Reading Suggestions: Curves

Recall that one possible engagement activity in 6.838 is to read a paper and comment about it; please refer to the course website for instructions. This document contains some pointers to papers that you might find interesting, which are relevant to the current set of lectures.

REMINDER: The goal of this assignment is to explore how shape analysis can be used in a discipline *you* find interesting. We'll provide a few suggestions for further reading, but do not let these stop you from exploring! The papers we link are not necessarily the best in their respective fields, just places to get you started.

Our focus in this class is on computational techniques, but for nearly everything we cover, centuries of research cover the continuous case. While future reading assignments will draw more from the computational/statistical literature, many of the readings below are drawn from other communities that are not necessarily computational.

Here are a few pointers on the mathematical underpinnings of the first few lectures:

- Chapter 1 of **Differential Geometry of Curves and Surfaces**, do Carmo, Courier Dover Publications, 2016
- Chapter 1 of **Curves and Surfaces**, Montiel and Ros, AMS, 2009 (second edition)
- Part I ("Rods") of **Elasticity and Geometry**, Audoly and Pomeau, Oxford University Press, 2010

In class, we discussed different ways to put a frame on every point on a curve. These papers provide some details:

- **There is more than one way to frame a curve**, Bishop, The American Mathematical Monthly, 82(3):246-251, 1975
- **The resultant electric moment of complex molecules**, Eyring, Physical Review, 39(4):746-748, 1932

The Frenet–Serret formulas also extend to define frames in higher dimensions via Gram–Schmidt orthogonalization. This (challenging) math paper discusses the most general case:

- **The Curvatures of Regular Curves and Euclidean Invariants of their Derivatives**, ArXiv 1007.2960, 2010

There may be some interesting applications of this machinery in analyzing time series data as curves in high-dimensional space.

In the first homework, you were responsible for implementing part of the **Discrete Elastic Rods** paper. If you are interested in extensions and applications of this work, take a look at any of these papers:

- **Discrete Viscous Threads**, Bergou et al., ACM SIGGRAPH, 2010
- **Unified Simulation of Elastic Rods, Shells, and Solids**, Martin et al., ACM SIGGRAPH, 2010
- **Realistic Hair Simulation: Animation and Rendering**, Bertails et al., SIGGRAPH courses, 2008
- **Position-based elastic rods**, Umetani et al., SCA, 2014
- **Contorting a Heavy and Naturally Curved Elastic Rod**, Lazarus et al., Soft Matter (The Royal Society of Chemistry), 34(9), 2013.
- Nearly any paper by Basile Audoly

Similar dynamics appear in molecule-scale physical systems:

- **Simulation of DNA Supercoil Relaxation**, Ivenso and Lillian, Biophysical Journal, 110(10):2176-2184, 2016
- **Dynamics of DNA Supercoils**, van Loenhout et al., Science, 338(6103): 94–97, 2012
- **Computer Simulation of DNA Double-helix Dynamics**, Levitt, Cold Spring Harbor Symposia on Quantitative Biology, 47: 251-262, 1983
- **An atomistic-continuum Cosserat rod model of carbon nanotubes**, Chandraseker et al., Journal of the Mechanics and Physics of Solids 57(6): 932-958, 2009

Architects use this machinery to simulate truss structures and other networks of flexible rods:

- **Cosserat Nets**, Spillmann et al., TVCG, 15(2): 325-338, 2008
- **Numerical aspects in the dynamic simulation of geometrically exact rods**, Lang and Arnold, Applied Numerical Mathematics, 62(10): 1411-1427, 2012
- **Nonlinear dynamics of elastic rods using the Cosserat theory: Modelling and simulation**, Cao and Tucker, International Journal of Solids and Structures, 45(2): 460-477, 2008
- **Investigation of bend and shear waves in a geometrically exact elastic rod model**, Bishop et al., Journal of Computational Physics, 193(2): 642-665, 2004

Discrete curves also appear in surgical simulation:

- **A robust and real-time vascular intervention simulation based on Kirchhoff elastic rod**, Luo et al., Computerized Medical Imaging and Graphics, 38(8): 735-743, 2014
- **A realistic elastic rod model for real-time simulation of minimally invasive vascular interventions**, Tang et al., The Visual Computer, 26(9): 1157-1165, 2010