Reading 3: Distances and Laplacians

Due March 19, 2019

REMINDER: The goal of this assignment is to explore how shape analysis can be used in a discipline *you* find interesting. We'll provide a few suggestions for further reading, but do not let these stop you from exploring! The papers we link are not necessarily the best in their respective fields, just places to get you started.

Geodesics: Smooth Theory

- Chapter 3 of **Riemannian Geometry**, do Carmo, Birkhauser Boston, 2013.
- Chapter 4 of **Differential Geometry of Curves and Surfaces**, do Carmo, Courier Dover Publications, 2016.

Geodesics: Discretization and Implementation

There are many ways to compute geodesics on discrete surfaces, with some generally better (or more broadly useful) than others. Here are just a few examples:

- **The Discrete Geodesic Problem**, Mitchell, Mount and Papadimitriou, SIAM J. Computing 1987.
- **Computing Geodesic Paths on Manifolds**, Kimmel and Seithan, PNAS 1998.
- **Computing Geodesic Distances on Triangular Meshes**, Novotni and Klein, WSCG 2002.
- **Fast Exact and Approximate Geodesics on Meshes**, Surazhsky et al., SIGGRAPH 2005.
- **Geodesics in Heat: A New Approach to Computing Distance Based on Heat flow**, Crane, Weischedel and Wardetzky, ToG 2013.

Geodesics: Variations and Applications

The following two papers tell you how to compute shortest paths when (1) your mesh is broken, or (2) when you want fast all-pairs queries.

- **Walking on Broken Mesh: Defect-Tolerant Geodesic Distances and Parameterizations**, Campen and Kobbelt, Eurographics 2011.
- **Constant-time All-pairs Geodesic Distance Query on Triangle Meshes**, Xin, Ying and He, I3D 2012.

In class we've talked about surfaces, which are 2-dimensional manifolds. We're going to take a leap of faith and go from 2 dimensions to higher-dimensional (and infinite-dimensional) spaces, with applications including transfer learning and animation:

- **Time-discrete Geodesics in the Space of Shells**, Heeren et al., SGP 2012.
- **Geodesic Flow Kernel for Unsupervised Domain Adaptation**, Gong et al., CVPR 2012.

Geodesics: Learning

In learning applications, we typically do not have access to the manifold representation of our data. One can ask if there is a way to build a representation of the manifold from a point cloud that preserves the metric of the manifold. The following papers show that this is not true of commonly used methods:

- **Shortest Path Distance in Random k-Nearest Neighbor Graphs**, Alamgir and von Luxburg, ICML 2012.
- **Geodesic Exponential Kernels: When Curvature and Linearity Conflict**, Feragen, Lauze and Hauberg, CVPR 2015.

Geodesics are also incorporated into machine learning algorithms that process meshes:

- **Geodesic Convolutional Neural Networks on Riemannian Manifolds**, Masci et al., ICCV 2015.

Sometimes the variables in a machine learning optimization problem live on a manifold rather than in $\mathbb{R}^n$, e.g. the space of rotations $\mathrm{SO}(3)$ or the set of positive (semi-)definite matrices. In this case, for example, the exponential map might be used in place of a gradient step in gradient descent. Some example papers discussing algorithms in this direction include the following:

- **Geometric Optimization in Machine Learning**, Sra and Hosseini, in *Algorithmic Advances in Riemannian Geometry and Applications* (2016).
- **Getting started with Manopt**: `https://www.manopt.org/tutorial.html` (Boumal et al., 2014)
- **Riemann Manifold Langevin and Hamiltonian Monte Carlo**, Girolami and Calderhead, *Journal of the Royal Statistical Society* 73.2, 123–214 (2011).

Laplace Operator

The Laplacian is a fundamental operator in differential geometry. For an overview of what the Laplacian is and how its computed, see the following papers:

- Chapter 6 of **Discrete Differential Geometry: An Applied Introduction**, Crane
- **Laplace-Beltrami: The Swiss Army Knife of Geometry Processing**, Solomon, Crane and Vouga
- **Analysis on Manifolds via the Laplacian**, Canzani, 2013

Laplace-Beltrami: Discretizations

The Laplacian is a smooth operator, and as we've seen several times in class, choosing a discretization can be difficult. Two commonly-used discretizations are presented in the following papers:

- *Uniform Laplacian*: **A Signal Processing Approach to Fair Surface Design**, Taubin, SIGGRAPH 1995

- *Cotangent Laplacian*: **Discrete Differential-Geometry Operators for Triangulated 2-Manifolds**, Meyer et al., Visualization and Mathematics III 2003

We would hope that discrete Laplacians preserve all of the properties of the smooth Laplacian, but this is not the case. In fact, there is a no free lunch theorem here that effectively says that it's *impossible* to come up with a discretization of the Laplacian operator without losing something:

- **Discrete Laplace operators: No free lunch**, Wardetzky et al., SGP 2007.

So far, we've discussed Laplacians on triangle meshes, but the Laplace operator can also be defined for discrete objects that are not triangle meshes.

- **A Discrete Laplace-Beltrami Operator for Simplicial Surfaces**, Bobenko and Springborn, 2005
- **Constructing the Laplace Operator from Point Clouds in $\mathbb{R}^d$**, Belkin, Sun and Wang, SODA 2009
- **Discrete Laplacians on General Polygonal Meshes**, Alexa and Wardetzky, ACM ToG 2011

Applications frequently involve solving a Poisson equation $\Delta\varphi = f$. On a triangle mesh, this is a (very large) linear system of equations. A strategy for solving such problems efficiently is presented in the following paper:

- **Efficient Preconditioning of Laplacian Matrices for Computer Graphics**, Krishnan, Fattal, and Szeliski, ACM ToG 2013.

Laplace-Beltrami: Applications

Laplacians have myriad applications including parameterization, pose transfer, segmentation, reconstruction, simulation, and interpolation. See these papers for a few examples:

- **Implicit Fairing of Irregular Meshes Using Diffusion and Curvature Flow**, Desbrun et al., SIGGRAPH 1999.
- **Laplace-Beltrami Spectra as Shape-DNA on Surfaces and Solids**, Reuter, Wolter and Peinecke, CAD 2006.
- **Laplace-Beltrami Eigenfunctions for Deformation Invariant Shape Representation**, Rustamov, SGP 2007.
- **Multiresolution Analysis of Arbitrary Meshes**, Eck et al., SIGGRAPH 1995.
- **Biharmonic Distance**, Lipman, Rustamov and Funkhouser, ACM ToG 2009.
- **Functional Maps: A Flexible Representation of Maps Between Shapes**, Ovsjanikov et al., SIGGRAPH 2012.
- **Geodesics in heat: A new approach to computing distance based on heat flow**, Crane, Weischedel, and Wardetzky, ACM ToG 2013.
- **Accelerated Quadratic Proxy for Geometric Optimization**, Kovalsky, Galun, and Lipman, SIGGRAPH 2016.