Edge*: **Primitives for the Manipulation of General Subdivisions and Computation of Voronoi Diagrams**, Guibas and Stolfi, ACM Transactions on Graphics, 1985
- *Directed Edge*: **Directed Edges: A Scalable Representation for Triangle Meshes**, Campagna, Kobbelt, and Seidel, Journal of Graphics, GPU, and Game Tools, 1998.
- *Volume meshes*: **OpenVolumeMesh - A Versatile Index-Based Data Structure for 3D Polytopal Complexes**, Kremer, Bommes, and Kobbelt.

Programming these data structures in a robust way is a delicate process. See this paper for one way to tackle this problem:

- **Using Generic Programming for Designing a Data Structure for Polyhedral Surfaces**, Kettner, Computational Geometry, 1999.

These data structures are designed for manifold meshes, and typically assume that you want access and control over all aspects of your mesh. For data structures designed for non-manifold meshes, see this paper:

- **Data Structures for Simplicial Complexes: An Analysis and a Comparison**, Eurographics Symposium on Geometry Processing, 2005.

Outside of CAD, non-manifold meshes also appear in fluid simulation:

- **Codimensional Surface Tension Flow on Simplicial Complexes**, Zhu et al., SIGGRAPH, 2014.

For a data structure that trades off low memory consumption with expressive power, see the following paper:

- **Succinct Representations of Planar Maps**, Aleardi, Devillers, and Schaeffer, Theoretical Computer Science, 2008.

Discrete Curvature and Applications

Curvature plays a central role in almost all geometry processing applications. These range from surface restoration, to parameterization, and fairing. Here are just a few of these applications:

- **Vertex normals and face curvatures of triangle meshes**, Sun et al., Advances in Discrete Differential Geometry, 2016
- **Discrete Willmore Flow**, Bobenko and Schroder, Eurographics, 2005.
- Mean Curvature Flow: **Implicit Fairing of Irregular Meshes Using Diffusion and Curvature Flow**, Desbrun et al., SIGGRAPH, 1999
- **Discrete Fairing**, Kobbelt, IMA Conference on the Mathematics of Surfaces, 1997
- **Anisotropic Diffusion of Surfaces and Functions on Surfaces**, Bajaj and Xu, ACM Transactions on Graphics, 2003
- **Curvature-Domain Shape Processing**, Eigensatz et al., Eurographics, 2009
- **Consistent Normal Orientation for Polygonal Meshes**, Borodin, Zachmann, and Klein, Proc. of Computer Graphics International, 2004
- **Conformal Flattening by Curvature Prescription and Metric Scaling**, Ben-Chen, Gotsman, and Bunin, Eurographics, 2008
- **Flexible Developable Surfaces**, Solomon et al., SGP, 2012
- **Curvatures of Smooth and Discrete Surfaces**, Sullivan, in *Discrete Differential Geometry*, 2008

Mathematicians have put serious effort into extending the definition of curvature to other domains, many of which are discrete. These often purely discrete notions of curvature have brought insight to challenging mathematical problems, and some recent research extracts practical applications from this work:

- **A visual introduction to Riemannian curvatures and some discrete generalizations**, Ollivier, 2013
- **Curvature notions on graphs**, Hua and Lin, Frontiers of Mathematics in China, 2016
- **Lecture notes on Geometric Group Theory**, Sisto, 2014

Learning on Geometric Data

Learning on 3D data is a very active field of research: The question of how to design a deep learning model that acts on a piece of geometry is largely unsolved. This paper gives a high level overview of the state of the field:

- **Geometric Deep Learning: Going Beyond Euclidean Data**, Bronstein et al., IEEE Signal Processing Magazine, 2017

A key challenge in geometric deep learning is the choice of shape representation; while most image processing techniques operate on pixel grids, there is no consensus regarding the best way to "communicate" a shape to a deep network. This is an extremely active research field with new ideas being proposed almost daily. The following papers present some approaches to learning on different types of geometric data, like point clouds, voxel grids, and graphs:

- **PointNet: Deep Learning on Point Sets for 3D Classification and Segmentation**, Qi and Su et al., CVPR, 2017
- **Geometric Deep Learning on Graphs and Manifolds using Mixture Model CNNs**, Monti et al., CVPR 2017
- **Frustum PointNets for 3D Object Detection from RGB-D Data**, Qi et al., CVPR, 2018
- **VoxNet: A 3D Convolutional Neural Network for Real-Time Object Recognition**, Maturana and Scherer, IROS, 2015
- **3d ShapeNets: A Deep Representation for Volumetric Shapes**, Wu et al., CVPR, 2015
- **Multi-view convolutional neural networks for 3d shape recognition**, Su et al., CVPR, 2015.
- **Deep learning 3d shape surfaces using geometry images**, Sinha et al., ECCV, 2016.
- **Deep kd-networks for the recognition of 3d point cloud models**, Klokov and Lempitsky, ICCV, 2017
- **Spectral networks and locally connected networks on graphs**, Bruna et al., 2013
- **Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks**, Boscaini et al., Computer Graphics Forum, 2015
- **Convolutional neural networks on surfaces via seamless toric covers**, Maron et al., SIGGRAPH, 2017