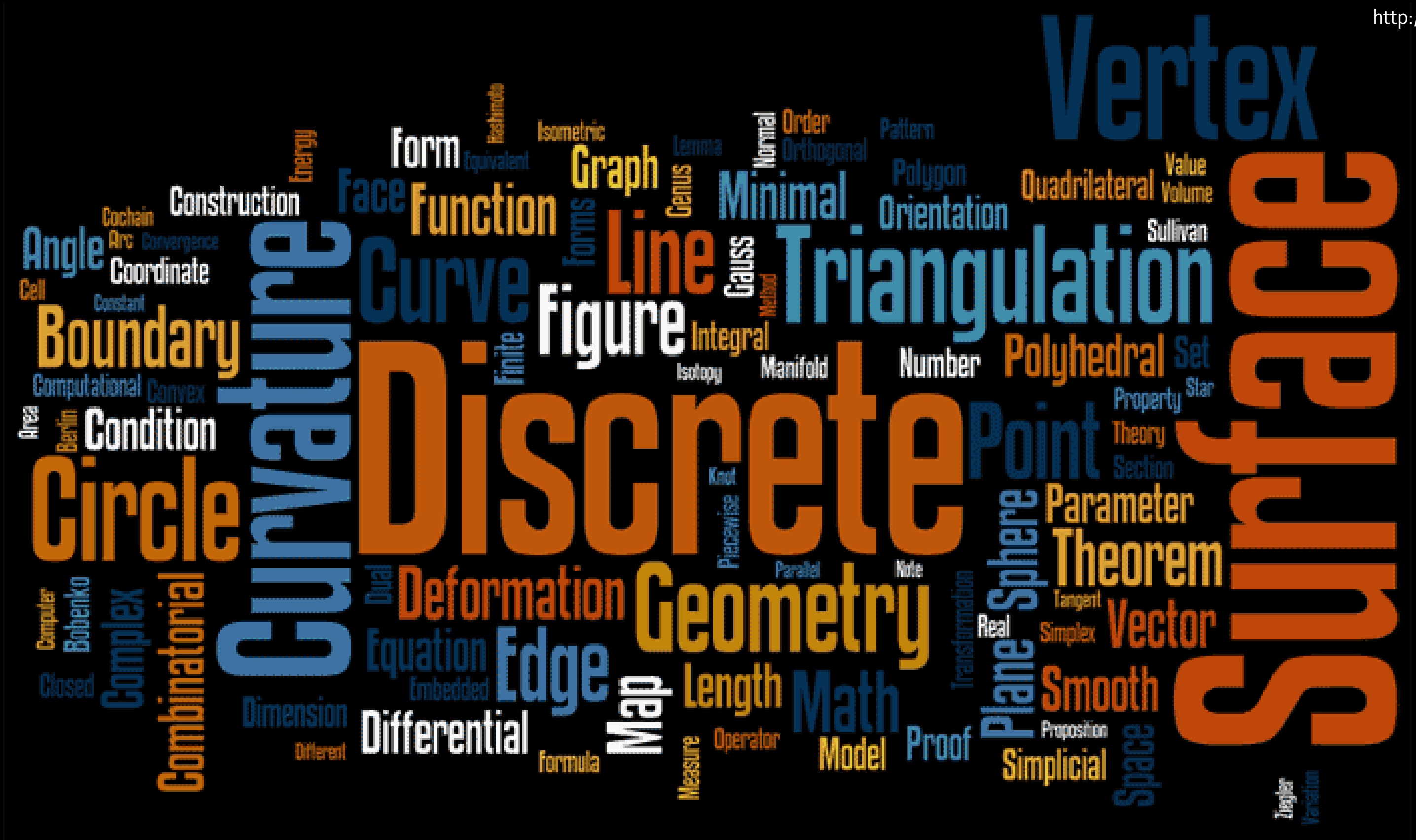
http://www.gogeometry.com



6.838: Shape Analysis

Justin Solomon

MIT, Spring 2019

MIT EECS

<administrative>

Course Instructor

Instructor: Justin Solomon
Email: jsolomon@mit.edu
Office: 32-D460
Office hours: Wednesdays, 10am-12pm

Geometric Data Processing Group:
http://gdp.csail.mit.edu

TA

TA: Sebastian Claici
Email: **s**claici@mit.edu
Office: 32-D475a
Office hours: Mondays/Wednesdays, 3pm-4pm

On the Web

`gdp.csail.mit.edu/6838_spring_2019.html`

\+ piazza

Register ASAP!

Deliverables

1. **Four homeworks (40%)**
   Written + coding
2. **One project (40%)**
   Instructions already online
3. **Biweekly nanoquizzes (10%)**
   Designed to be easy!
4. **Biweekly readings (10%)**
   New experiment: Paragraph summary + ?’s

Late Days

Three days.

- After: Lose 25% / day
- Cannot use for project
- S^3 must approve exceptions

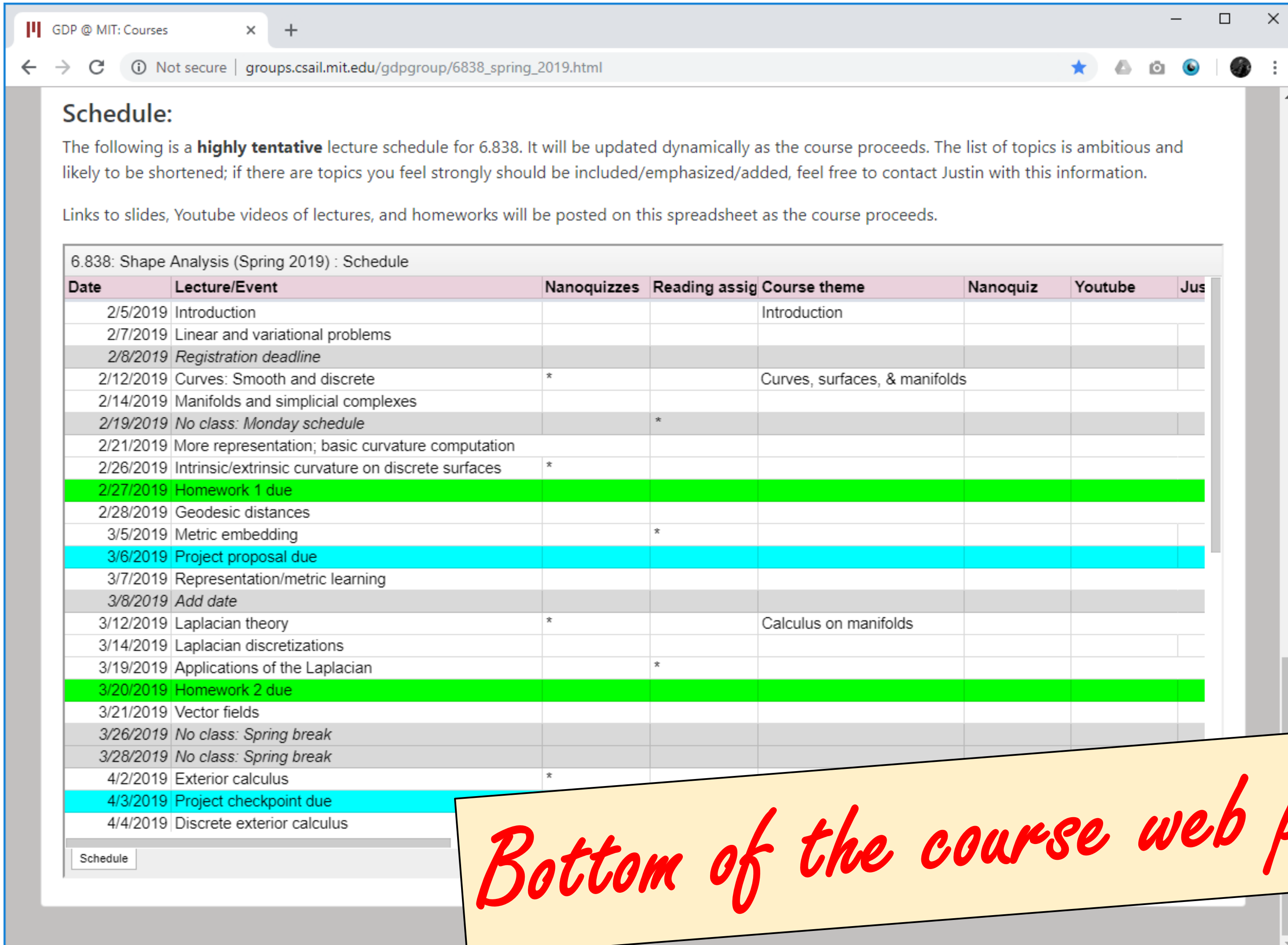
Course Project

- Instructions on course website
- Individual or groups of two
- **Implement** and **extend** a relevant technique

- Milestones:
  - Proposal (500 words)
  - Checkpoint (≤2 pages)
  - Writeup (6-10 pages)
  - Presentation (8-10 minutes)

Start early:
We will help you!

Schedule



GDP @ MIT: Courses

Not secure | groups.csail.mit.edu/gdpgroup/6838_spring_2019.html

Schedule:

The following is a **highly tentative** lecture schedule for 6.838. It will be updated dynamically as the course proceeds. The list of topics is ambitious and likely to be shortened; if there are topics you feel strongly should be included/emphasized/added, feel free to contact Justin with this information.

Links to slides, Youtube videos of lectures, and homeworks will be posted on this spreadsheet as the course proceeds.

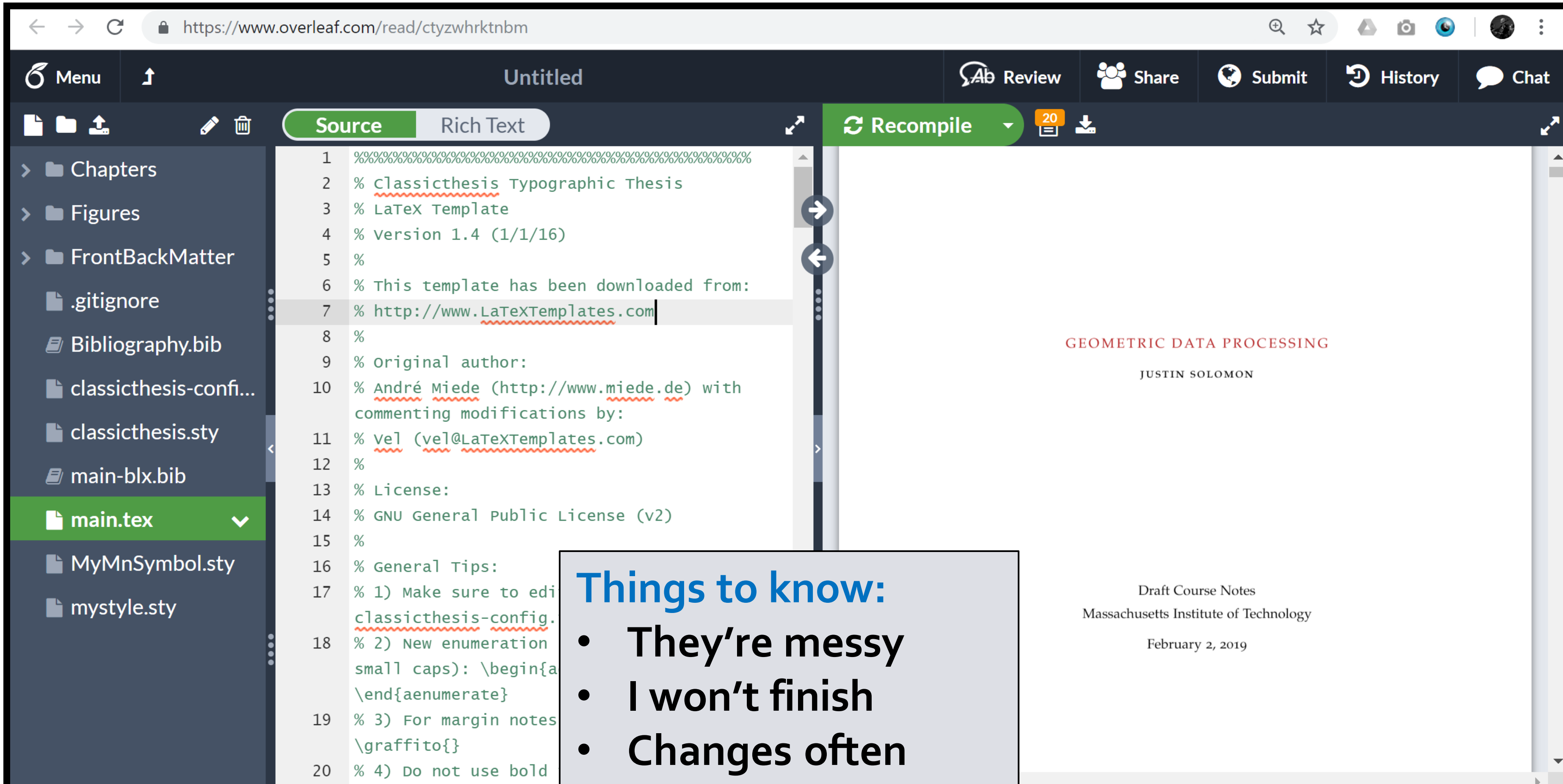
6.838: Shape Analysis (Spring 2019) : Schedule

Date	Lecture/Event	Nanoquizzes	Reading assig	Course theme	Nanoquiz	Youtube	Jus
2/5/2019	Introduction			Introduction			
2/7/2019	Linear and variational problems						
2/8/2019	*Registration deadline*						
2/12/2019	Curves: Smooth and discrete	*		Curves, surfaces, & manifolds			
2/14/2019	Manifolds and simplicial complexes						
2/19/2019	*No class: Monday schedule*		*				
2/21/2019	More representation; basic curvature computation						
2/26/2019	Intrinsic/extrinsic curvature on discrete surfaces	*					
2/27/2019	Homework 1 due						
2/28/2019	Geodesic distances						
3/5/2019	Metric embedding		*				
3/6/2019	Project proposal due						
3/7/2019	Representation/metric learning						
3/8/2019	*Add date*						
3/12/2019	Laplacian theory	*		Calculus on manifolds			
3/14/2019	Laplacian discretizations						
3/19/2019	Applications of the Laplacian		*				
3/20/2019	Homework 2 due						
3/21/2019	Vector fields						
3/26/2019	*No class: Spring break*						
3/28/2019	*No class: Spring break*						
4/2/2019	Exterior calculus	*					
4/3/2019	Project checkpoint due						
4/4/2019	Discrete exterior calculus						

Schedule

Bottom of the course web page!

Experiment: Course Notes



Things to know:

- They're messy
- I won't finish
- Changes often
- Typos & comments encouraged

Prerequisites

- **Coding**
  Julia or Matlab preferred
- **Math**
  Fluency in linear algebra and multivariable calculus
- **Not required (won't hurt):**
  Graphics, differential geometry, numerics, ML

New-ish Course

- Counts as a TQE (AI)
- Schedule is too ambitious!
- Contact Justin with suggestions, must-cover topics, questions, etc.
- Video on Youtube
  (unreliable!)

Philosophy

I <u>want</u> you to take this course!

Assignments intended to be <u>interesting</u>

(may be unintentionally easy/hard!)

Will be generous with support/grading

Quick Survey

Degree

Undergraduate

M.Eng.

M.Sc./PhD

Quick Survey

Background

EECS

Math

Engineering

Elsewhere

</administrative>

Theme

1. $\underbrace{\textit{Geometric data}}_{\text{Modifier}}\ \underbrace{\textit{analysis:}}_{\text{Noun}}$ The analysis of geometric data

2. $\underbrace{\textit{Geometric}}_{\text{Modifier}}\ \underbrace{\textit{data analysis:}}_{\text{Noun}}$ Data analysis using geometric techniques

Applied Geometry

I. Theoretical toolbox

II. Computational toolbox

III. Application areas

Mostly a picture book!

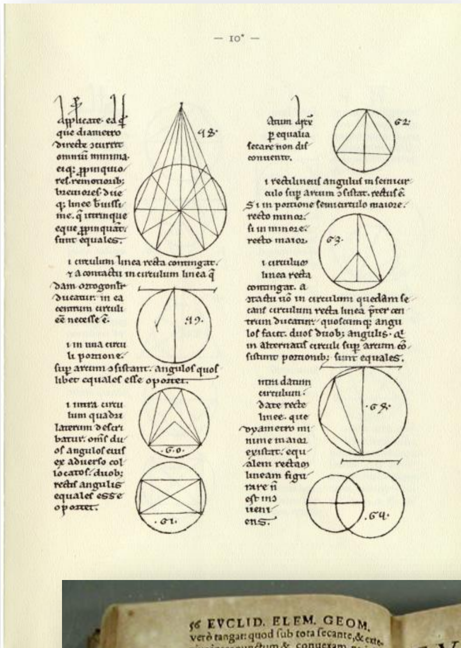
Applied Geometry

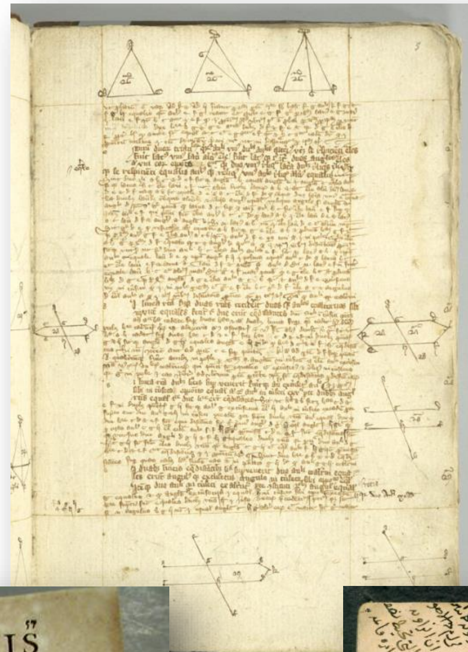
I. Theoretical toolbox

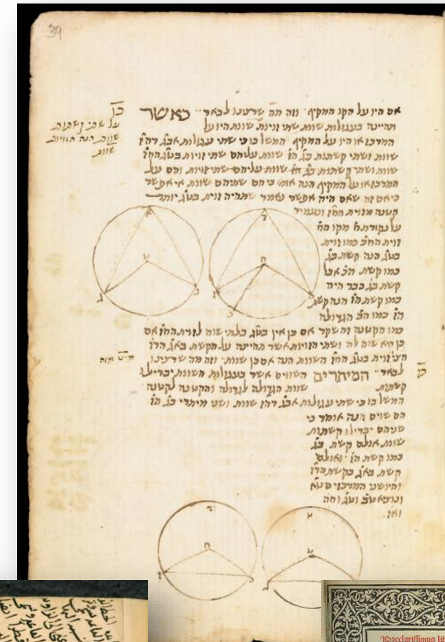
II. Computational toolbox

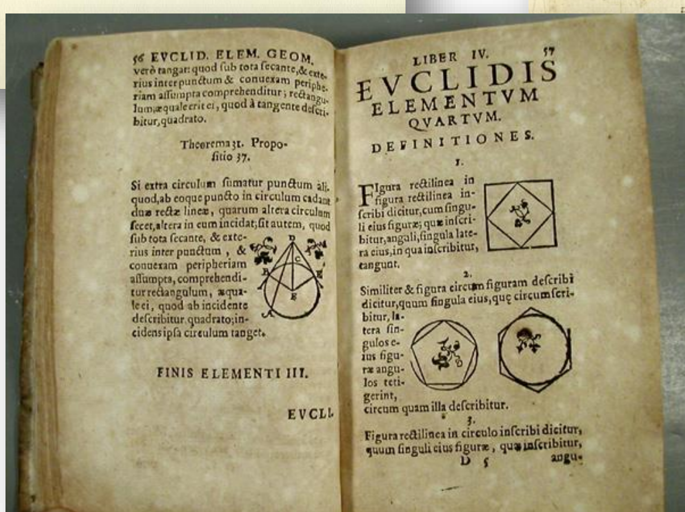
III. Application areas

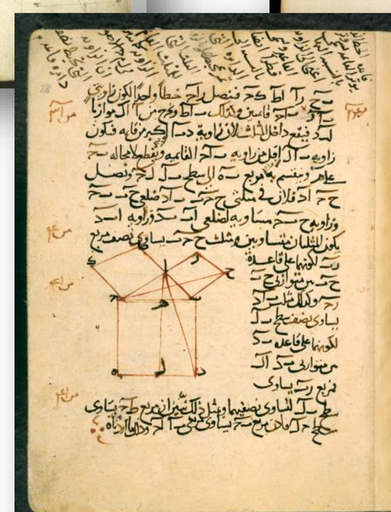
Euclidean Geometry

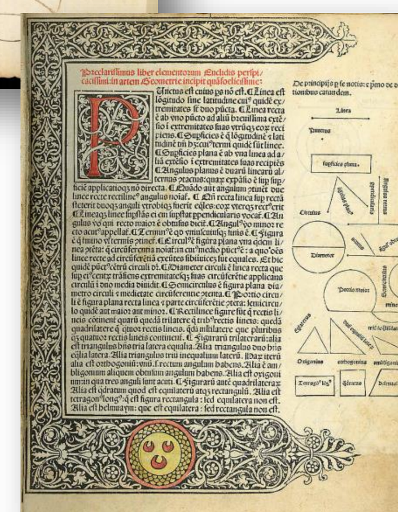






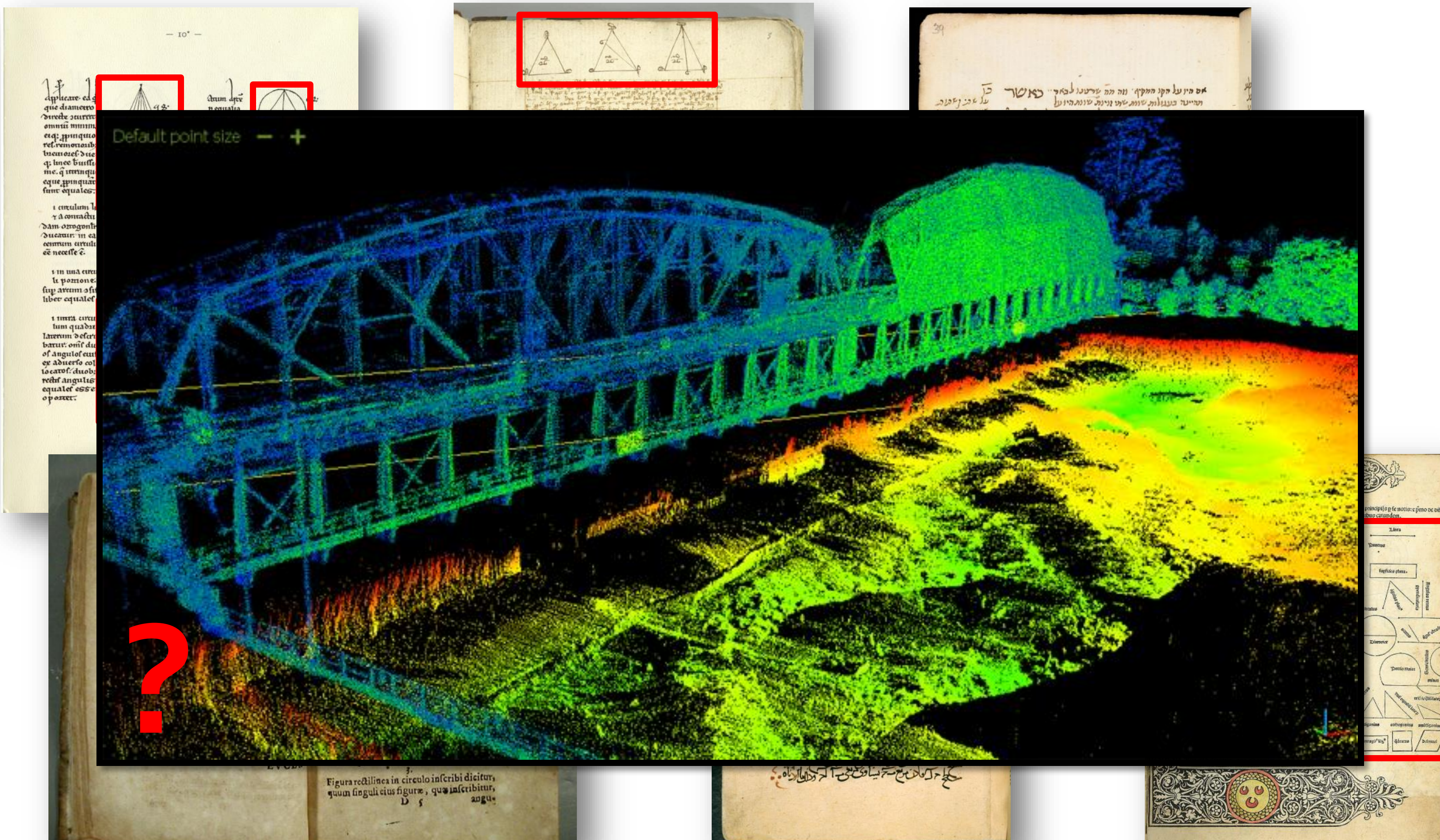






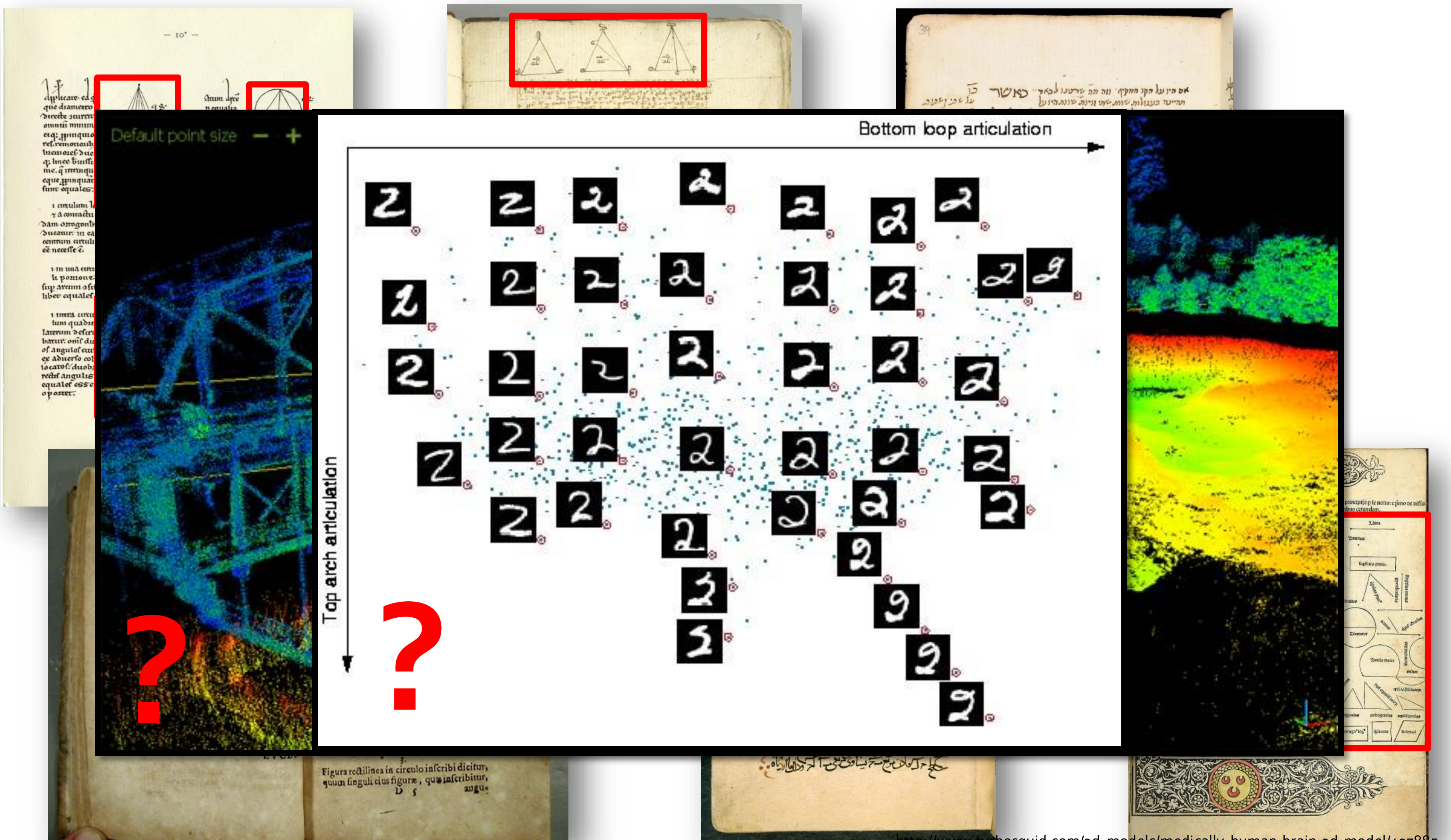
Euclidean Geometry

Euclidean Geometry



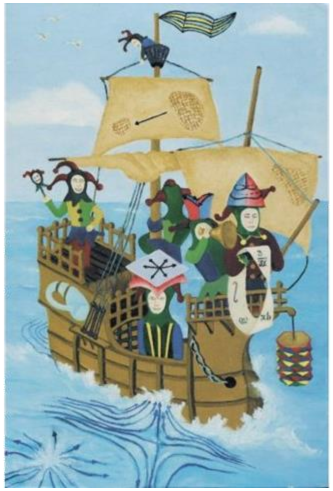
http://www.turbosquid.com/3d-models/medically-human-brain-3d-model/437885

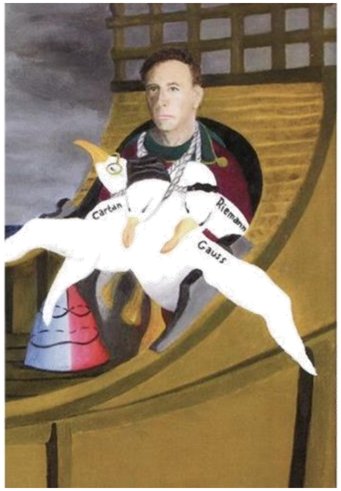
Euclidean Geometry

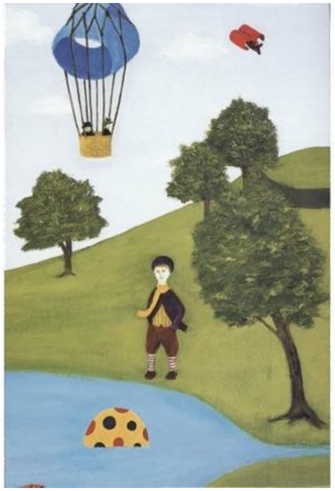


http://www.turbosquid.com/3d-models/medically-human-brain-3d-model/437885

Differential Geometry

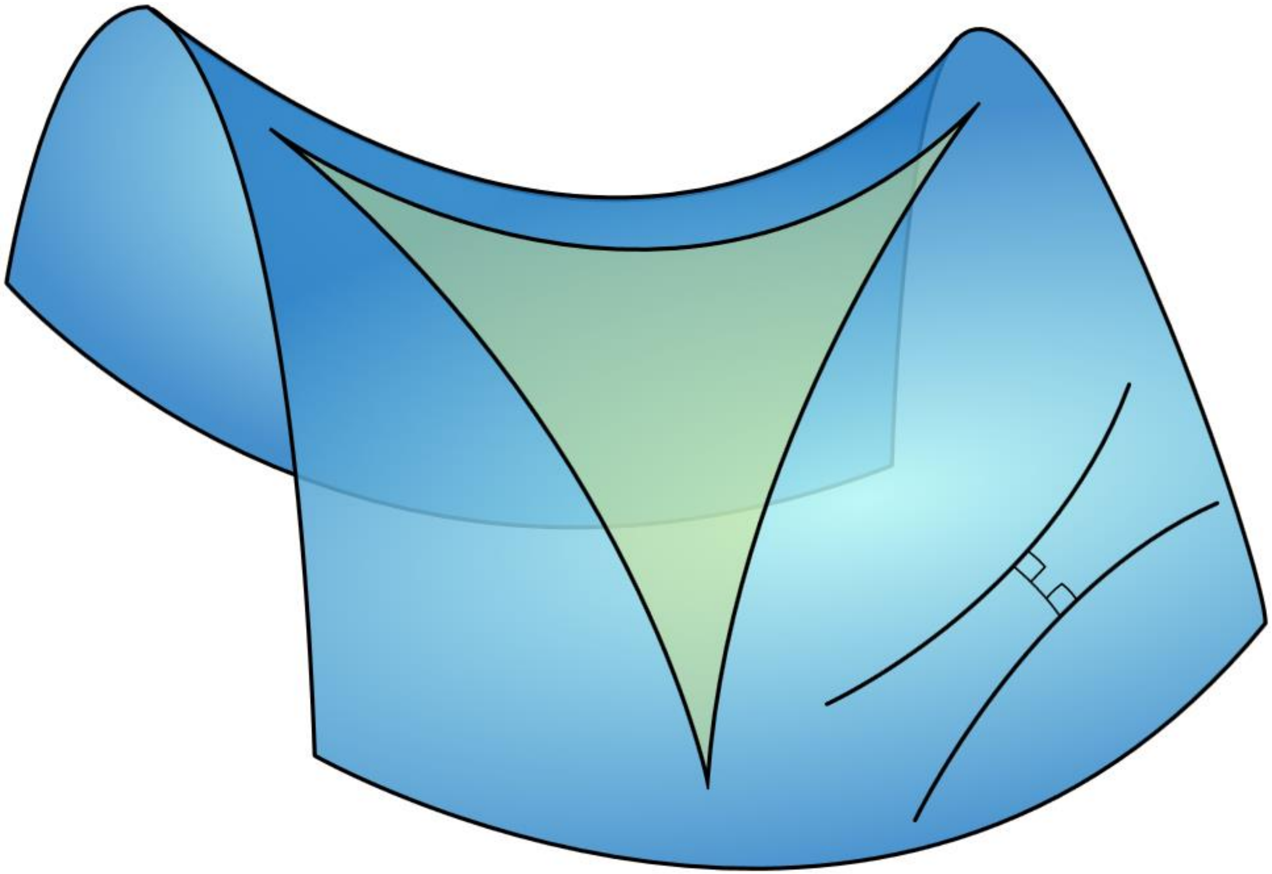






Spivak: *A Comprehensive Introduction to Differential Geometry*

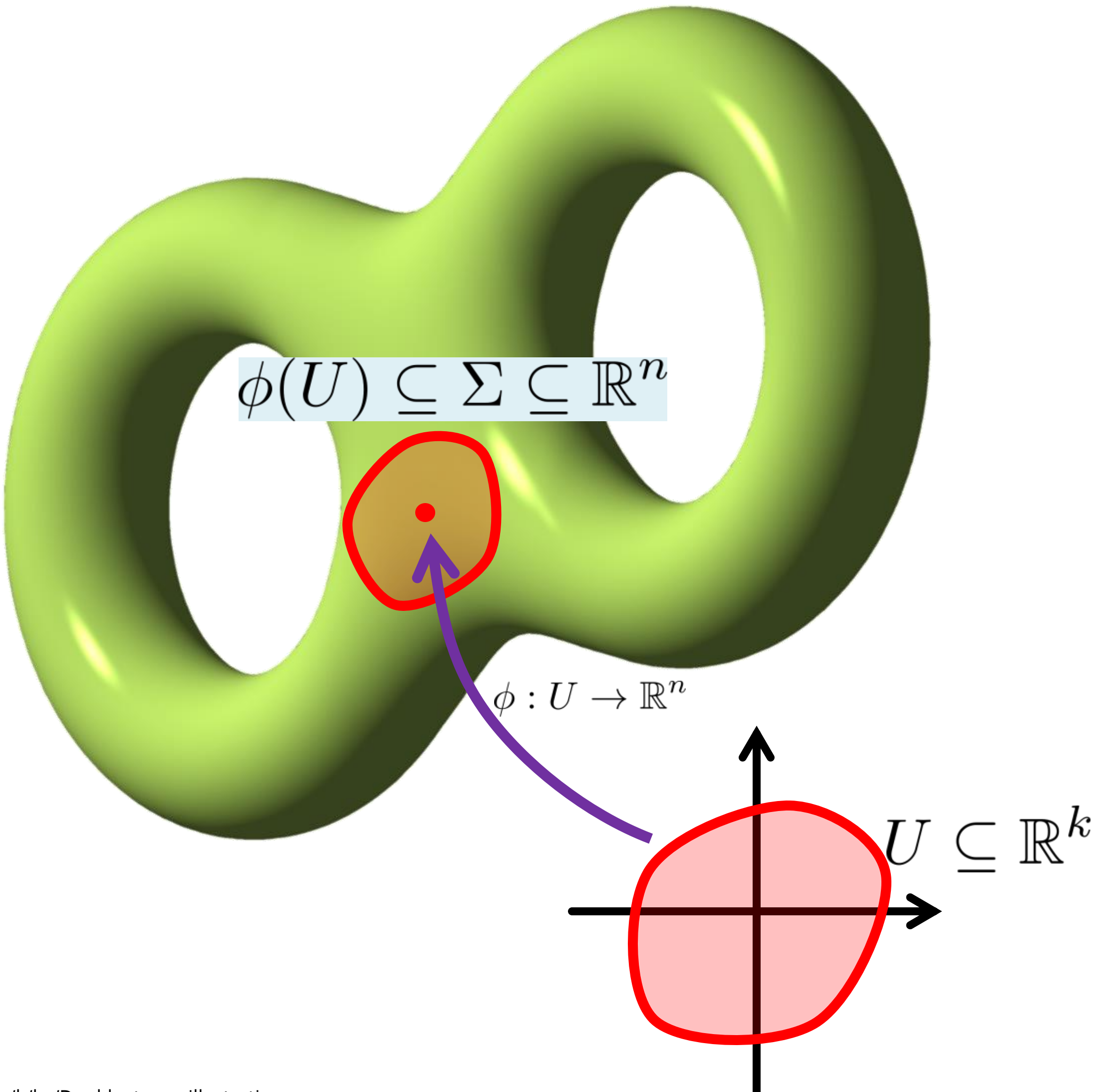
Differential Geometry



http://en.wikipedia.org/wiki/Differential_geometry

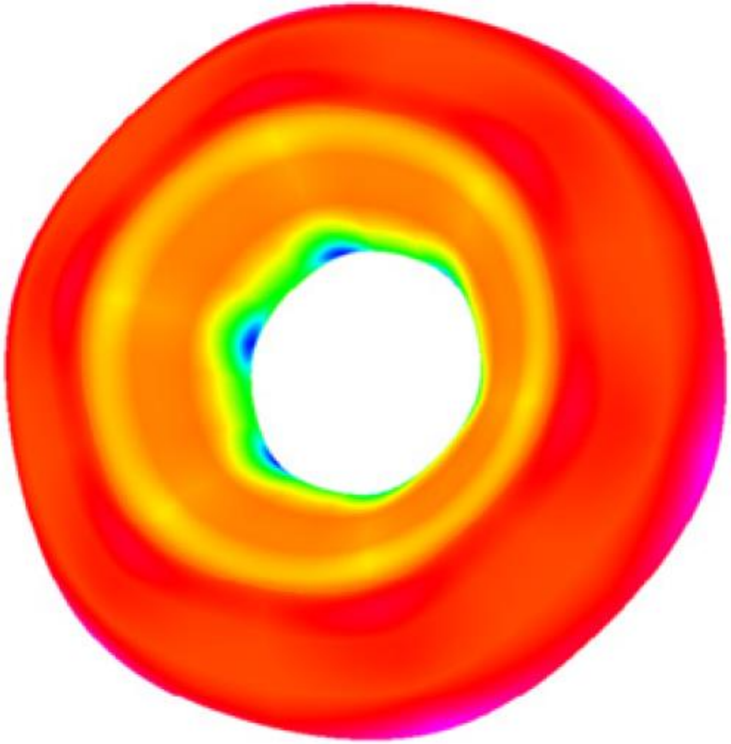
Study of smooth manifolds

Manifold

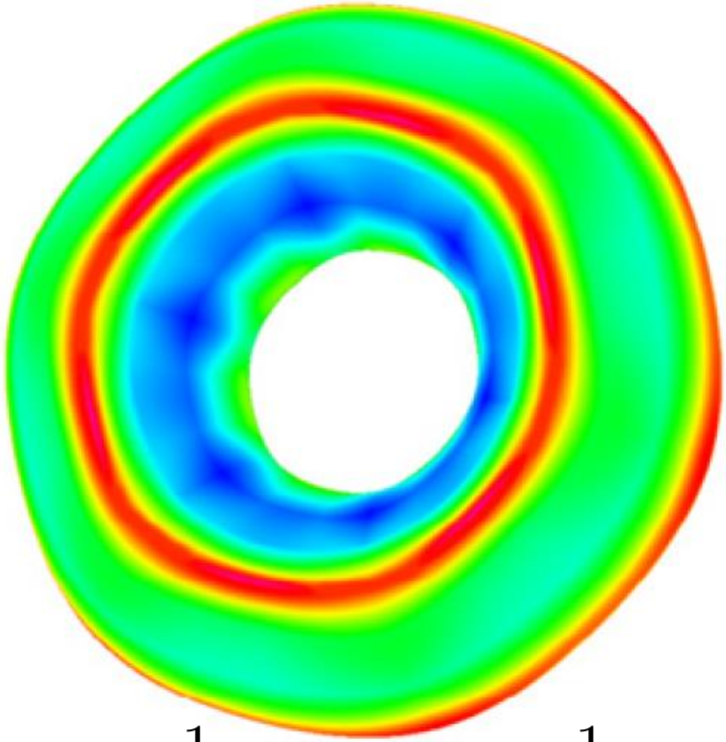


http://upload.wikimedia.org/wikipedia/commons/b/bc/Double_torus_illustration.png

Differential Geometry Toolbox



$$K := \kappa_1 \kappa_2 = \det \mathrm{I\!I}$$

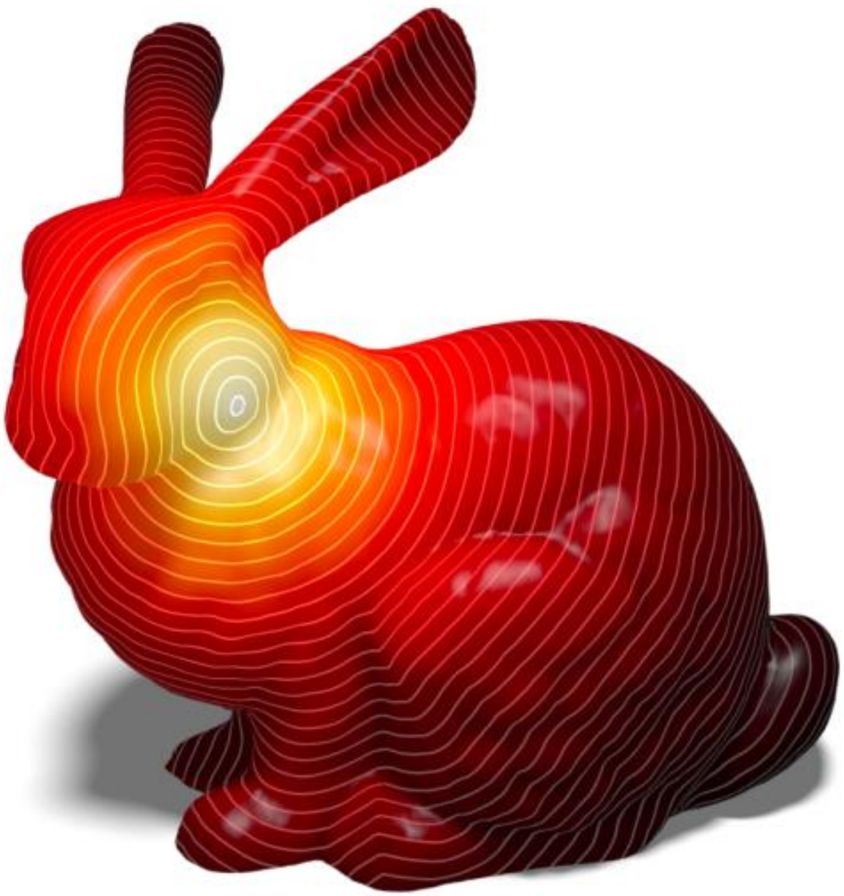


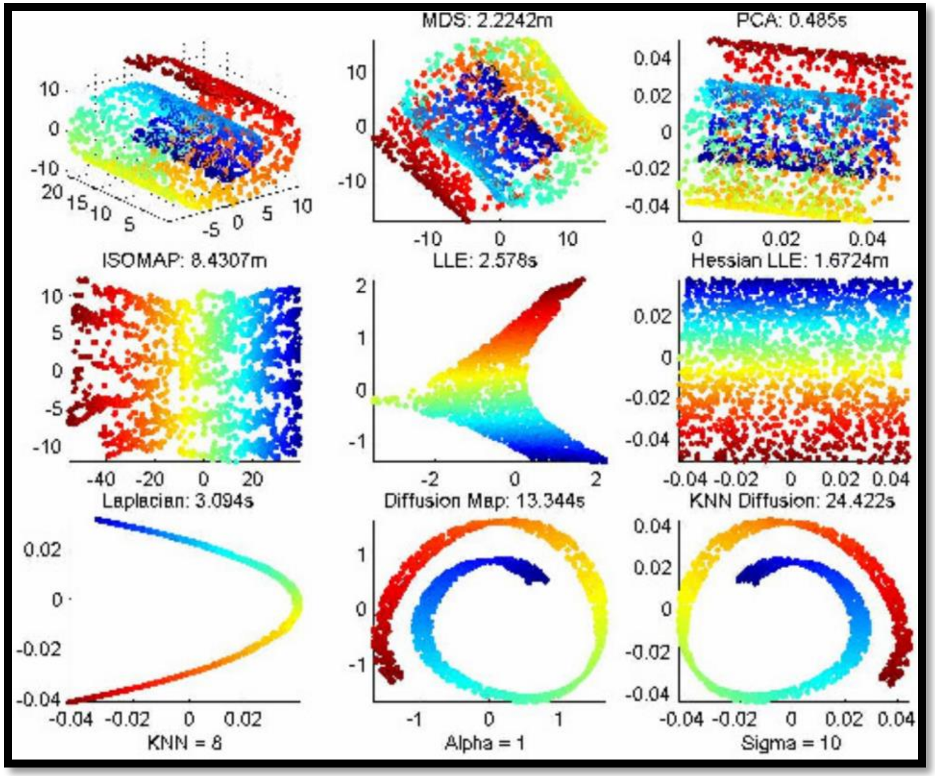
$$H := \frac{1}{2}(\kappa_1 + \kappa_2) = \frac{1}{2}\mathrm{tr}\,\mathrm{I\!I}$$

http://www.sciencedirect.com/science/article/pii/S0010448510001983

Curvature and shape properties

Differential Geometry Toolbox

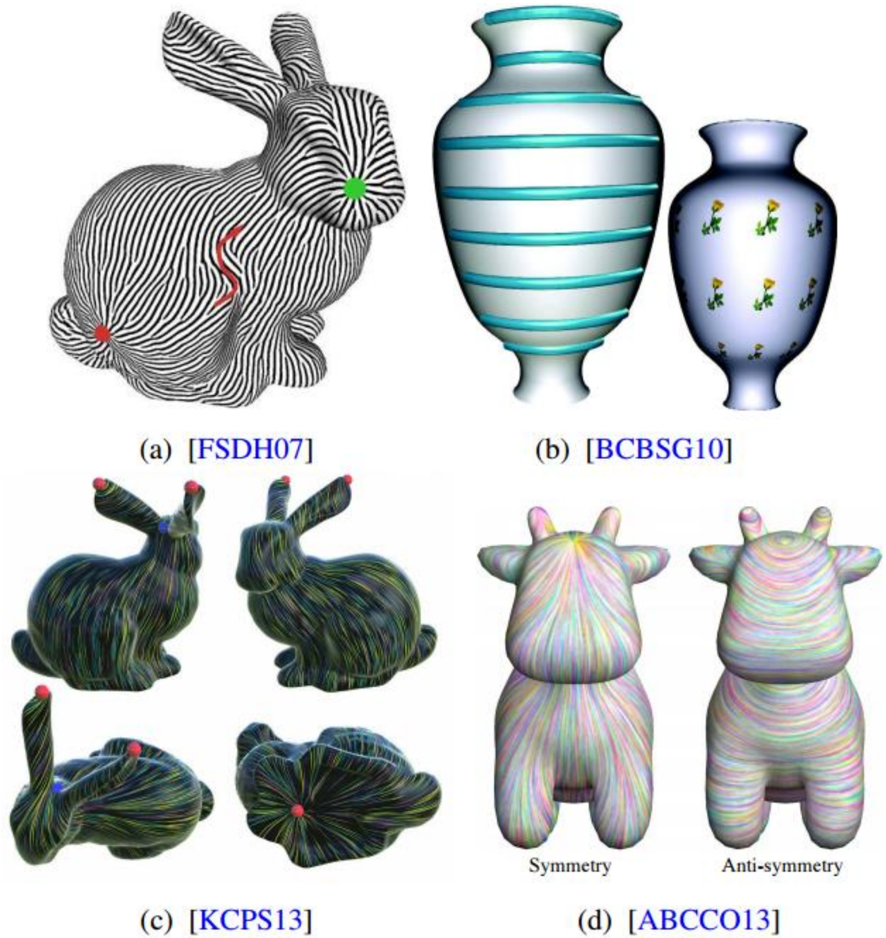




Crane, Weischedel, Wardetzky. *Geodesics in heat*. TOG 2013.
Wittman. Manifold learning techniques.

Distances

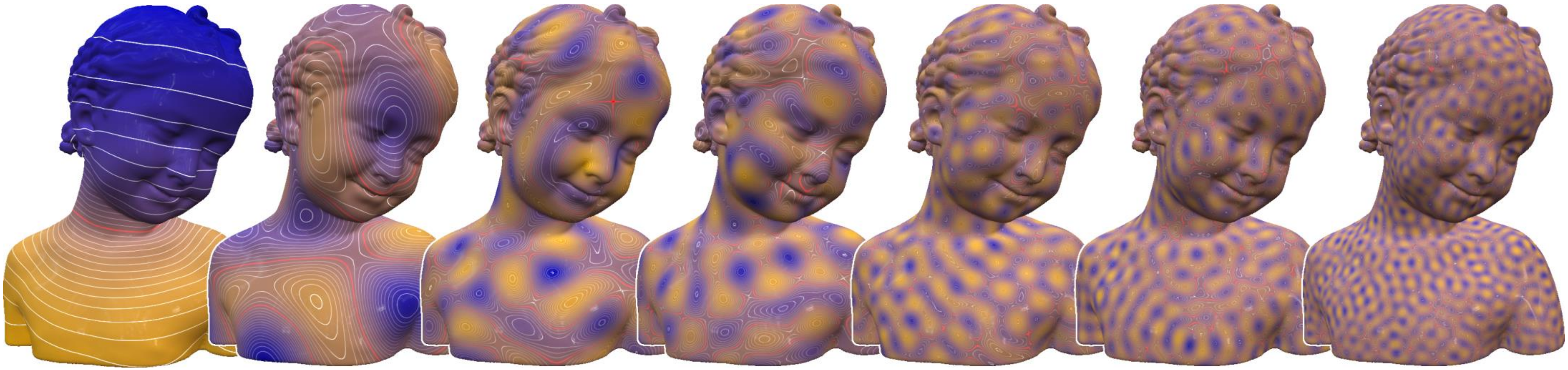
Differential Geometry Toolbox



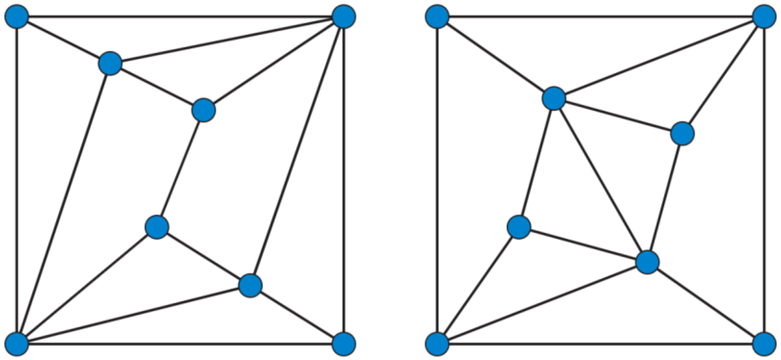
Vaxman et al.
Directional field synthesis, design, and processing.
EG STAR 2016.

Flows and vector fields

Differential Geometry Toolbox



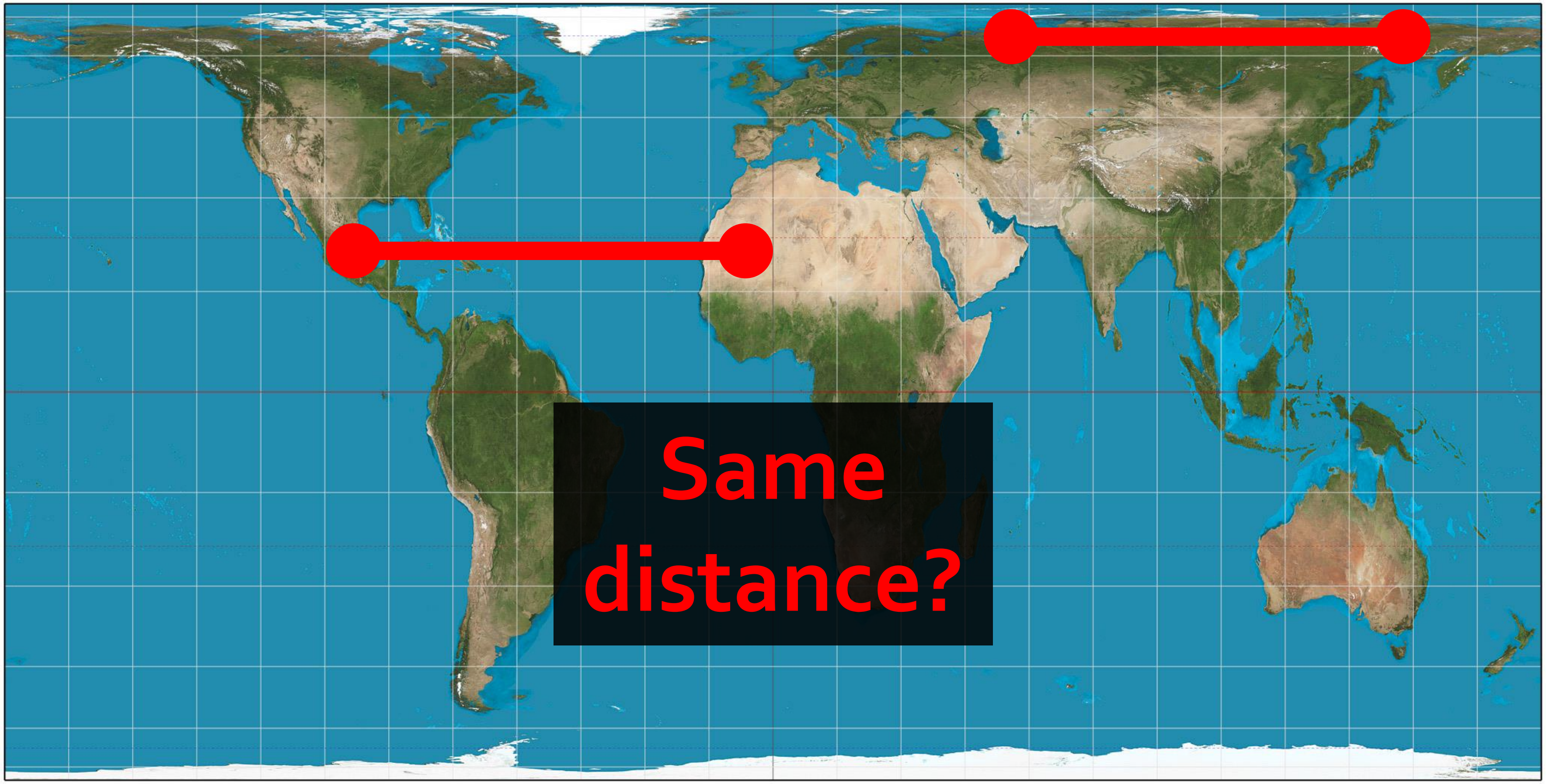
Vallet and Lévy. *Spectral Geometry Processing with Manifold Harmonics*. EG 2008



"Enneahedra"

Differential operators

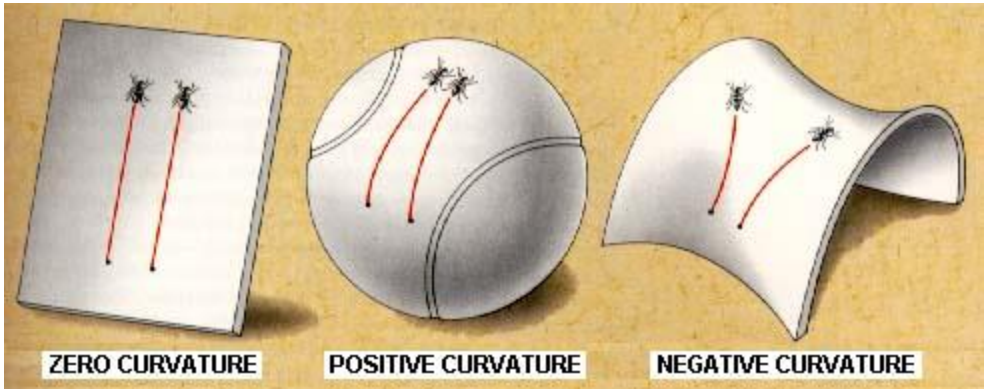
Riemannian Viewpoint

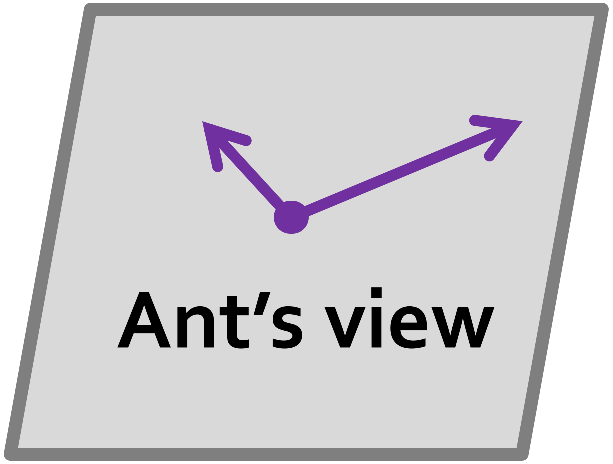


http://upload.wikimedia.org/wikipedia/commons/2/2c/Hobo%E2%80%93Dyer_projection_SW.jpg

Only need angles and distances

Riemannian Viewpoint

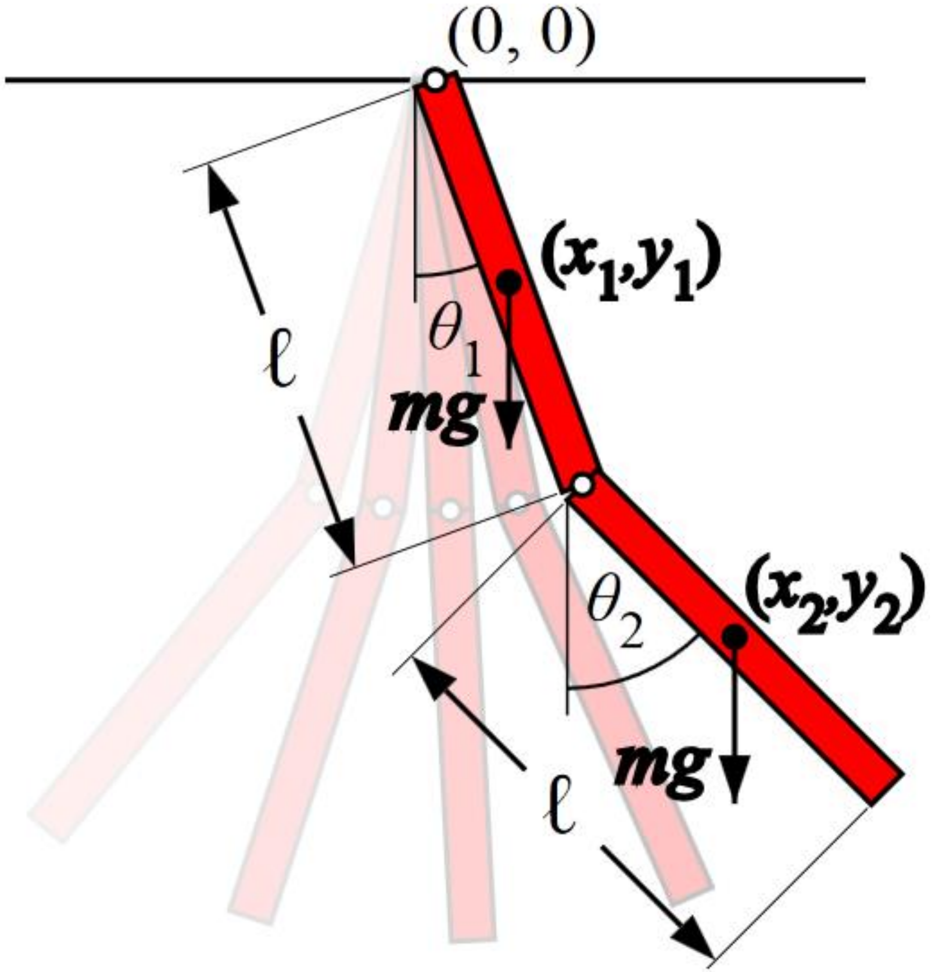


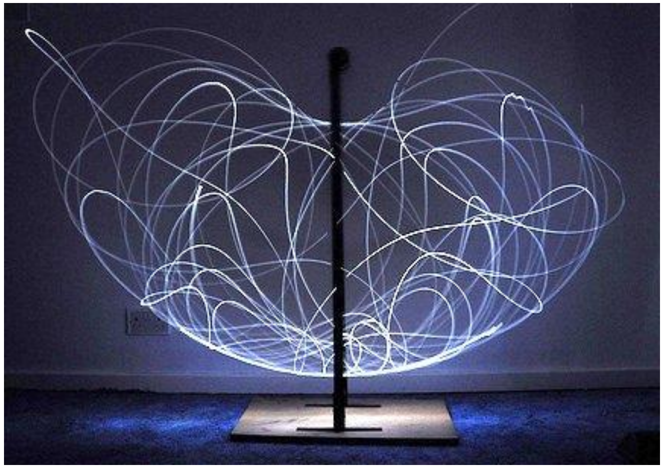


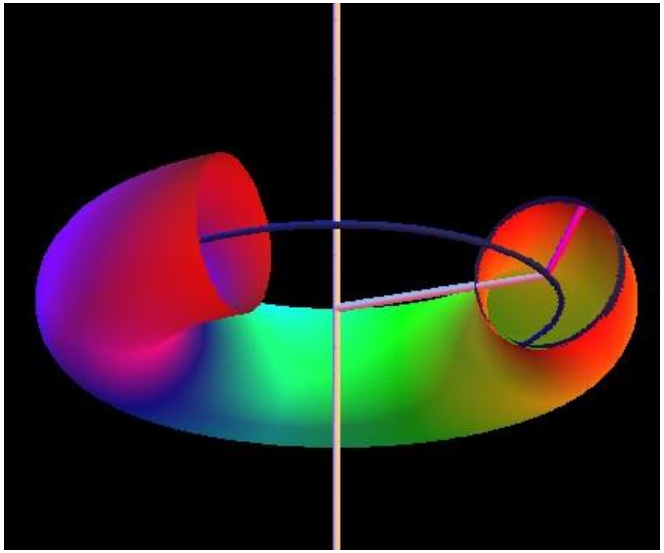
http://www.phy.syr.edu/courses/modules/LIGHTCONE/pics/curved.jpg

Only need angles and distances

Geometric Mechanics

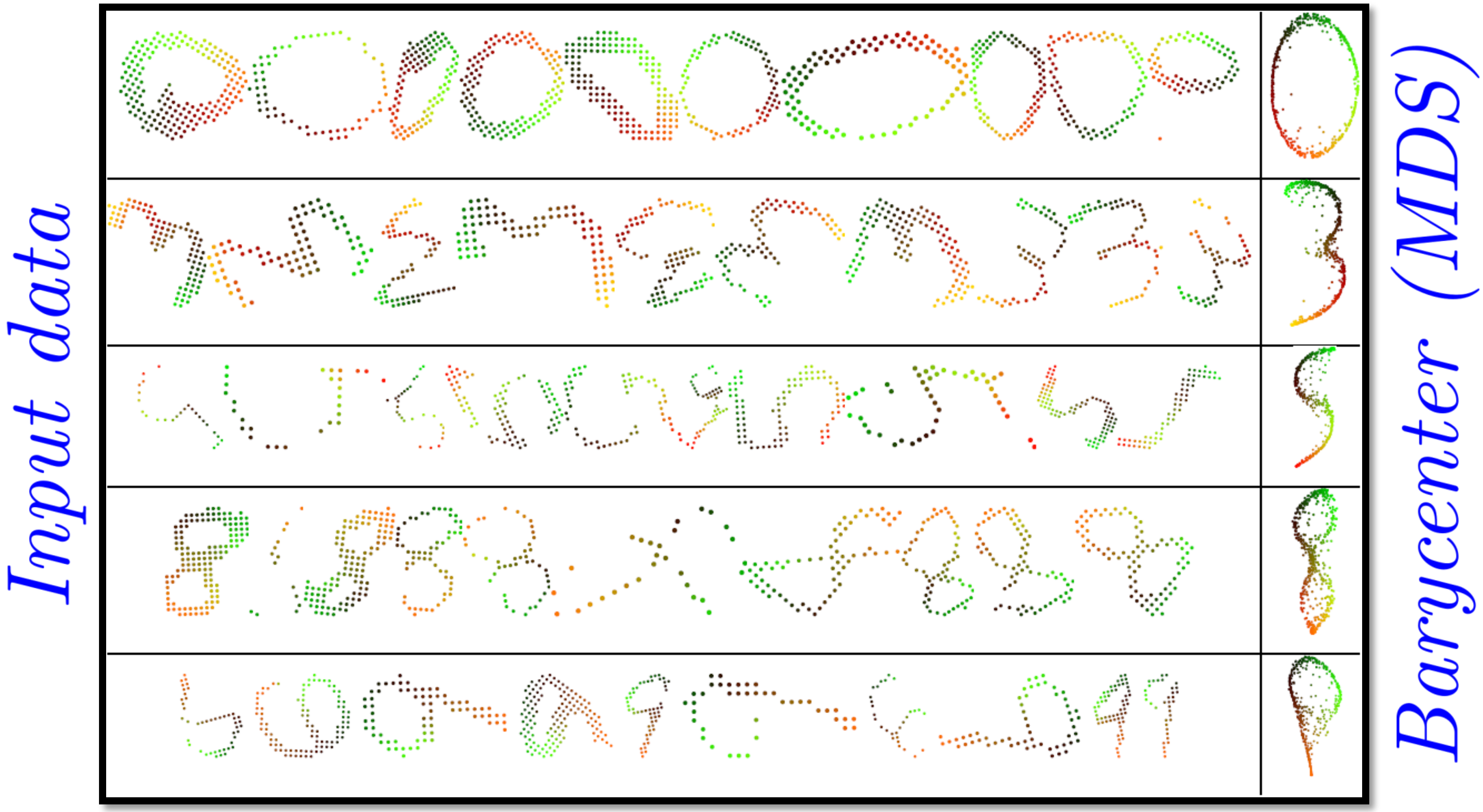






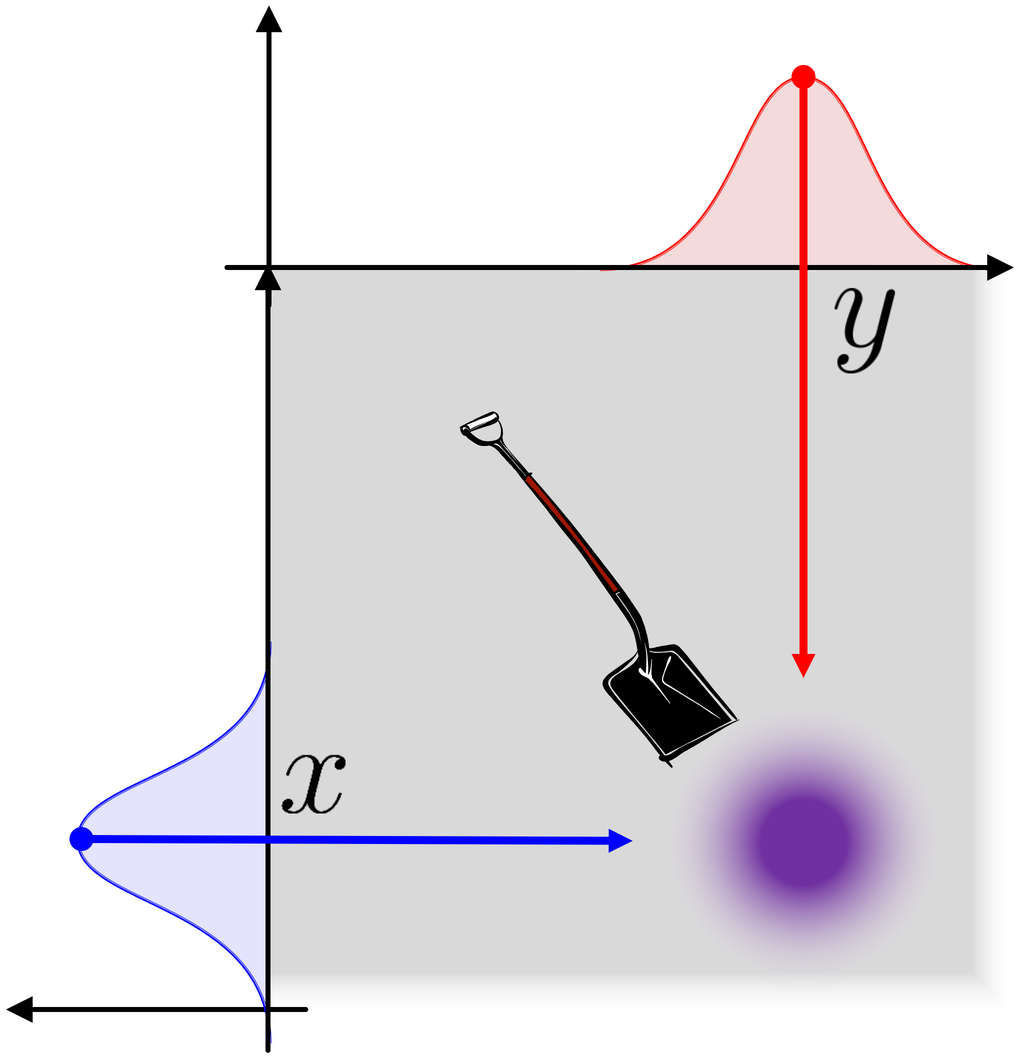
http://en.wikipedia.org/wiki/Double_pendulum
http://www.ualberta.ca/dept/math/gauss/fcm/BscIdeas/SpcDmnsn/pndlm2.htm

Metric Geometry

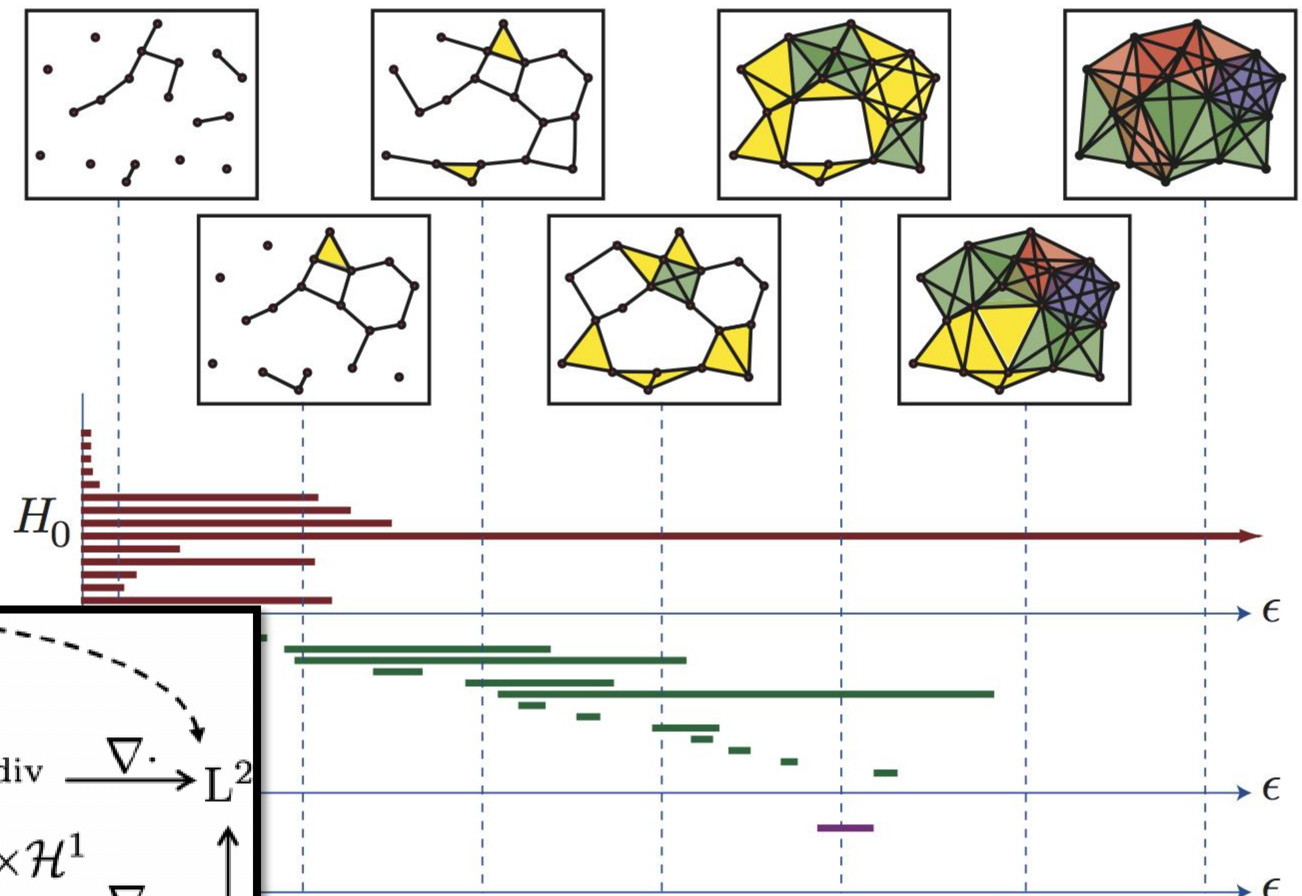


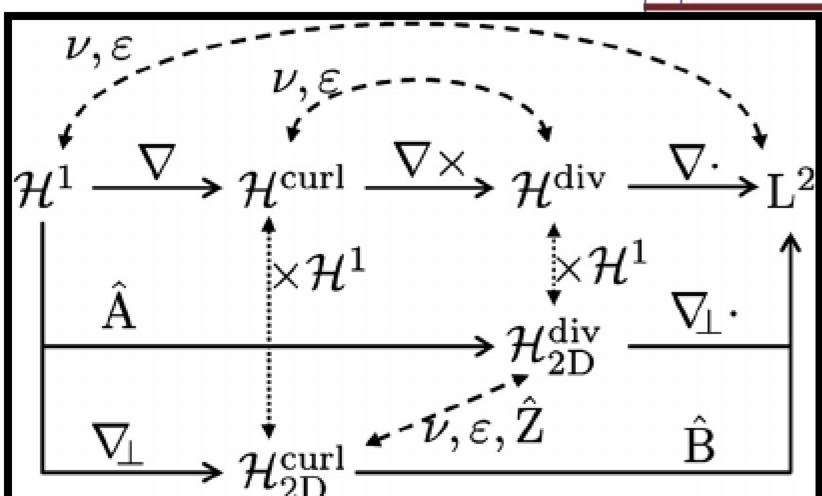
Peyré, Cuturi, and Solomon.
Gromov-Wasserstein Averaging of Kernel and Distance Matrices.
ICML 2016.

Optimal Transport



{Differential/Morse/Persistent/...} Topology





https://www.researchgate.net/publication/274071948_Space_charge_and_resistive_wall_impedance_computation_in_the_frequency_domain_using_the_finite_element_method
http://blog.forty.to/wp-content/uploads/2013/05/Bars1.jpg

Plan for Today

I. Theoretical toolbox

II. Computational toolbox

III. Application areas

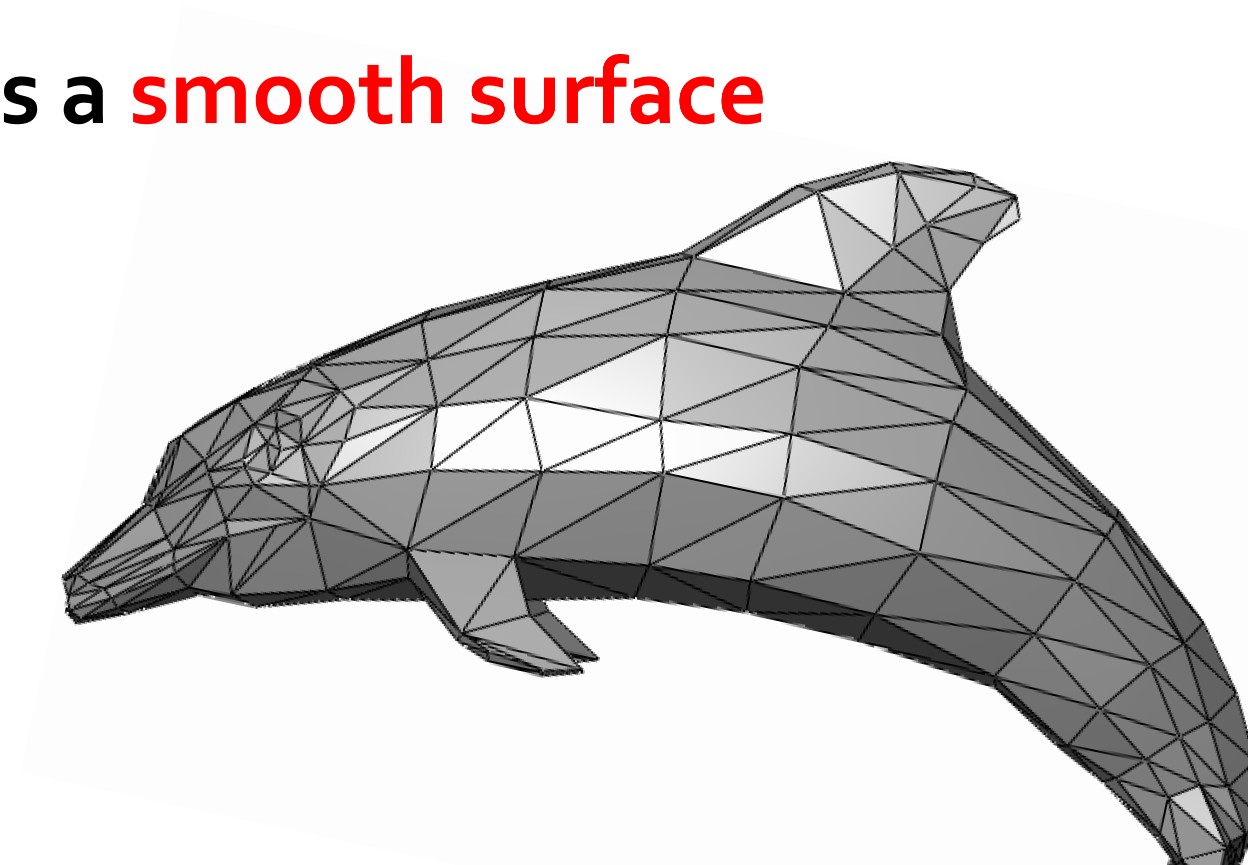
Many Notions of Shape

- Triangle mesh
- Triangle soup
- Graph
- Point cloud
- Pairwise distance matrix
- Dataset
- Network embedding

Nearly anything with a notion of proximity/distance/curvature/…

Typical issue:
Euclidean? Riemannian?

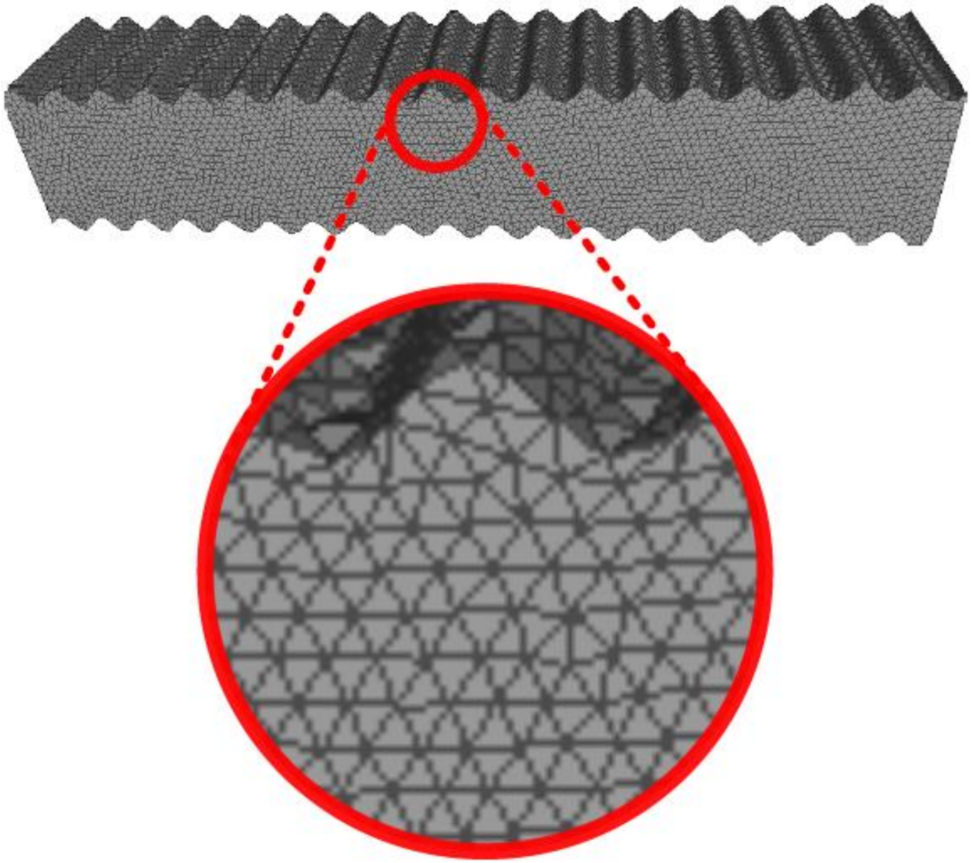
- Collection of flat triangles
- Approximates a smooth surface



Q:

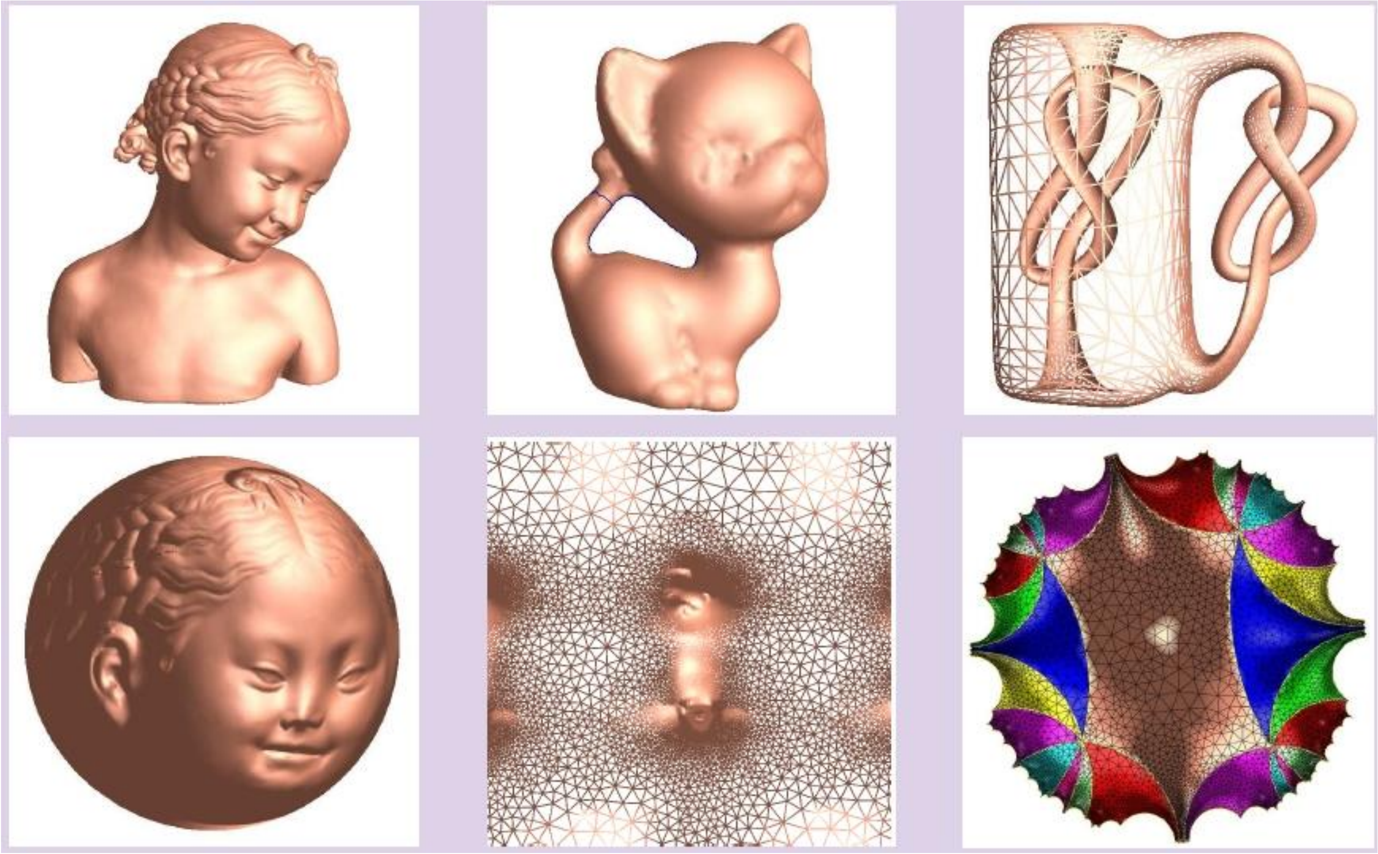
Can a triangle mesh have curvature?

Jack of All Trades



Combine smooth and discrete

Example:
Discrete Differential Geometry



http://www.geomtop.org/teaching/conformal_2012Summer/title_image.jpg

Modern Approach

Discrete

vs.

Discretized

Discrete Differential Geometry

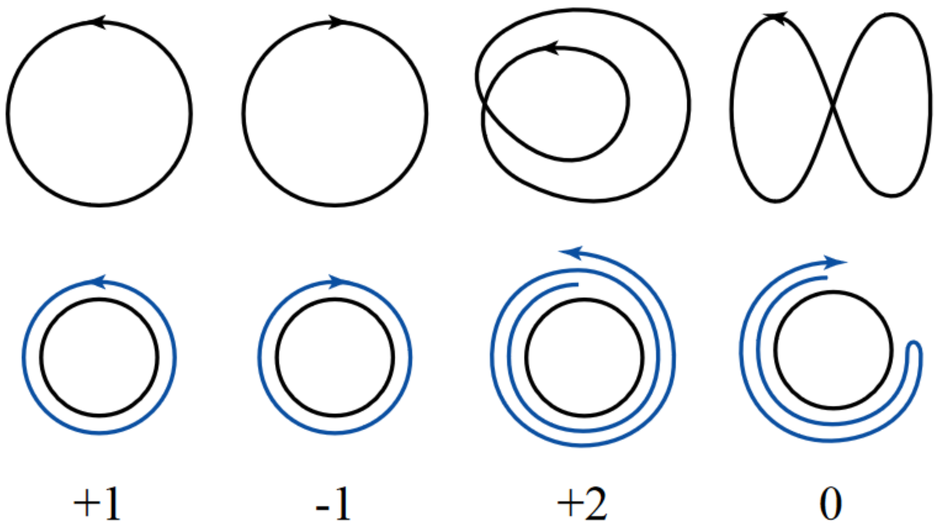
Discrete theory *paralleling* differential geometry.

Structure preservation

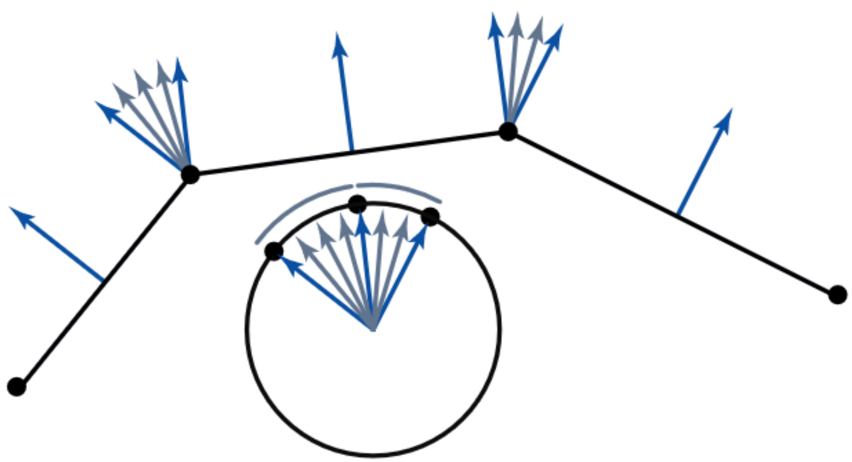
[**struhk**-cher pre-zur-**vey**-sh*uh* n]**:**

Keeping properties from the continuous abstraction exactly true in a discretization.

Example: Turning Numbers



$$\int_{\Omega} \kappa \, ds = 2\pi k$$



$$\sum_i \alpha_i = 2\pi k$$

Images from: Grinspun and Secord, "The Geometry of Plane Curves" (SIGGRAPH 2006)

Convergence

[k*uh* n-**vur**-j*uh* ns]**:**

Increasing approximation quality as a discretization is refined.

Convergence *and* Structure

Can you have it all?

Fey et al. *30 Rock* (2006-2013).

Disappointing Result

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

Max Wardetzky[1] Saurabh Mathur[2] Felix Kälberer[1] Eitan Grinspun[2 †]

[1]Freie Universität Berlin, Germany [2]Columbia University, USA

Abstract

Discrete Laplace operators are ubiquitous in applications spanning geometric modeling to simulation. For robustness and efficiency, many applications require discrete operators that retain key structural properties inherent to the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

In applications one often requires certain structural prop-

1.1. Properties of smooth Laplacians

Consider a smooth surface S, possibly with boundary, equipped with a Riemannian metric, *i.e.*, an intrinsic notion of distance. Let the intrinsic L^2 inner product of functions u and v on S be denoted by $(u,v)_{L^2} = \int_S uv\, dA$, and let $\Delta = -\mathrm{div}\,\mathrm{grad}$ denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

(NULL) $\Delta u = 0$ whenever u is constant.

Disappointing Result

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

Max Wardetzky[1] Saurabh Mathur[2] Felix Kälberer[1] Eitan Grinspun[2 †]

[1]Freie Universität Berlin, Germany [2]Columbia University, USA

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operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians can- *not satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators.* *Finally, we present a family of operators that includes and extends well-known and widely-used operators.*

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

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(NULL) $\Delta u = 0$ whenever u is constant

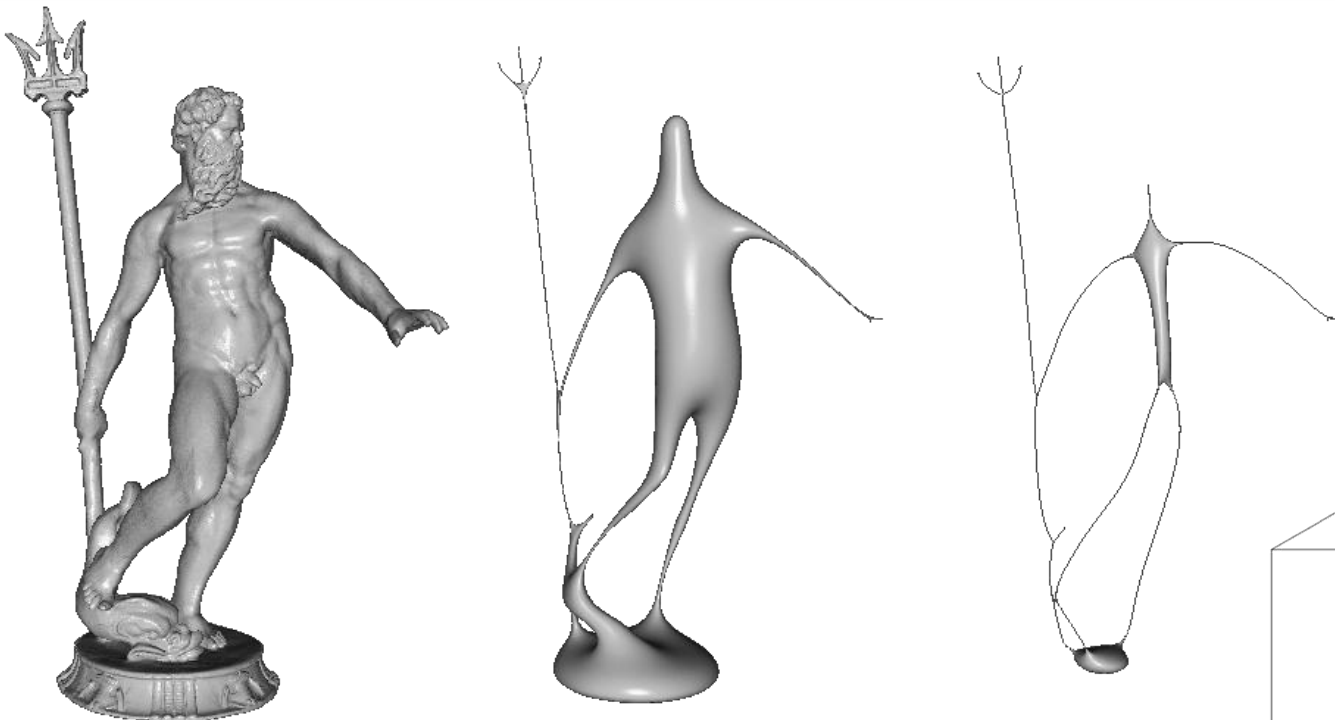
Theme

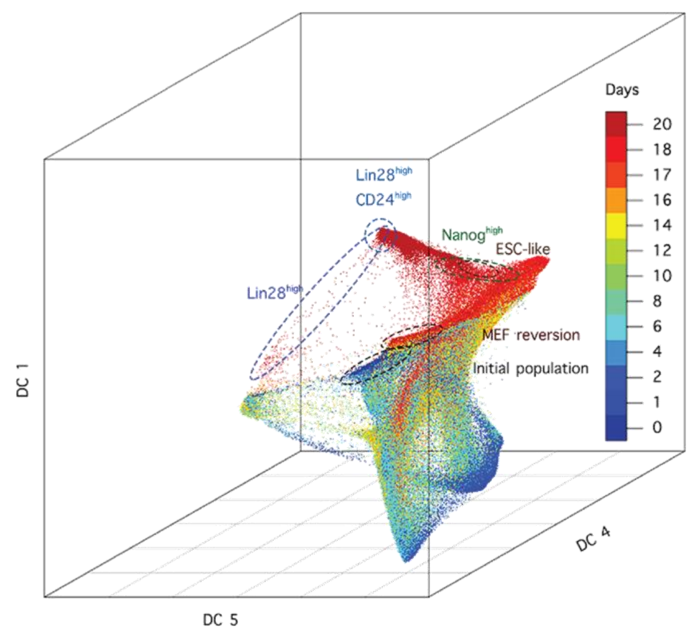
Pick and choose

which properties you need.

But there is a huge toolbox to draw from!

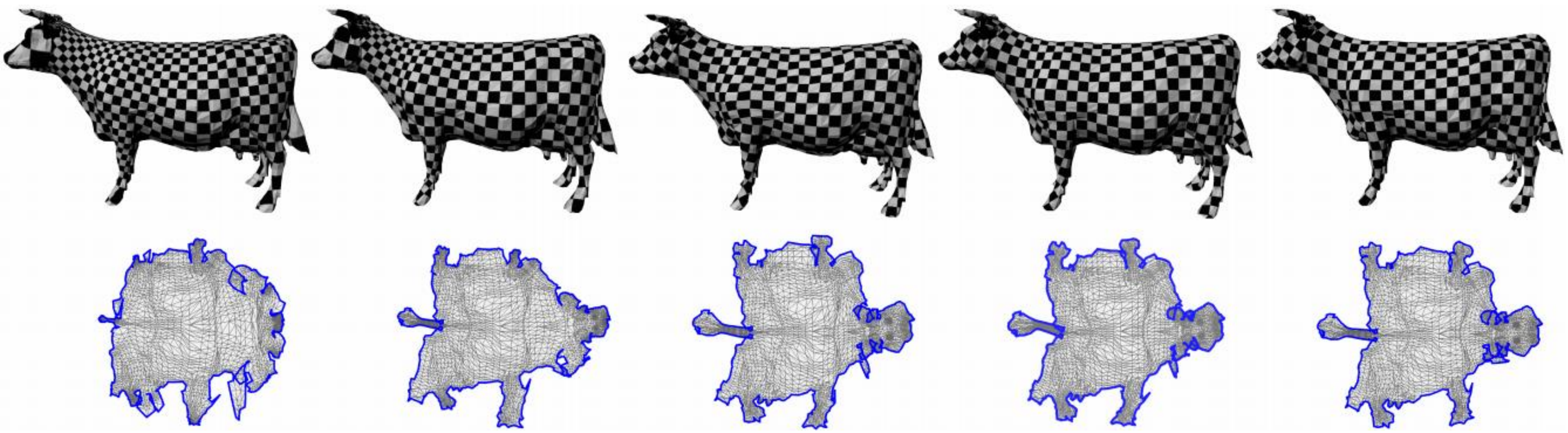
Numerical PDE





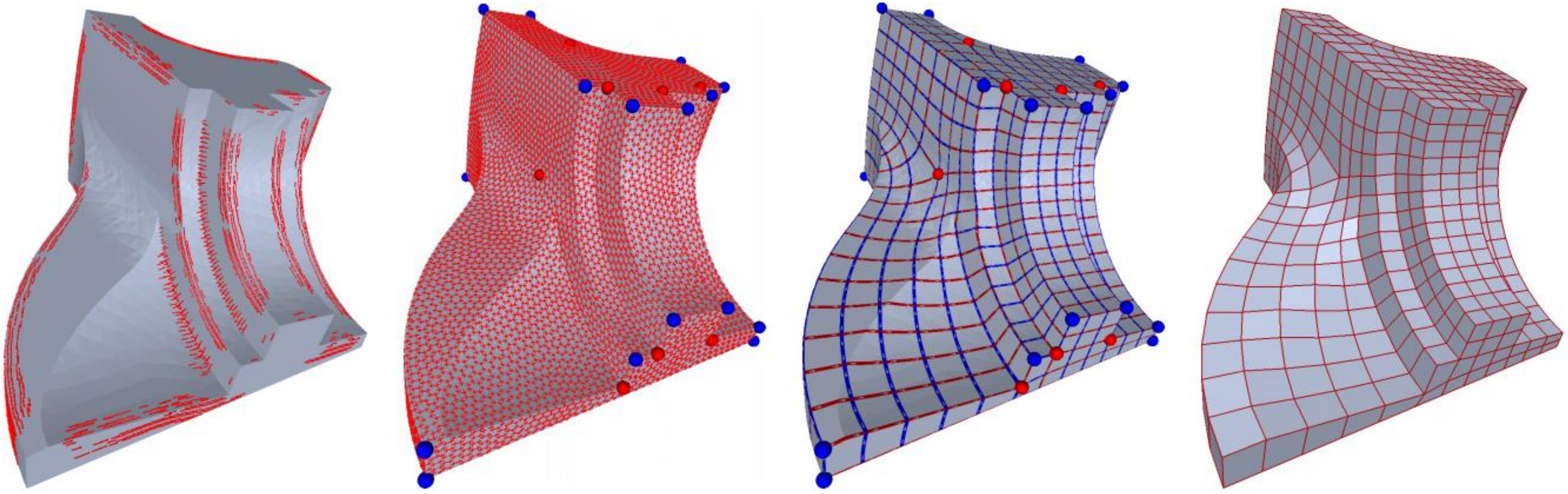
Chuang and Kazhdan. *Fast Mean-Curvature Flow via Finite-Elements Tracking*. CGF 2011.
Coifman & Lafon. *Diffusion Maps*. ACHA 2006.

Smooth Optimization



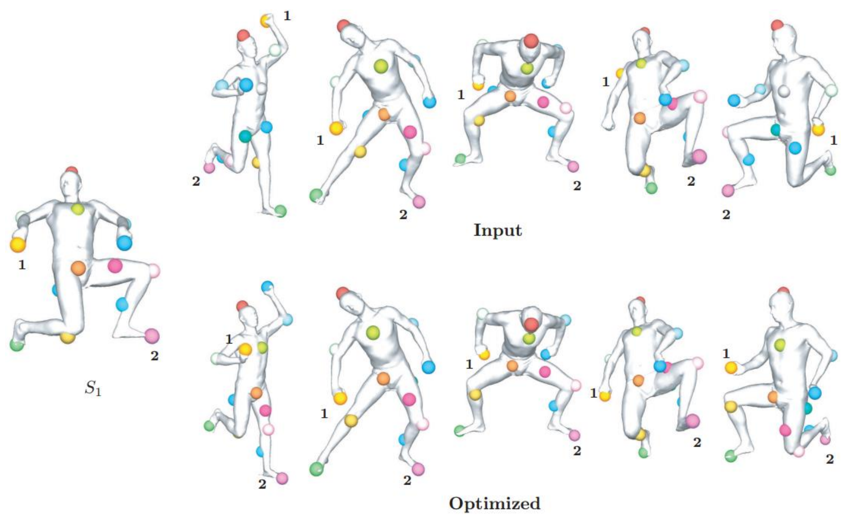
Smith and Schaefer. *Bijective parameterization with free boundaries*. SIGGRAPH 2015.

Discrete Optimization

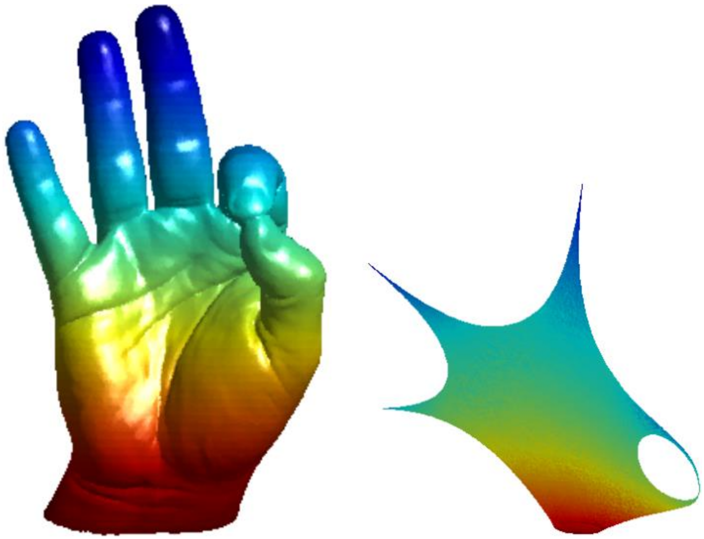


Bommes, Zimmer, Kobbelt. *Mixed-integer quadrangulation*. SIGGRAPH 2009.

Linear Algebra

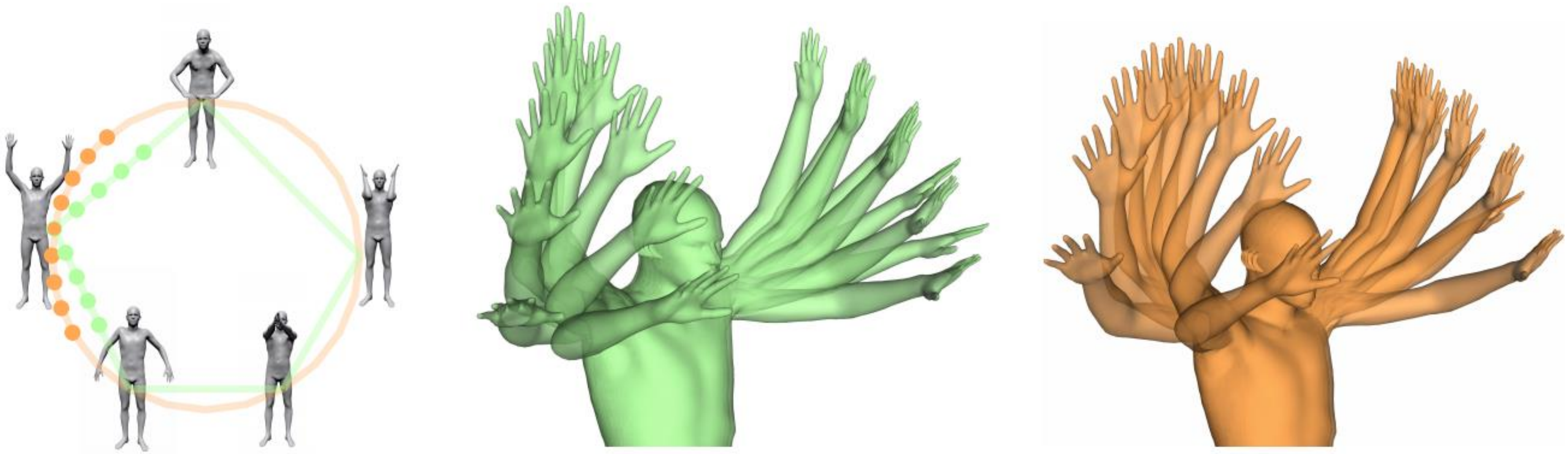


Huang, Guibas. *Consistent shape maps via semidefinite programming*. SGP 2013.



Krishnan, Fattal, Szeliski. *Efficient preconditioning of Laplacian matrices for computer graphics*. SIGGRAPH 2013.

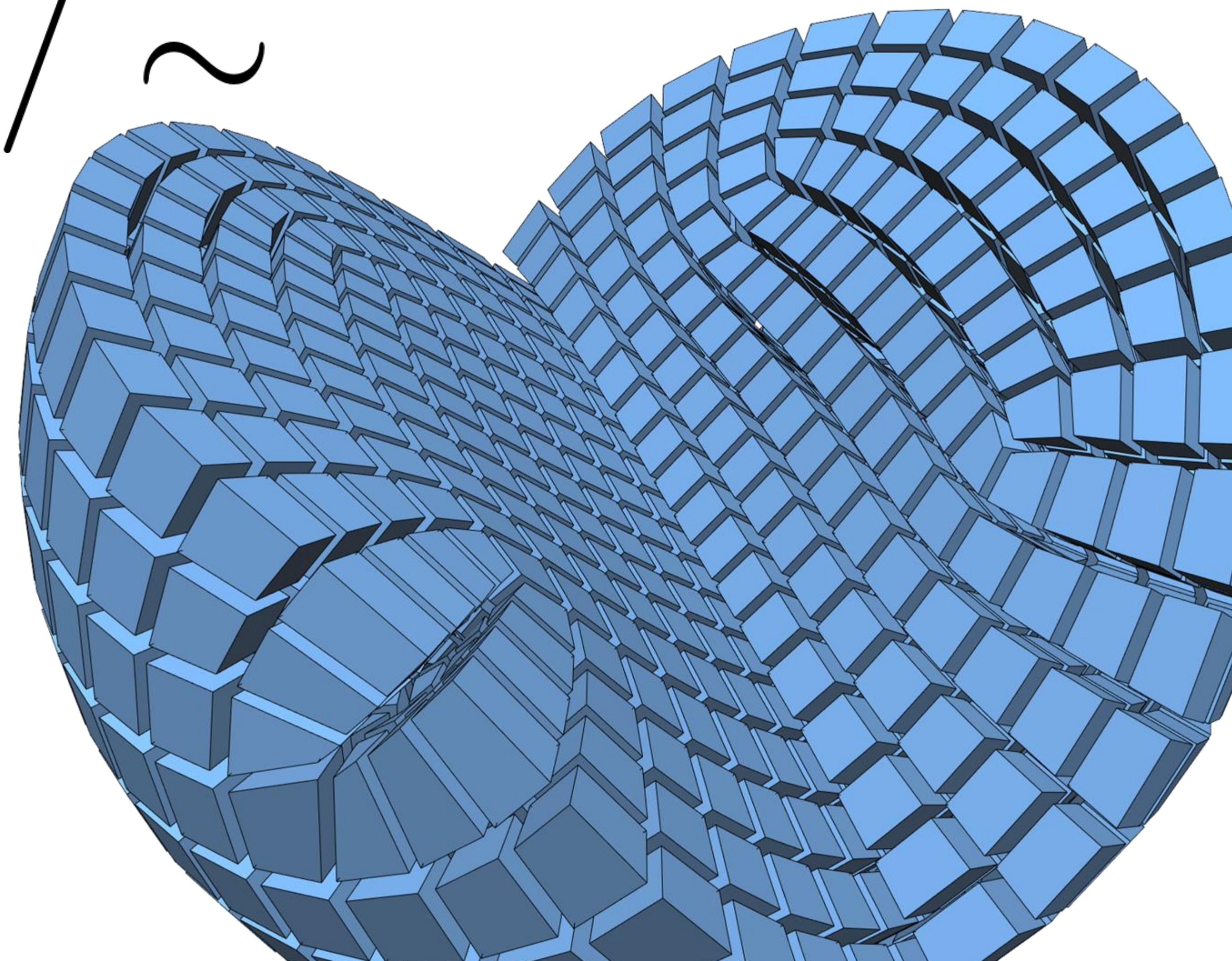
The "Geometry of Geometry"



Heeren et al. *Splines in the space of shells*. SGP 2016.

Algebra & Representation Theory

$$\mathrm{SO}(3)/\sim$$



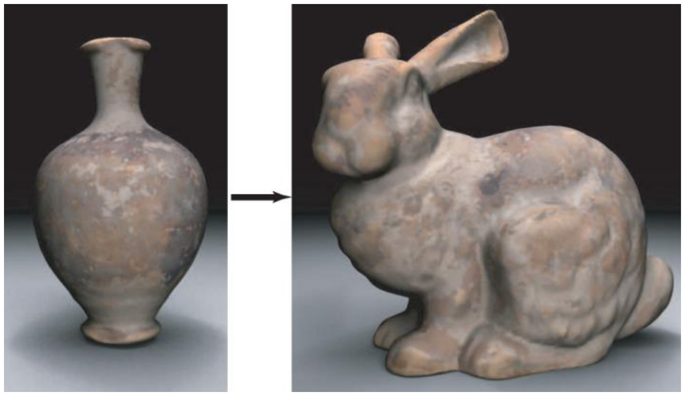
Plan for Today

I. Theoretical toolbox

II. Computational toolbox

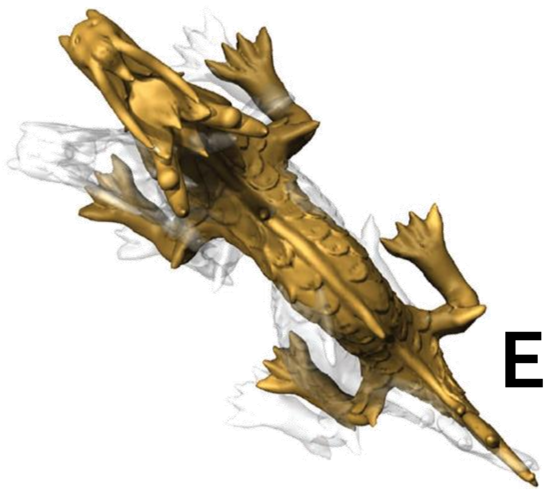
III. Application areas

Applications

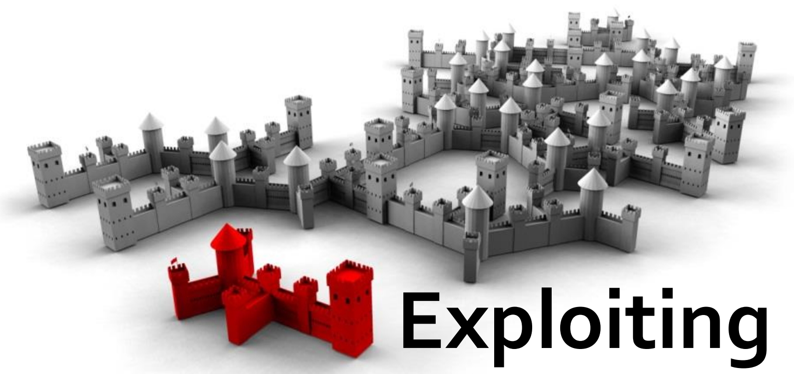


Transfer

Retrieval



Editing

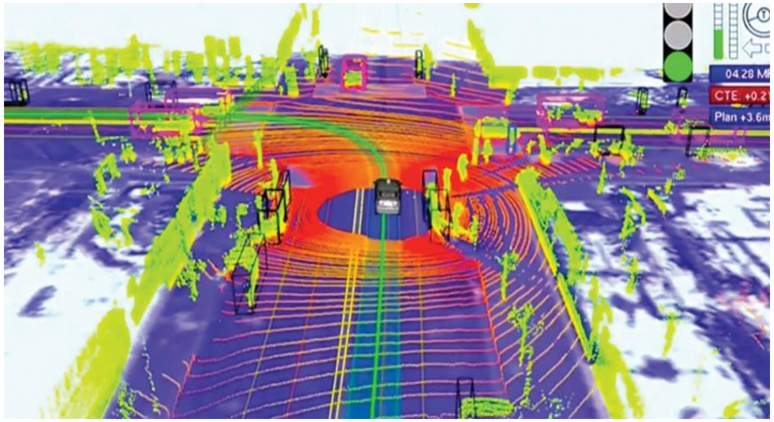


Exploiting patterns

http://people.csail.mit.edu/tmertens/papers/textransfer_electronic.pdf
http://graphics.stanford.edu/~mdfisher/Data/Context.pdf
http://graphics.stanford.edu/~niloy/research/symmetrization/paper_docs/symmetrization_sig_07.pdf
http://www.mpi-inf.mpg.de/~mbokeloh/project_dockingSites.html

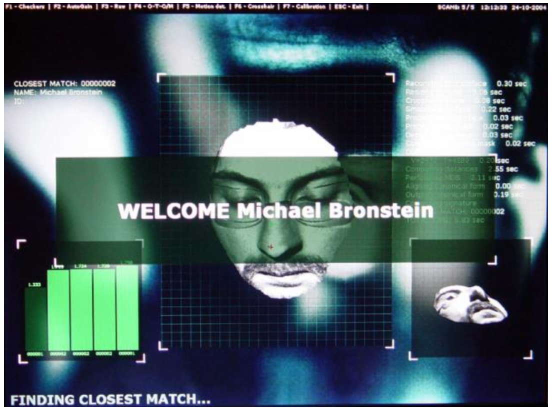
Graphics

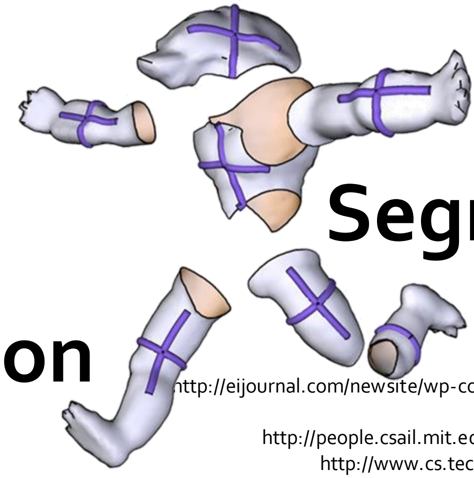
Applications



Recognition

Navigation





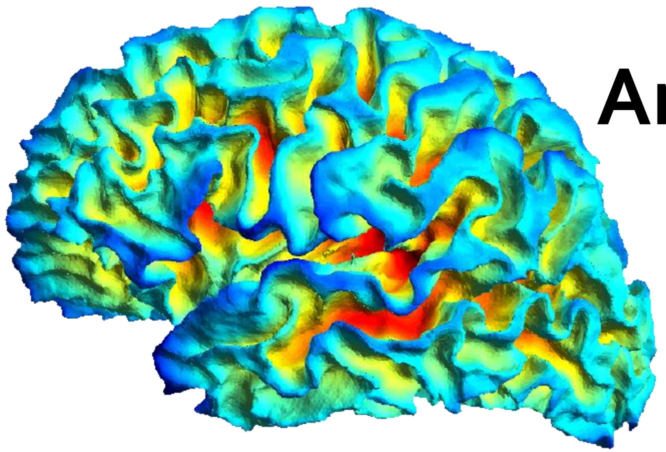
Segmentation

Reconstruction

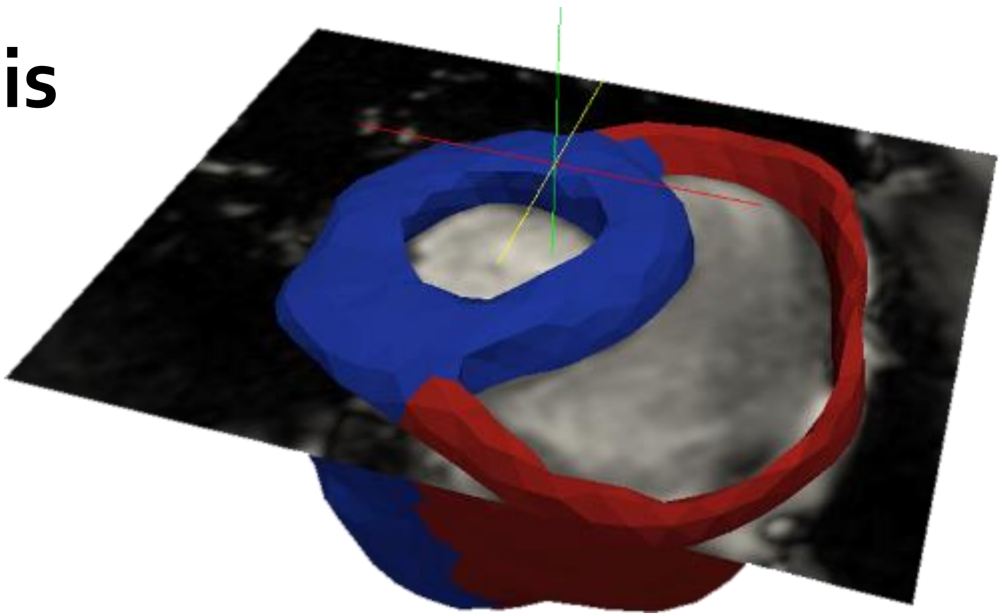
http://eijournal.com/newsite/wp-content/uploads/2012/01/VELODYNE-IMAGE.jpg
http://www.stanford.edu/~jinhae/iccv09/
http://people.csail.mit.edu/jsolomon/assets/intrinsic_part_discovery.pdf
http://www.cs.technion.ac.il/~ron/PAPERS/BroBroKimIJCV05.pdf

Vision

Applications



Analysis



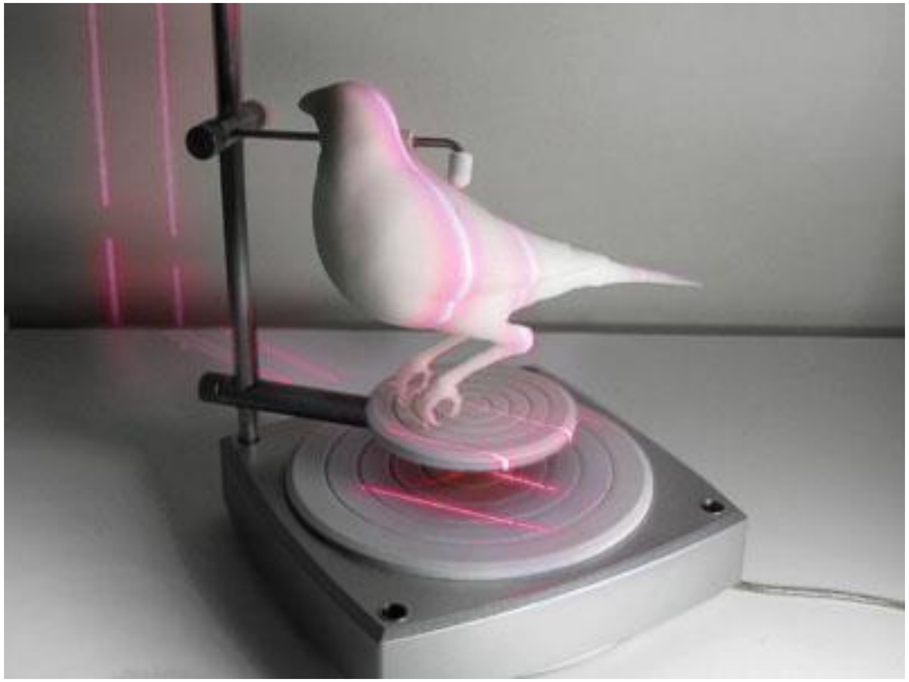
Segmentation

Registration

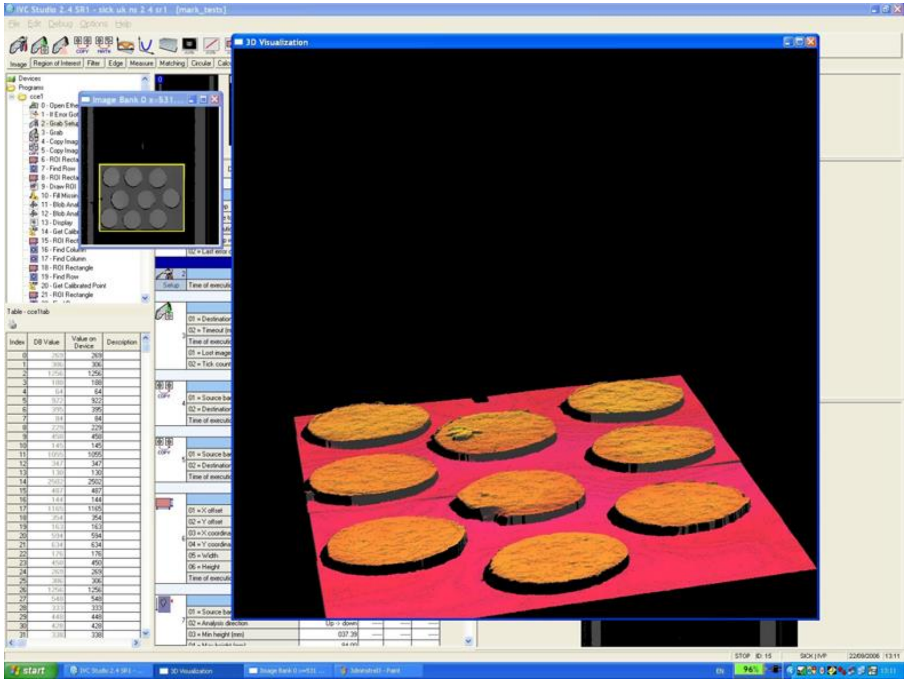
http://dmfr.birjournals.org/content/33/4/226/F3.large.jpg
http://www-sop.inria.fr/asclepios/software/inriaviz4d/SphericalImTransp.png
http://www.creatis.insa-lyon.fr/site/sites/default/files/segm2.png

Medical Imaging

Applications



Scanning

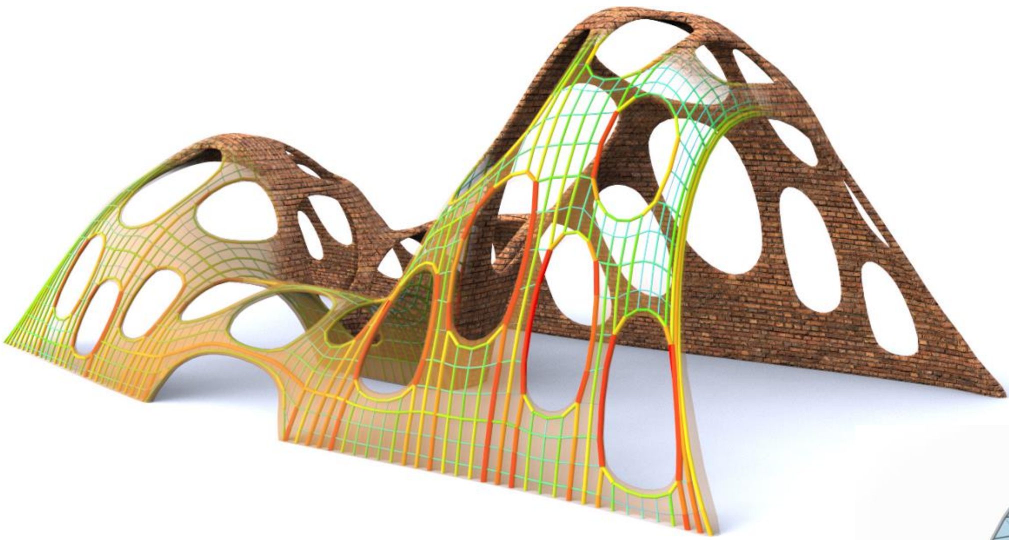


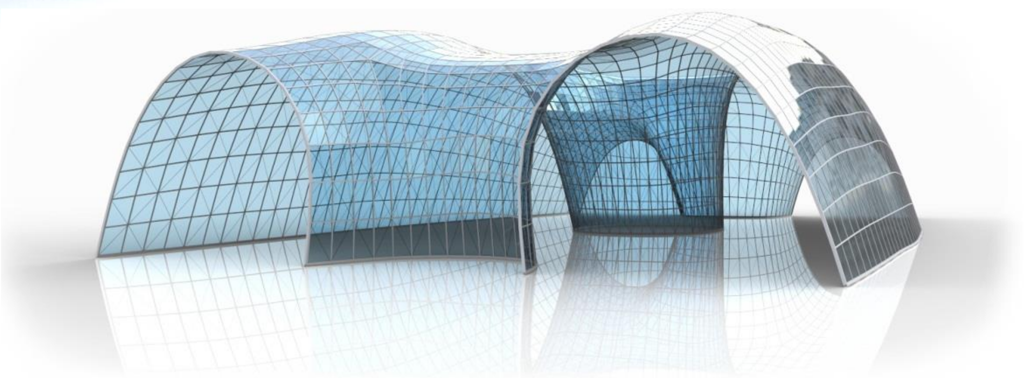
Defect detection

http://www.conduitprojects.com/php/images/scan.jpg
http://www.emeraldinsight.com/content_images/fig/0330290204005.png

Manufacturing and Fabrication

Applications



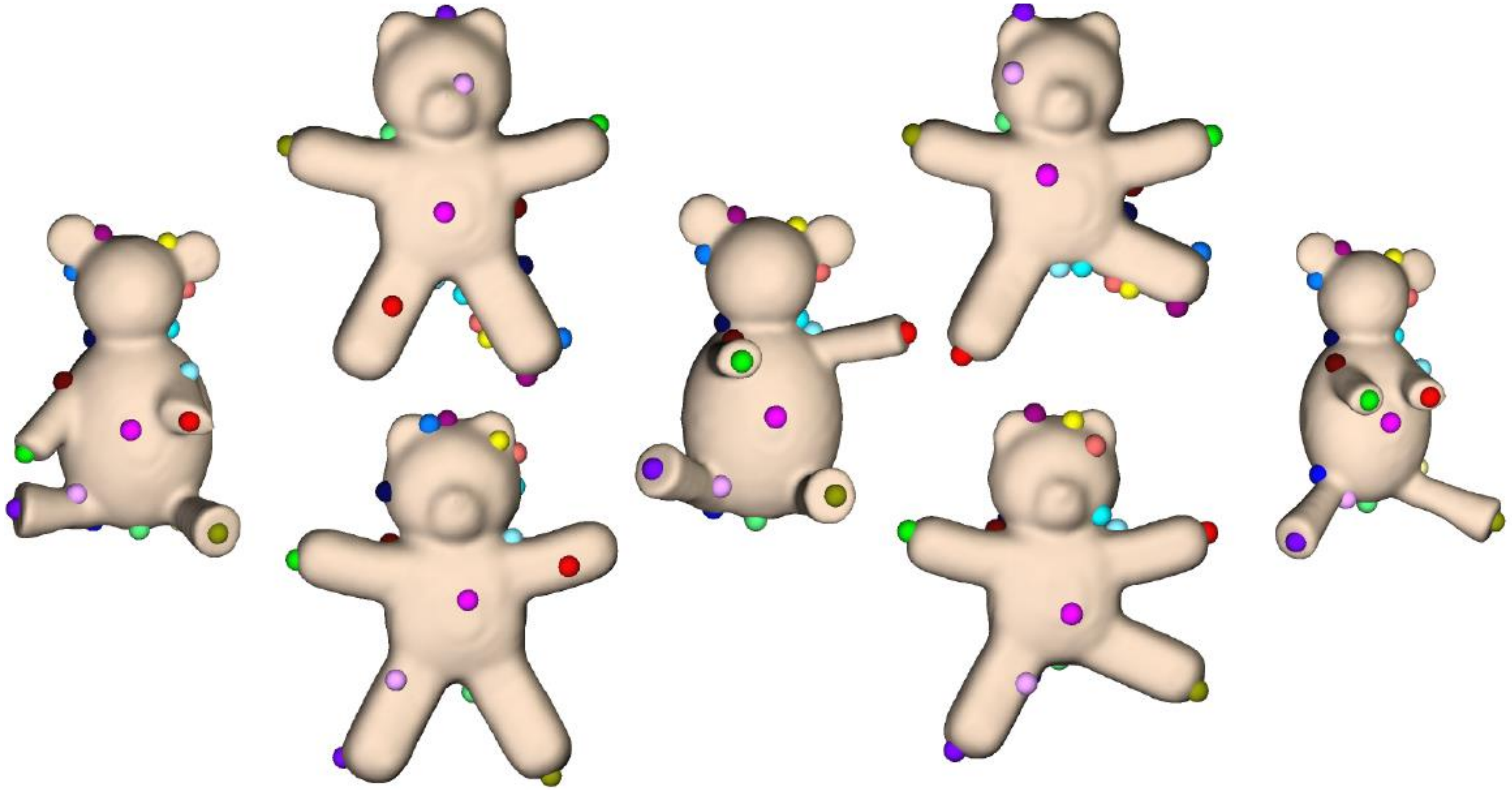


Design and analysis

http://gmsv.kaust.edu.sa/people/faculty/pottmann/pottmann_pdf/selfsupporting.pdf

Architecture

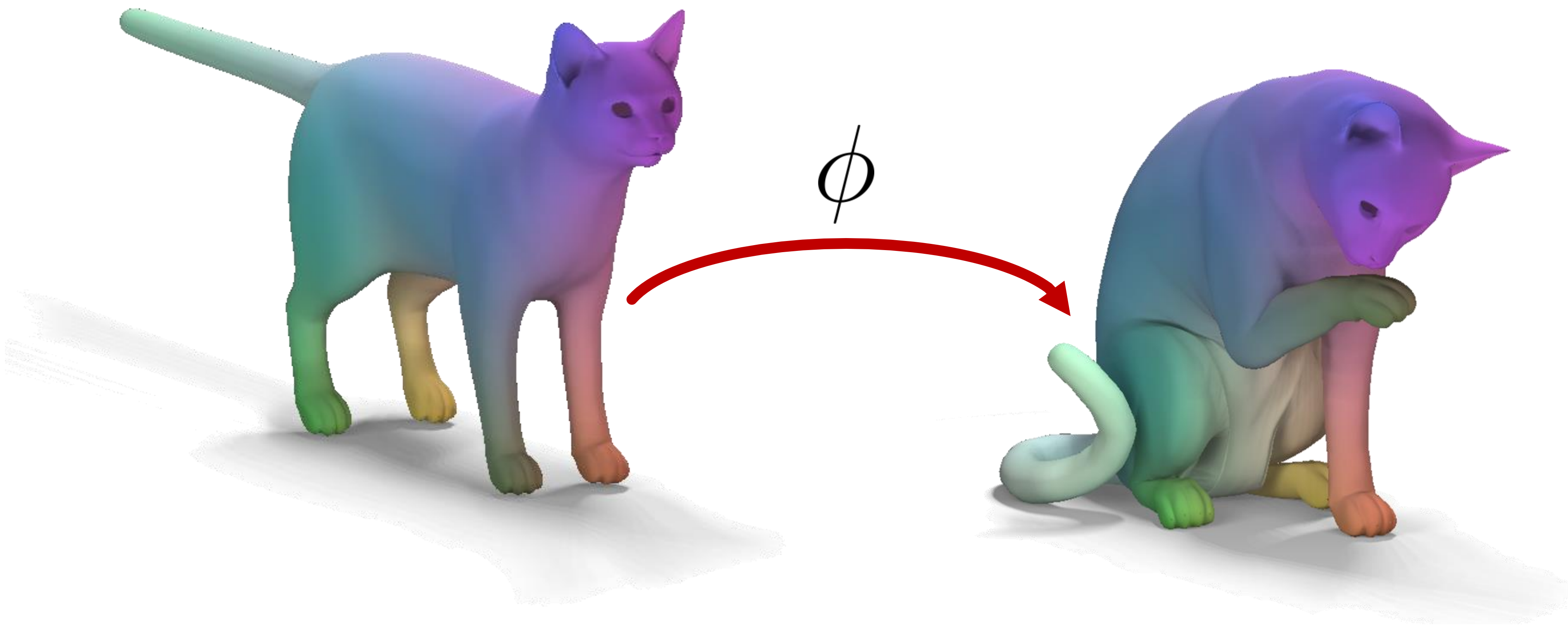
Applications



http://graphics.stanford.edu/projects/lgl/papers/nbwyg-oaicsm-11/nbwyg-oaicsm-11.pdf

Shape collections

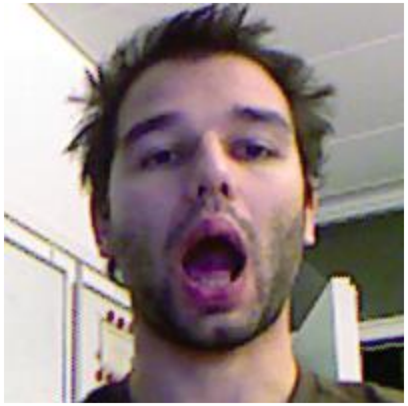
Applications

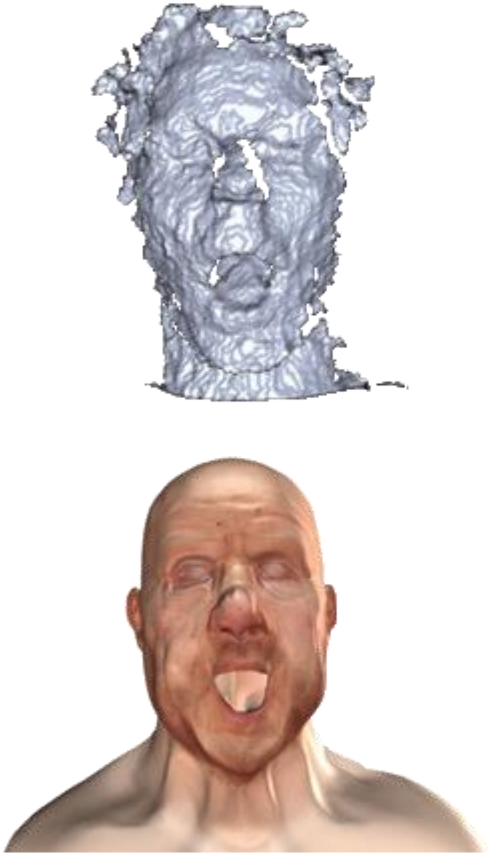


http://people.csail.mit.edu/jsolomon/assets/fmaps.pdf

Correspondence

Applications

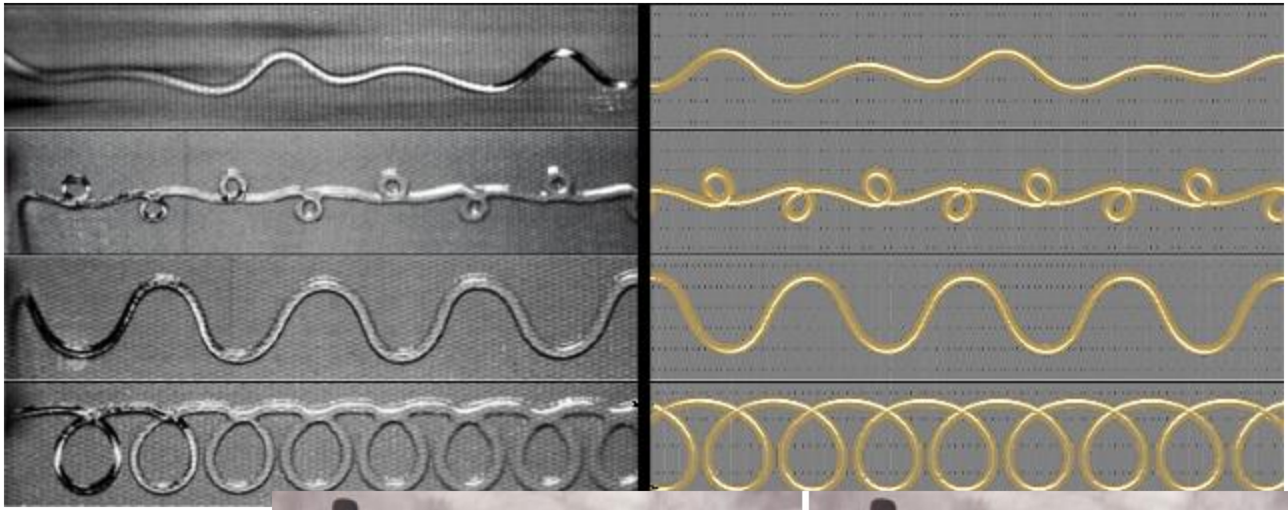




http://www.hao-li.com/publications/papers/siggraph2011RPBFA.pdf

Deformation transfer

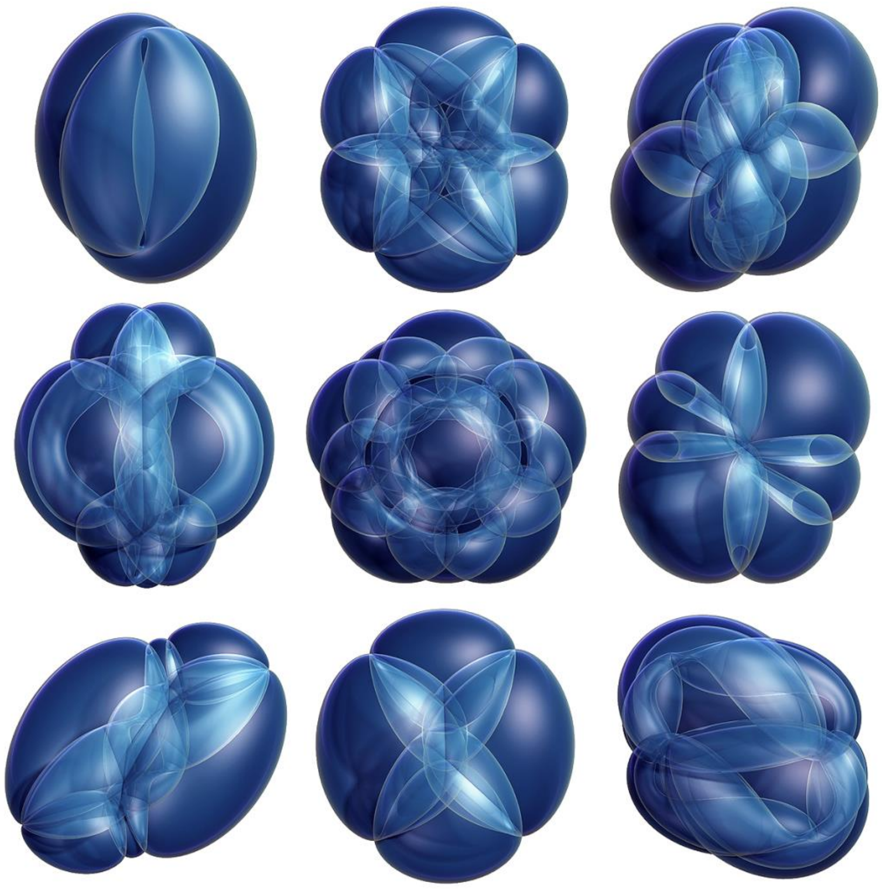
Applications



http://www.cs.columbia.edu/cg/threads/ http://mbergou.com/

Simulation

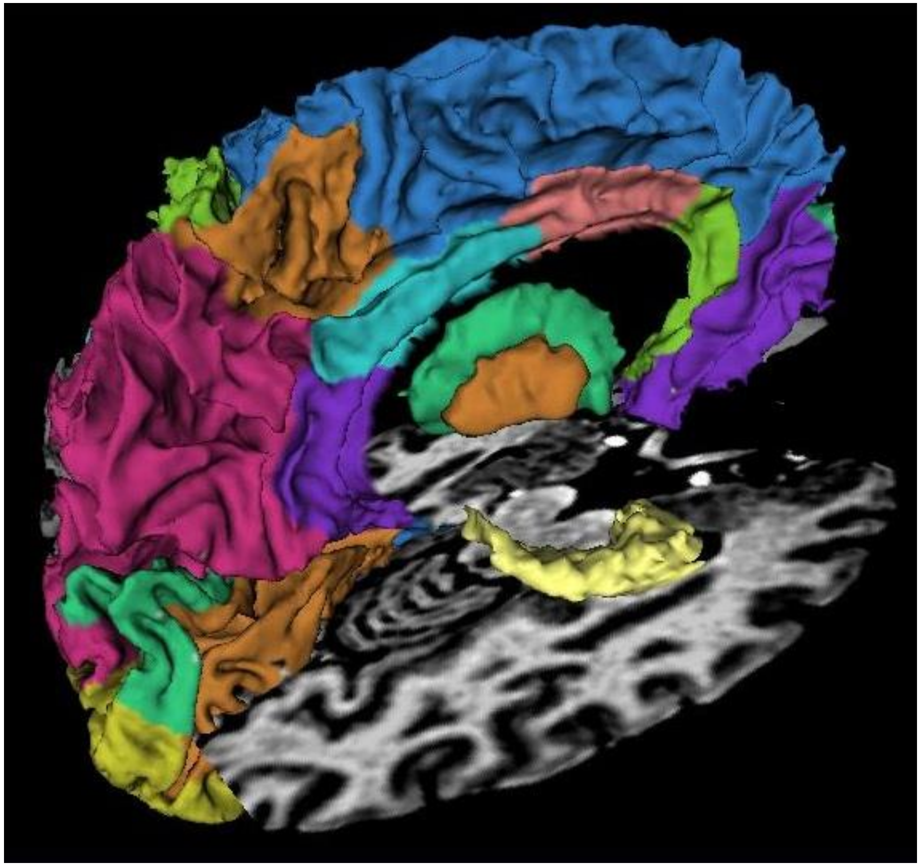
Applications

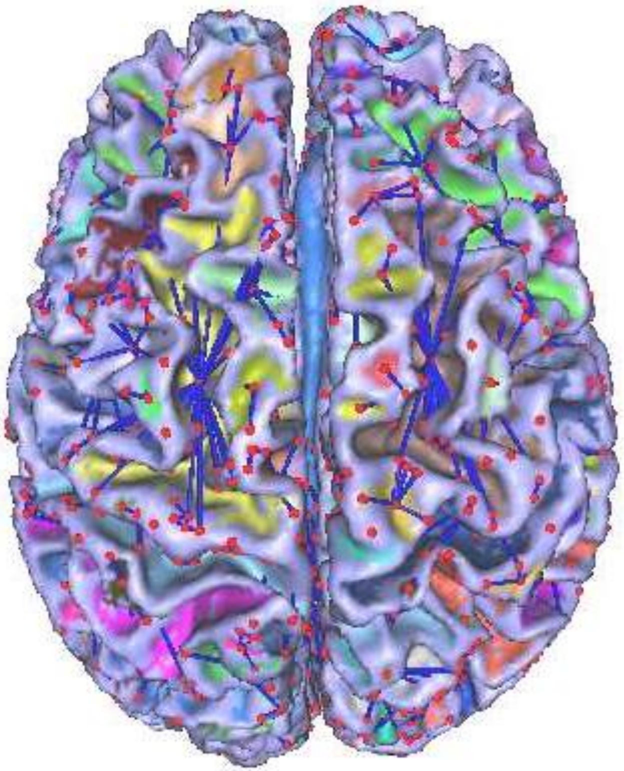


http://multires.caltech.edu/~keenan/pdf/spinxform.pdf

Scientific visualization

Applications

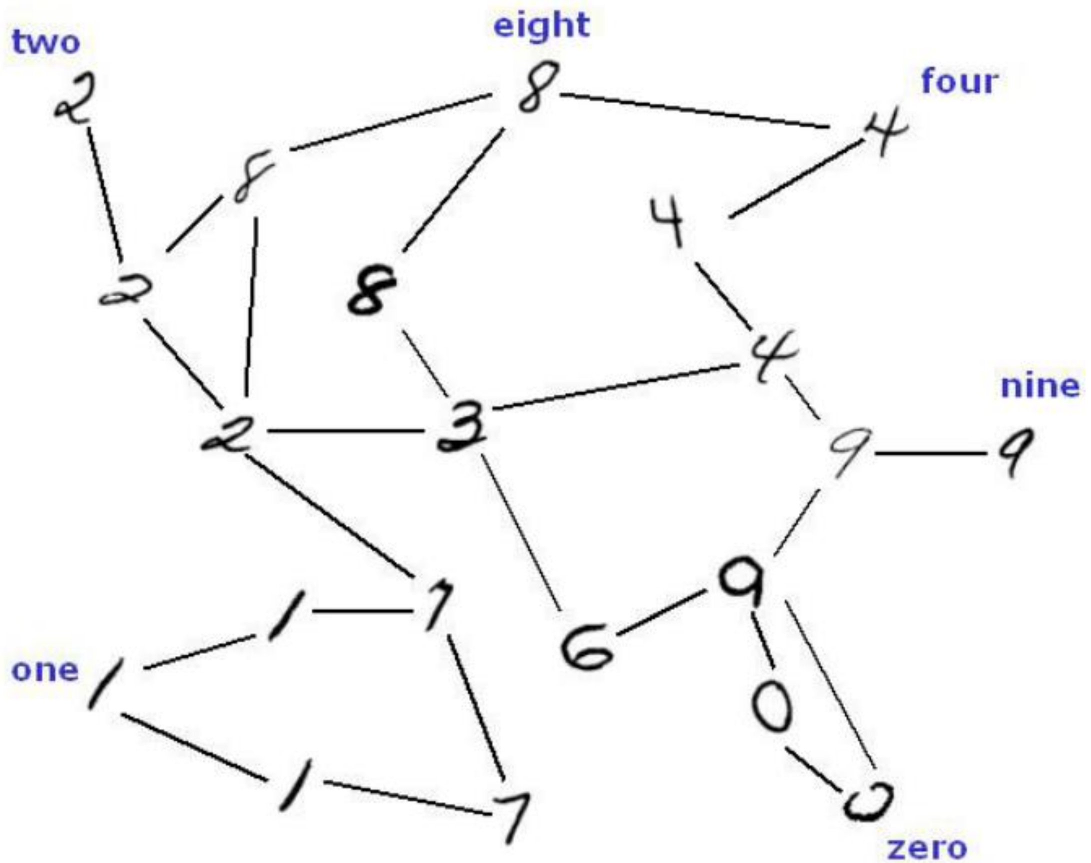




http://www.bioinformaticslaboratory.nl/twiki/pub/EBioScience/News/freesurfer-3d.jpg
http://hal.inria.fr/docs/00/40/21/30/IMG/vivodtzev_et_al-Dagstuhl03.jpg

Segmentation

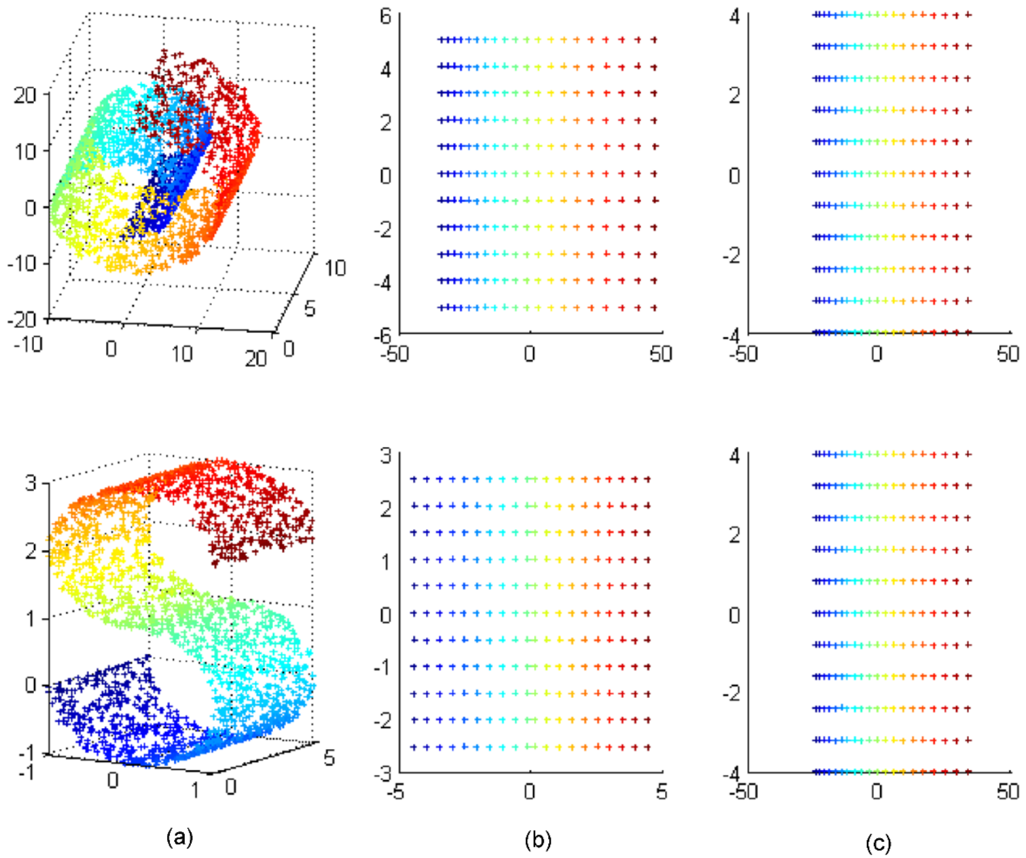
Applications



Zhu et al. *Semi-Supervised Learning Using Gaussian Fields and Harmonic Functions*. ICML 2003.

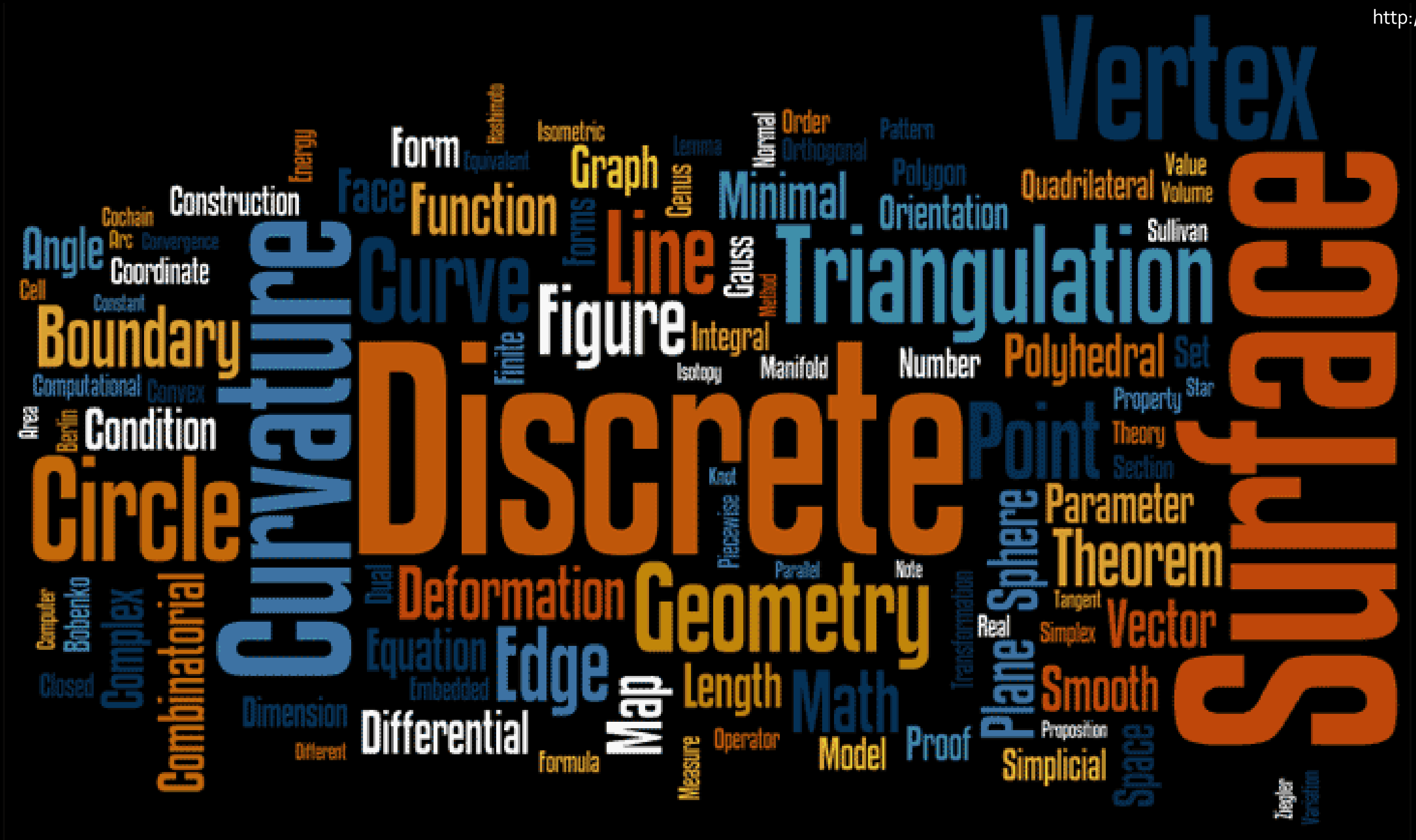
Machine learning

Applications



Hou et al. *Novel semisupervised high-dimensional correspondences learning method*. Opt. Eng. 2008.

http://www.gogeometry.com



6.838: Shape Analysis

Justin Solomon

MIT, Spring 2019

MIT EECS