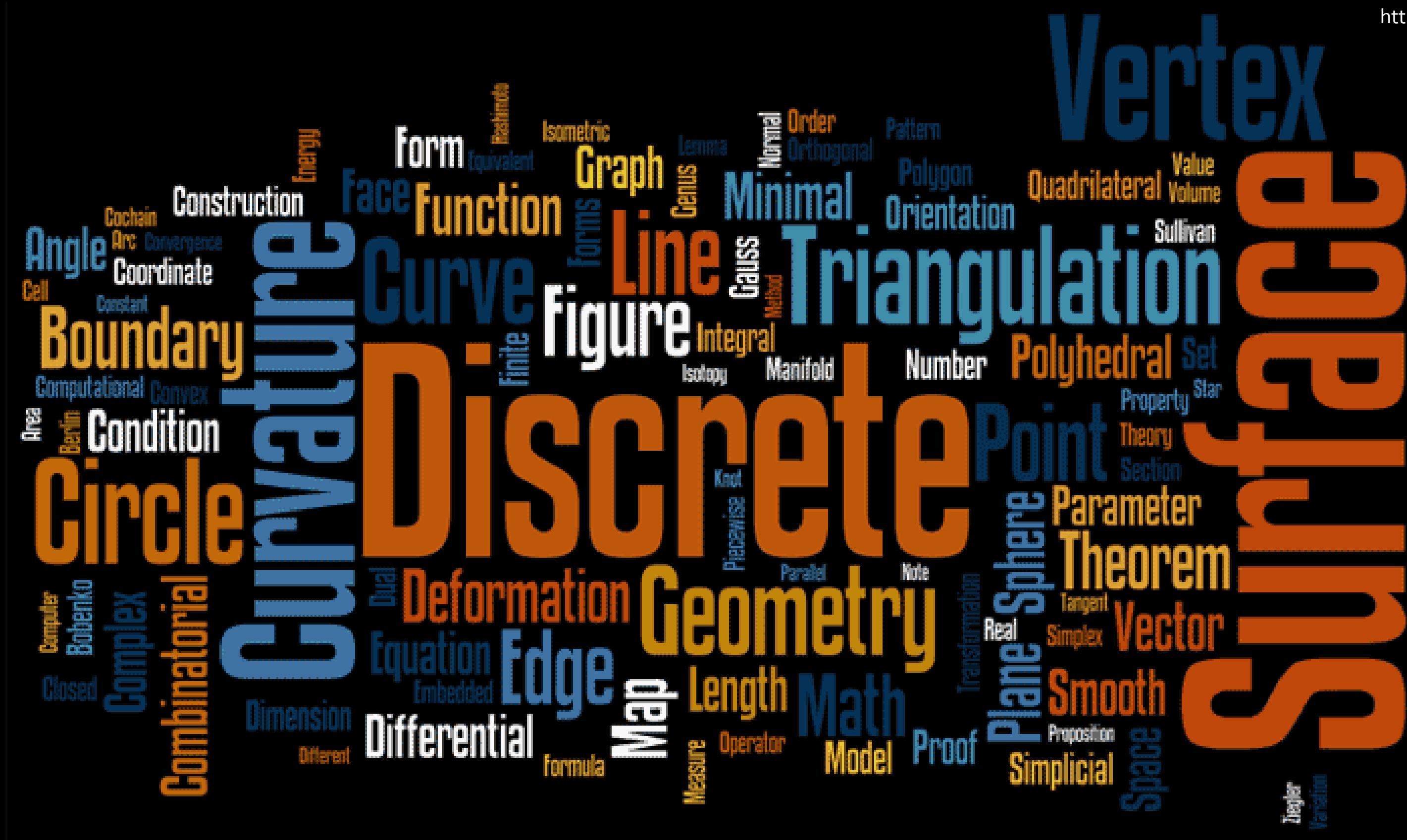
http://www.gogeometry.com



6.838: Shape Analysis

Justin Solomon
MIT, Spring 2017

MIT EECS

<administrative>

Course Instructor

Instructor: Justin Solomon
Email: jsolomon@mit.edu
Office: 32-D460
Office hours: Wednesdays, 1pm-3pm

Geometric Data Processing Group:
http://gdp.csail.mit.edu

TA

TA: Abhishek Bajpayee
Email: ab9@mit.edu
Office: 1-225
Office hours: Thursdays, 3pm-5pm

On the Web

gdp.csail.mit.edu/6838_spring_2017.html

+ piazza

Register ASAP!

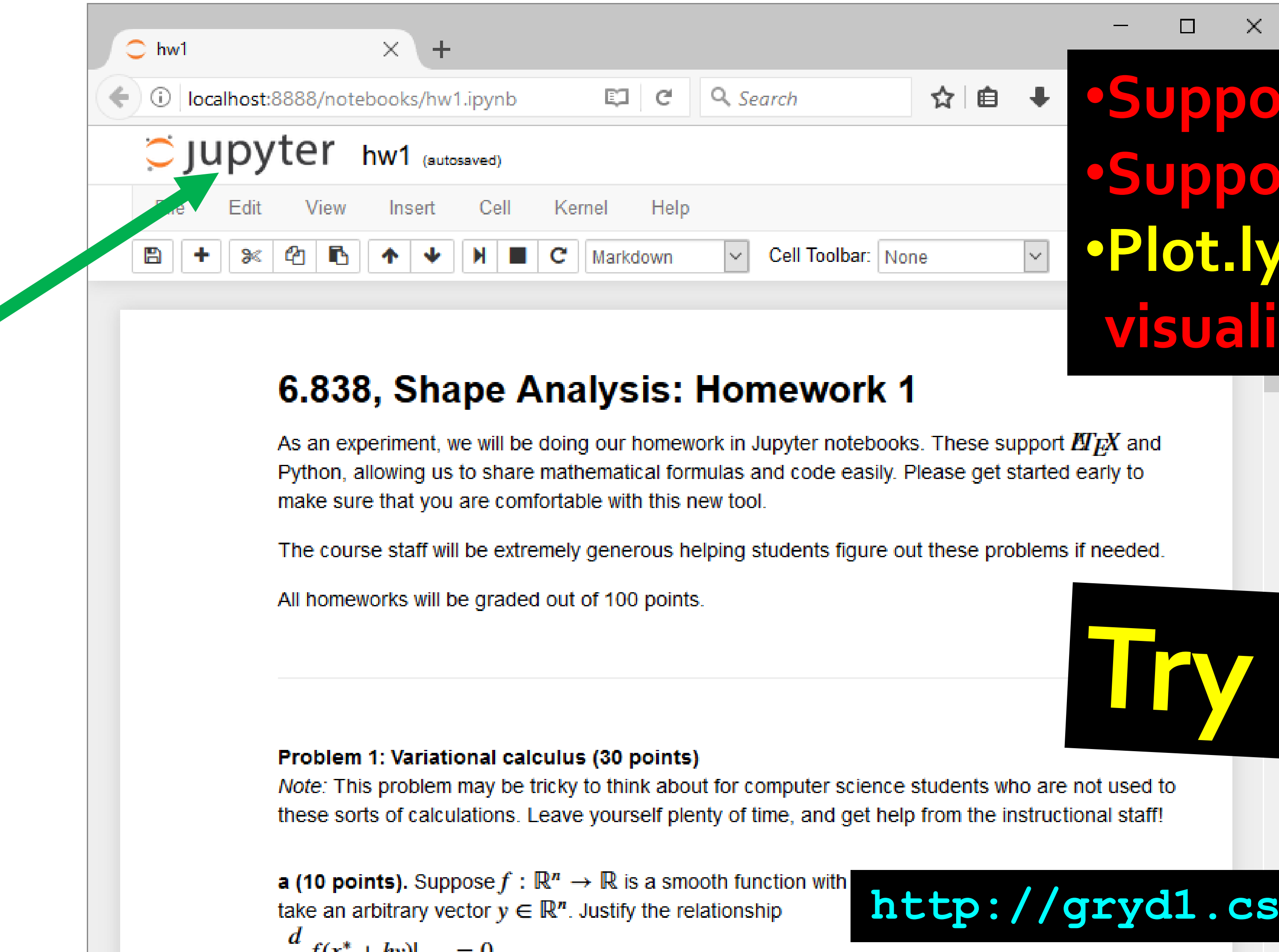
Deliverables

1. **Four homeworks (40%)**
   Written + coding
2. **One project (50%)**
   Instructions already online
3. **Biweekly nanoquizzes (10%)**
   Designed to be easy!

Prerequisites

- **Coding**
  Python or Matlab preferred
- **Math**
  Fluency in linear algebra and multivariable calculus
- **Not required (won't hurt):**
  Graphics, differential geometry, numerics

Experiment



- Supports LaTeX
- Supports Python
- Plot.ly for visualization

Try early!

http://gryd1.csail.mit.edu/

New Course

- Schedule is too ambitious!
- Contact Justin with suggestions, must-cover topics, questions, etc.
- Experiment: Video
  (unreliable!)

Philosophy

I want you to take this course!

Assignments intended to be interesting

(may be unintentionally easy/hard!)

Will be generous with support/grading

Quick Survey

Degree

Undergraduate

M.Eng.

M.Sc./PhD

Quick Survey

Background

EECS

Math

Engineering

Elsewhere

</administrative>

Theme

1. $\underbrace{\textit{Geometric data}}_{\text{Modifier}}\ \underbrace{\textit{analysis:}}_{\text{Noun}}$ The analysis of geometric data

2. $\underbrace{\textit{Geometric}}_{\text{Modifier}}\ \underbrace{\textit{data analysis:}}_{\text{Noun}}$ Data analysis using geometric techniques

Applied Geometry

I. Theoretical toolbox

II. Computational toolbox

III. Application areas

Mostly a picture book!

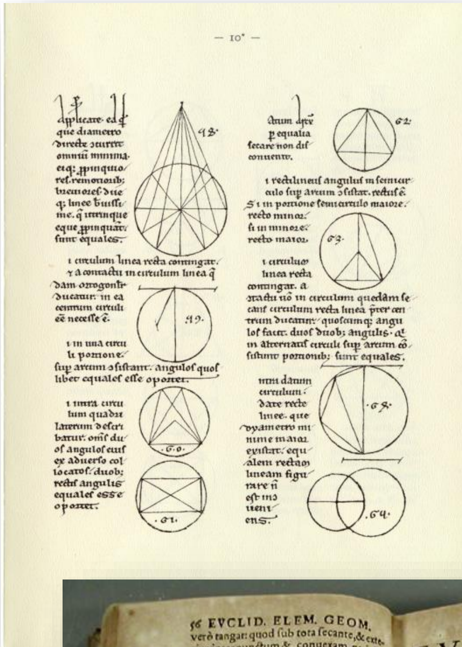
Applied Geometry

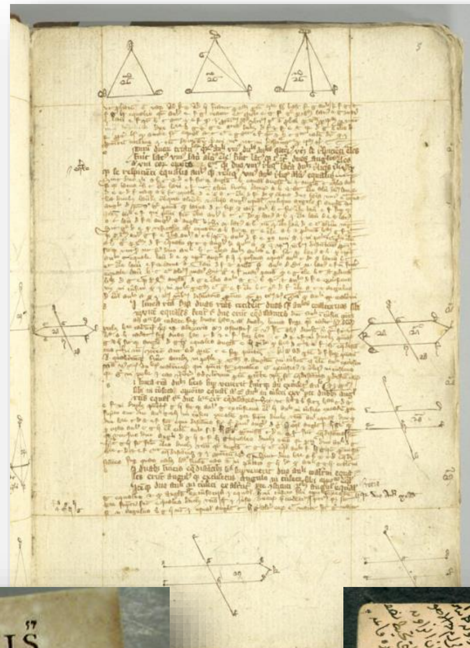
I. Theoretical toolbox

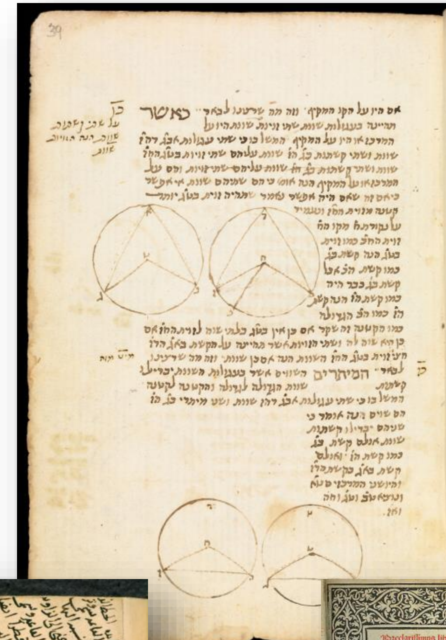
II. Computational toolbox

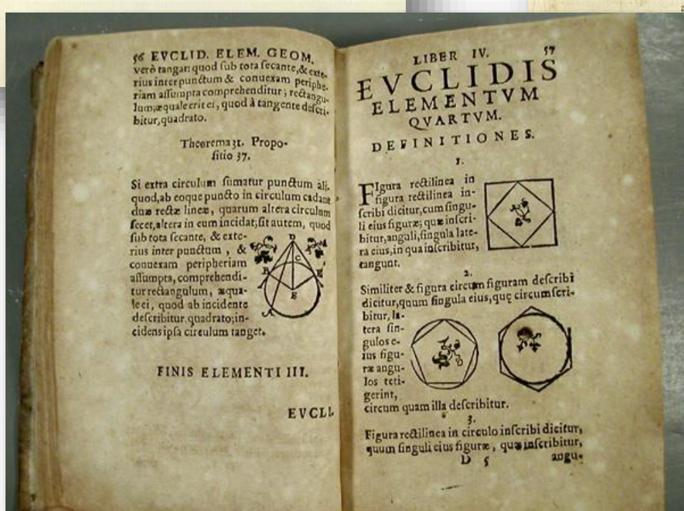
III. Application areas

Euclidean Geometry








56 EVCLID. ELEM. GEOM.
verò tangat: quod ſub tota ſecante, & exterius inter punctum & conuexam peripheriam aſſumpta comprehenditur; rectangulum æquale erit ei, quod à tangente deſcribitur, quadrato.

Theorema 31. Propoſitio 37.

Si extra circulum ſumatur punctum aliquod, ab eoque puncto in circulum cadant duæ rectæ lineæ, quarum altera circulum ſecet, altera in eum incidat; ſit autem, quod ſub tota ſecante, & exterius inter punctum, & conuexam peripheriam aſſumpta, comprehenditur rectangulum, æquale ei, quod ab incidente deſcribitur quadrato; incidens ipſa circulum tanget.

FINIS ELEMENTI III.

EVCLI.

LIBER IV. 57

EVCLIDIS ELEMENTVM QVARTVM.

DEFINITIONES.

1.

Figura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli eius figuræ; quæ inſcribitur, anguli, ſingula latera eius, in qua inſcribitur, tangunt.

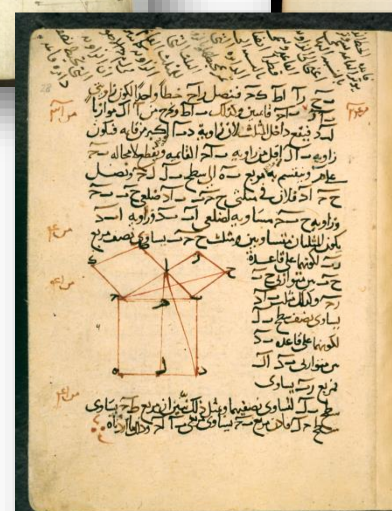
2.

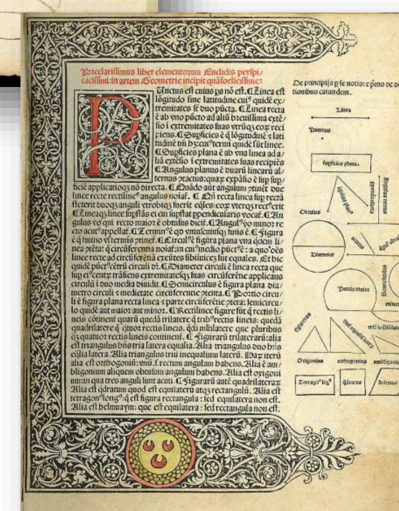
Similiter & figura circum figuram deſcribi dicitur, quum ſingula eius, quę circumſcribitur, latera ſingulos eius figuræ angulos tetigerint, circum quam illa deſcribitur.

3.

Figura rectilinea in circulo inſcribi dicitur, quum ſinguli eius figuræ, quæ inſcribitur, angu-

D 5



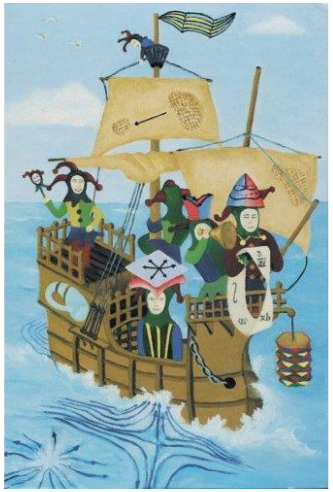


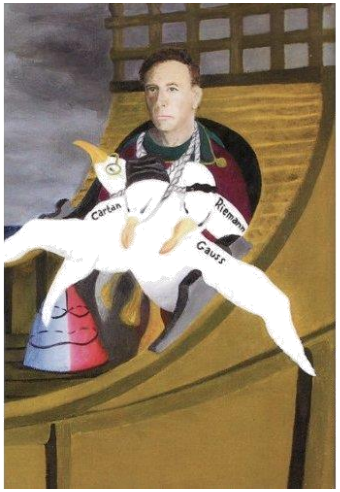
Euclidean Geometry

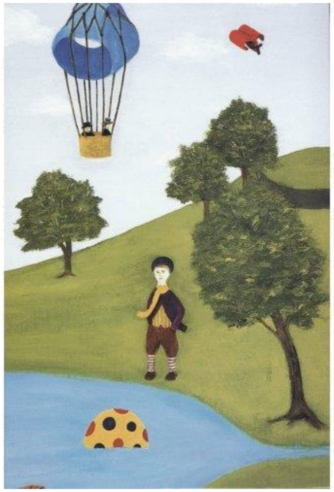
Euclidean Geometry

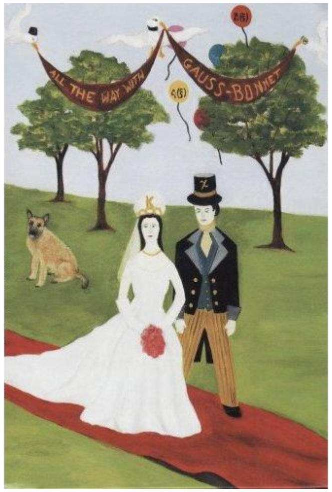
http://www.turbosquid.com/3d-models/medically-human-brain-3d-model/437885

Differential Geometry



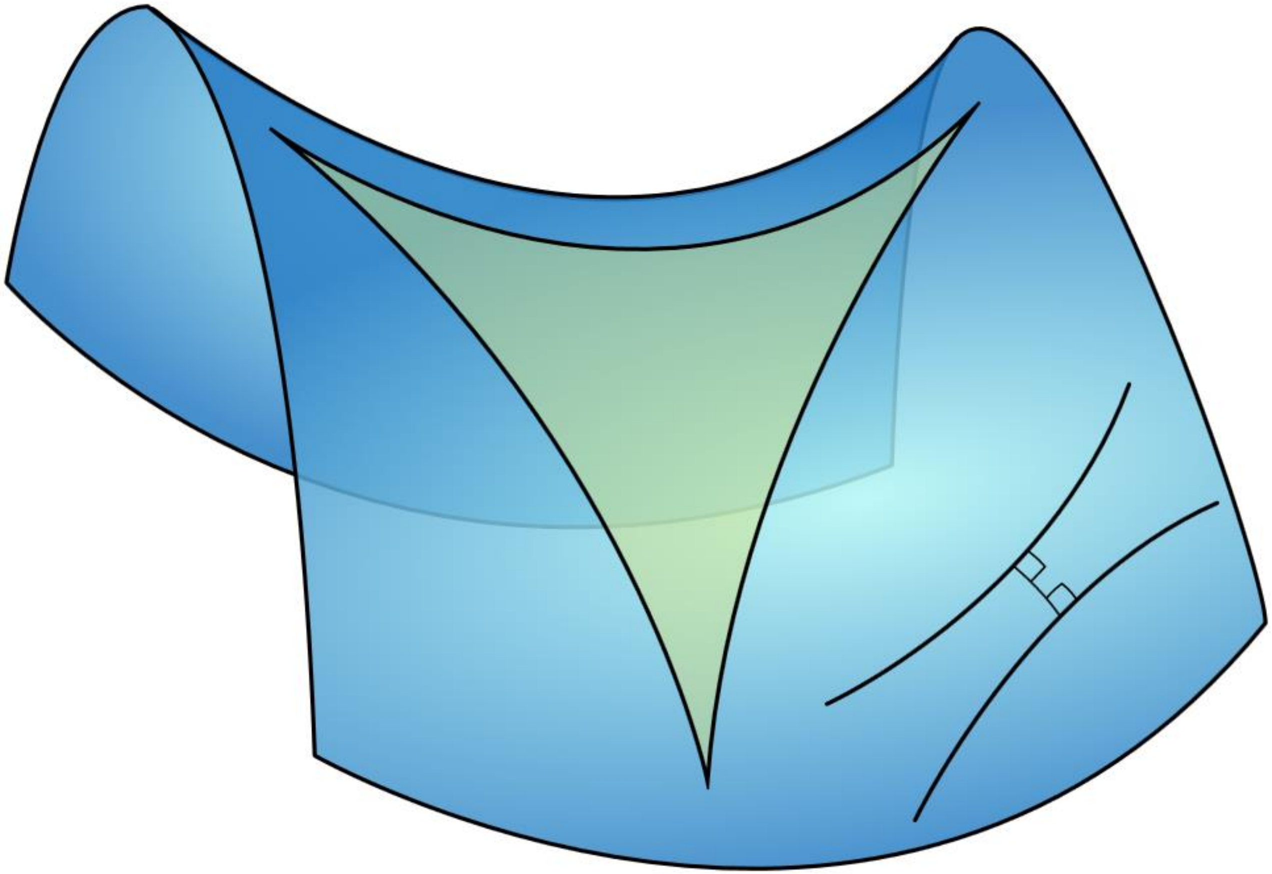






Spivak: *A Comprehensive Introduction to Differential Geometry*

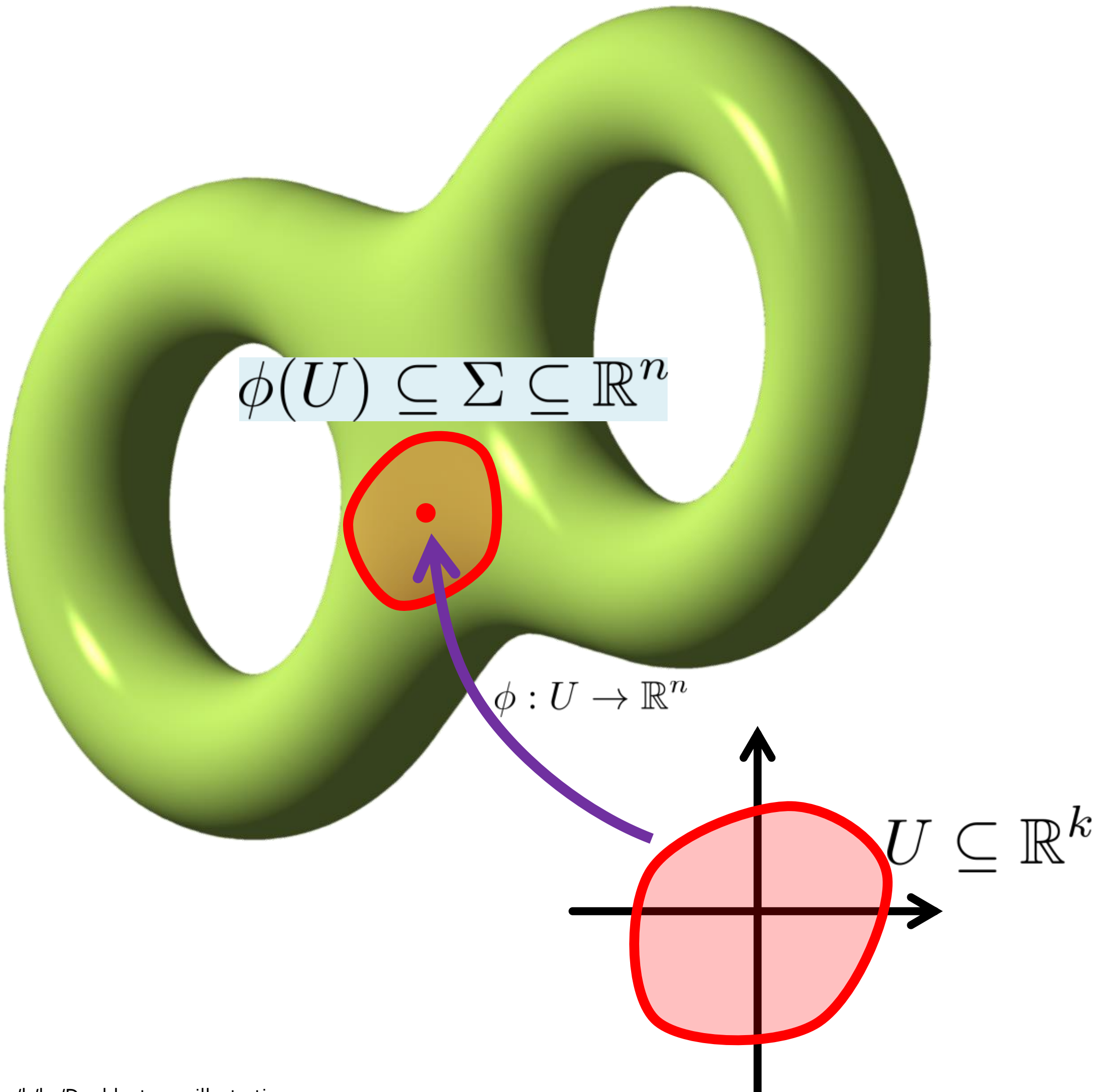
Differential Geometry



http://en.wikipedia.org/wiki/Differential_geometry

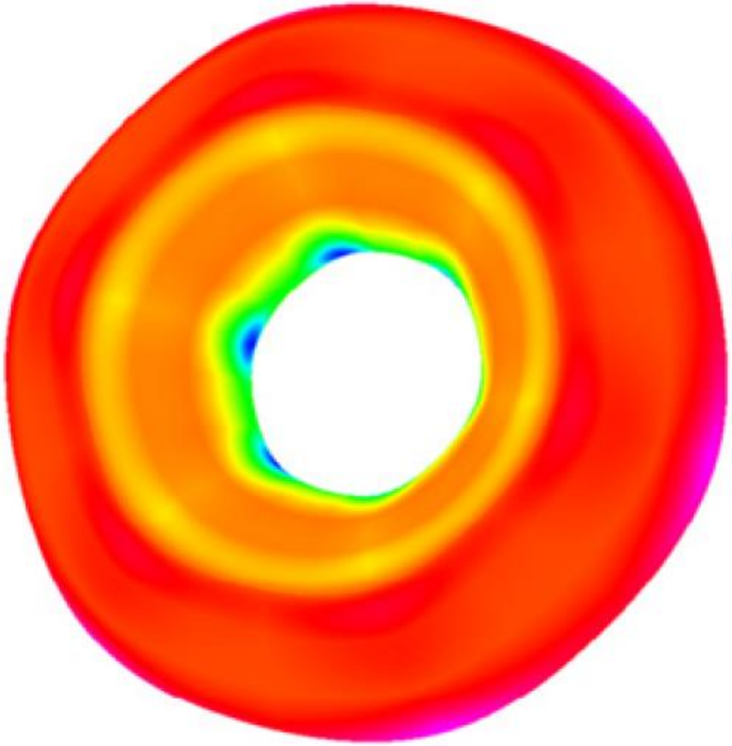
Study of smooth manifolds

Manifold

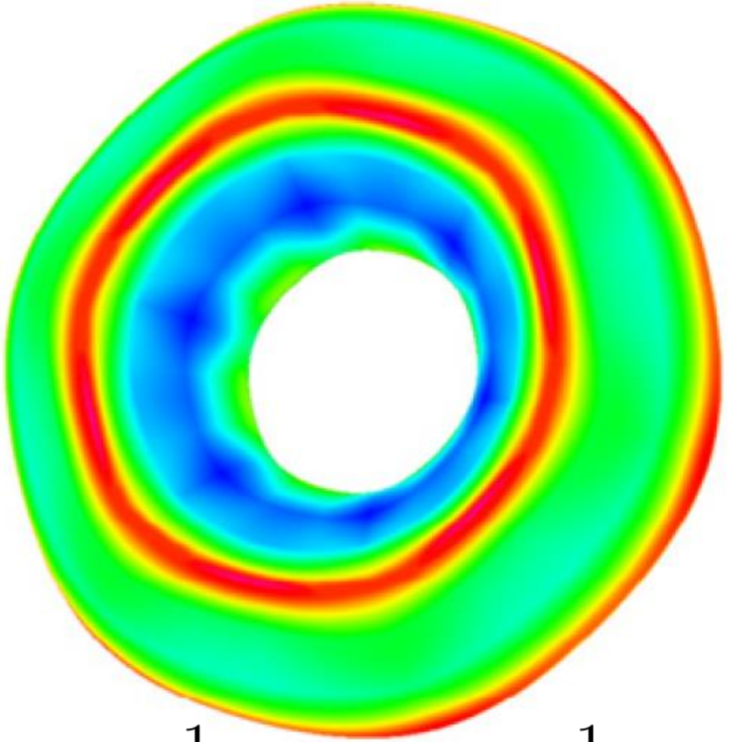


http://upload.wikimedia.org/wikipedia/commons/b/bc/Double_torus_illustration.png

Differential Geometry Toolbox



$$K := \kappa_1 \kappa_2 = \det \mathrm{I\!I}$$

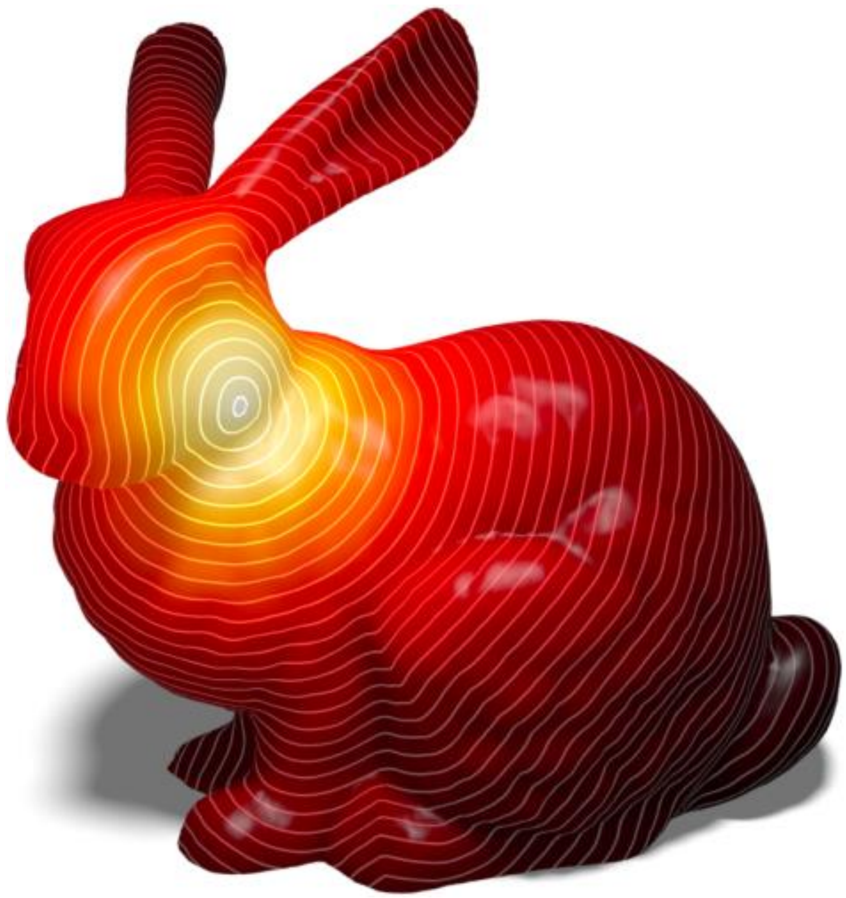


$$H := \frac{1}{2}(\kappa_1 + \kappa_2) = \frac{1}{2} \mathrm{tr}\, \mathrm{I\!I}$$

http://www.sciencedirect.com/science/article/pii/S0010448510001983

Curvature and shape properties

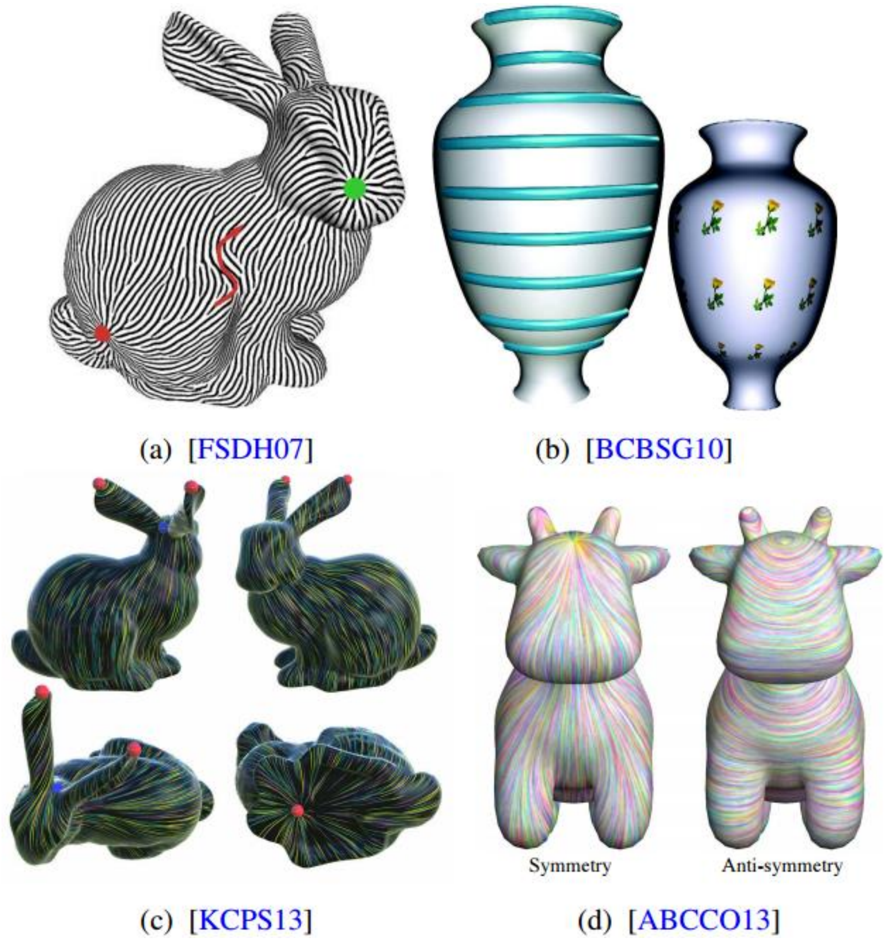
Differential Geometry Toolbox



Crane, Weischedel, Wardetzky.
Geodesics in heat. TOG 2013.

Distances

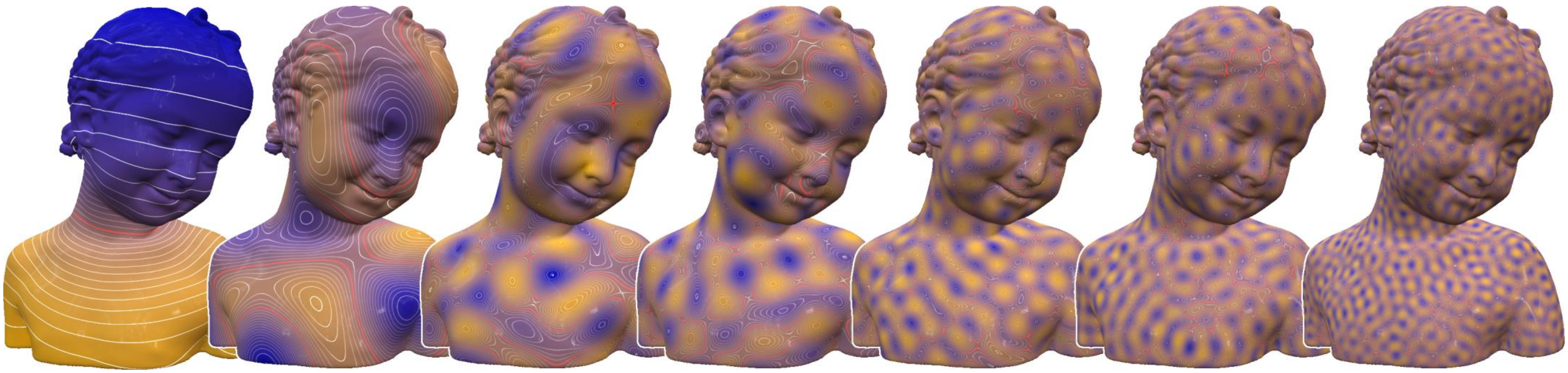
Differential Geometry Toolbox



Vaxman et al.
Directional field synthesis, design, and processing.
EG STAR 2016.

Flows and vector fields

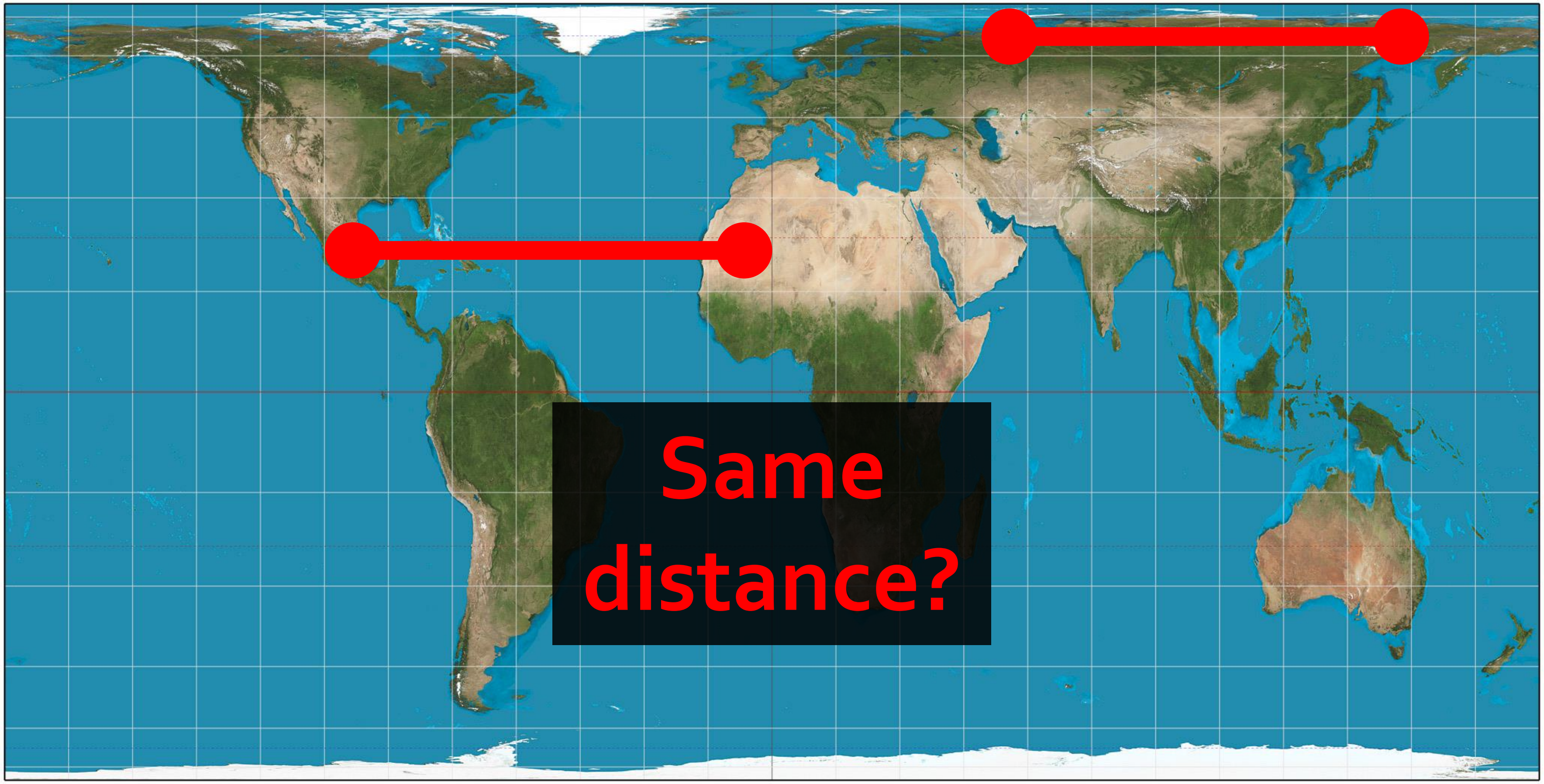
Differential Geometry Toolbox



Vallet and Lévy.
Spectral Geometry Processing with Manifold Harmonics. EG 2008

Differential operators

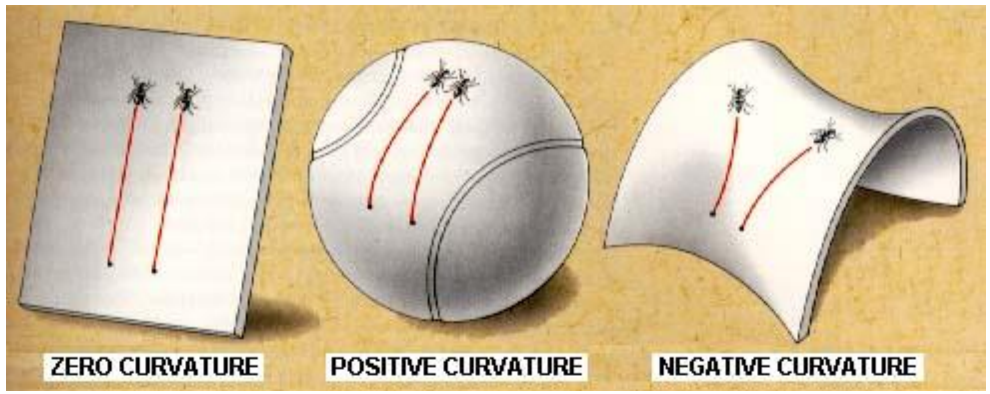
Riemannian Viewpoint

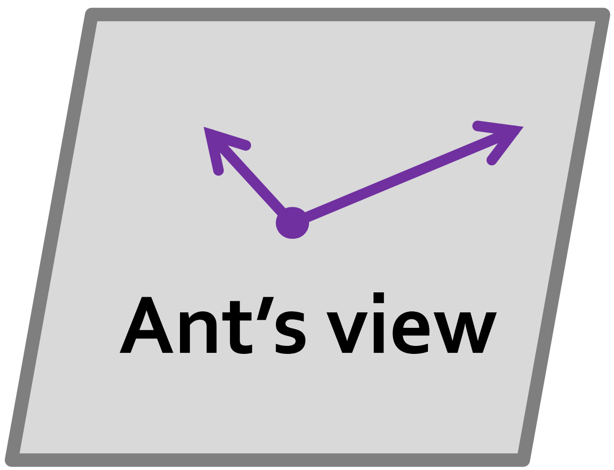


http://upload.wikimedia.org/wikipedia/commons/2/2c/Hobo%E2%80%93Dyer_projection_SW.jpg

Only need angles and distances

Riemannian Viewpoint

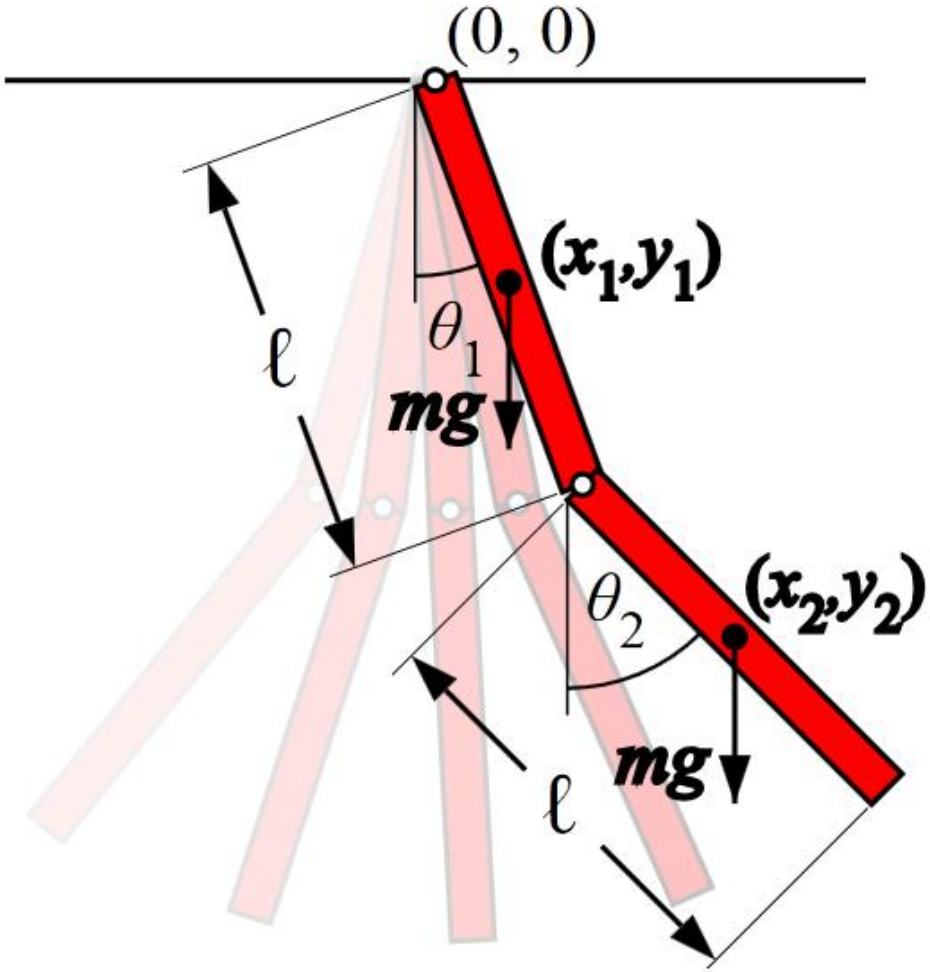


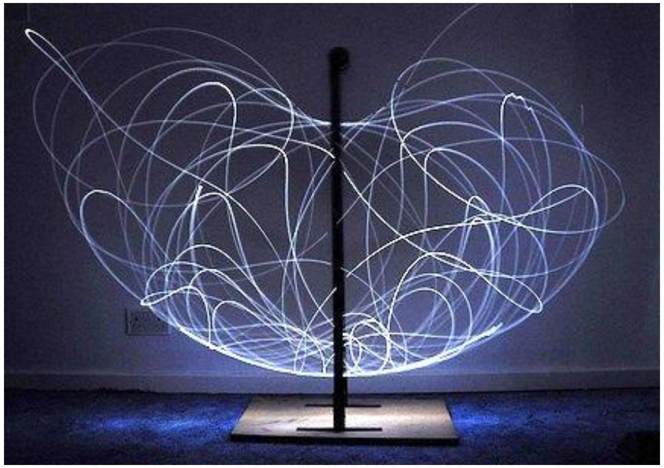


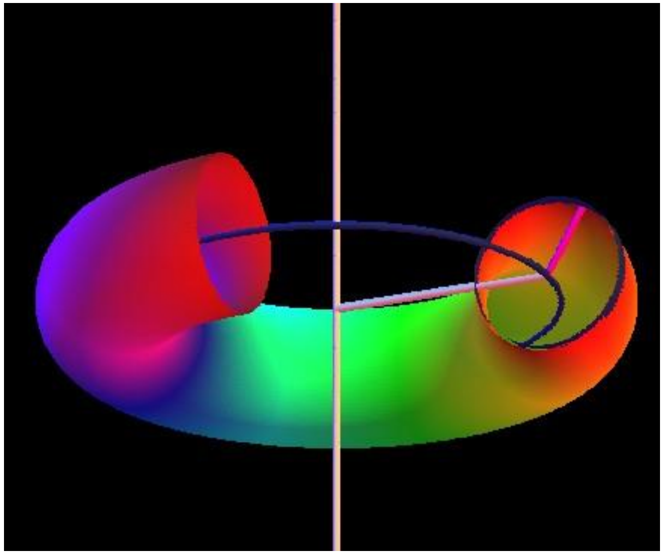
http://www.phy.syr.edu/courses/modules/LIGHTCONE/pics/curved.jpg

Only need angles and distances

Geometric Mechanics

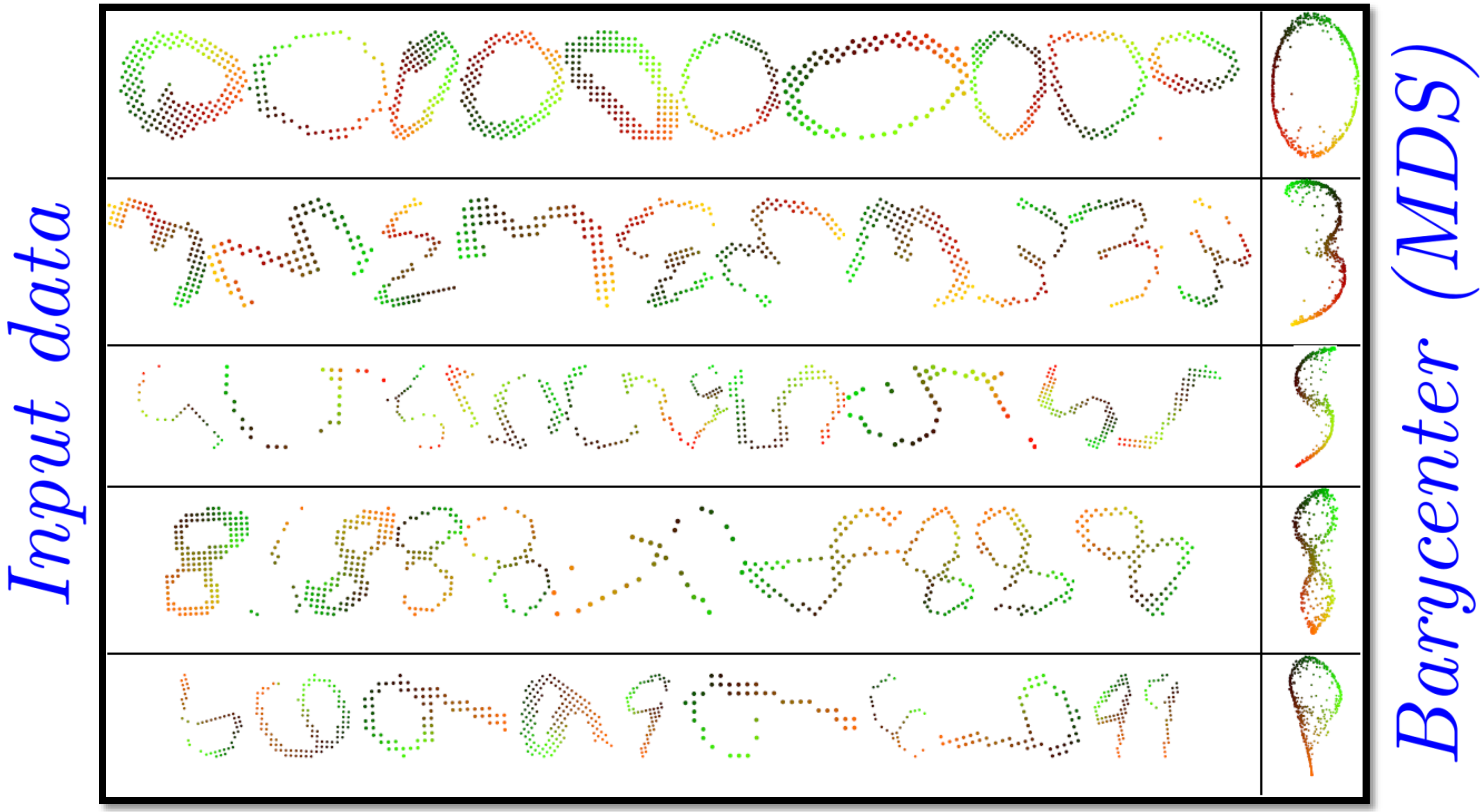






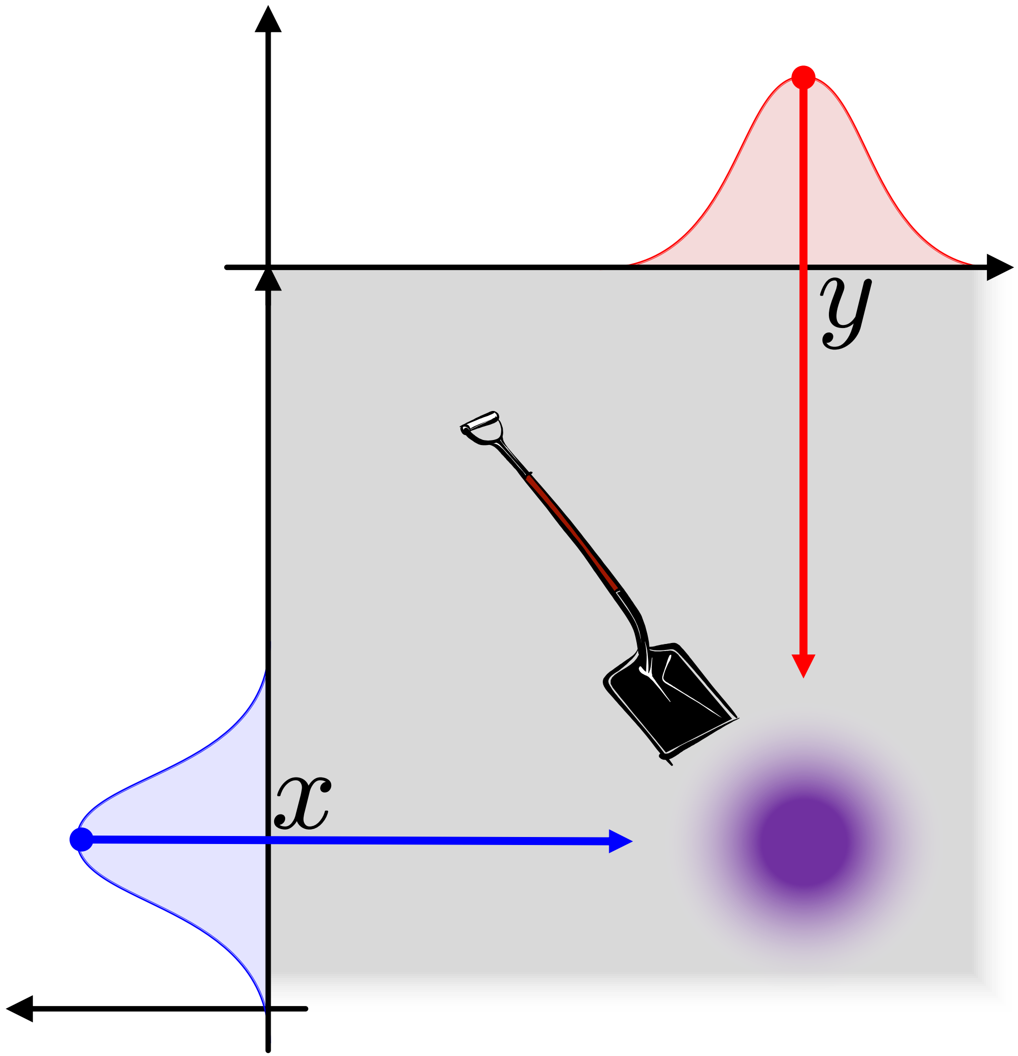
http://en.wikipedia.org/wiki/Double_pendulum
http://www.ualberta.ca/dept/math/gauss/fcm/BscIdeas/SpcDmnsn/pndlm2.htm

Metric Geometry

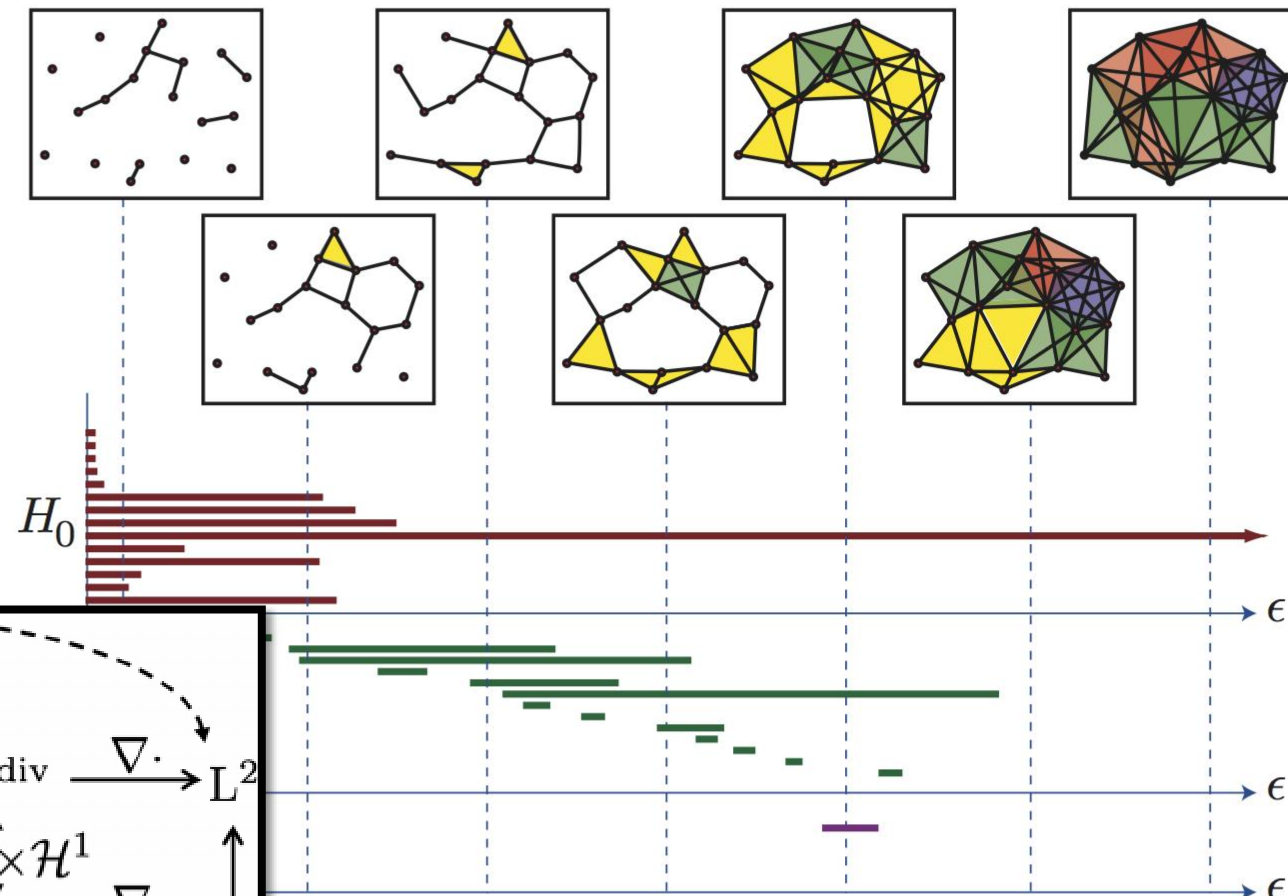


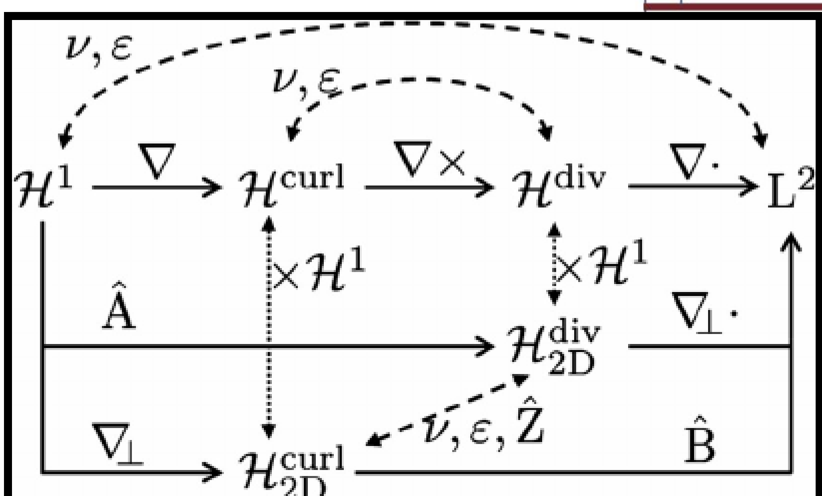
Peyré, Cuturi, and Solomon.
Gromov-Wasserstein Averaging of Kernel and Distance Matrices.
ICML 2016.

Optimal Transport



{Differential/Morse/Persistent/…} Topology





https://www.researchgate.net/publication/274071948_Space_charge_and_resistive_wall_impedance_computation_in_the_frequency_domain_using_the_finite_element_method
http://blog.forty.to/wp-content/uploads/2013/05/Bars1.jpg

Plan for Today

I. Theoretical toolbox

II. **Computational toolbox**

III. Application areas

Many Notions of Shape

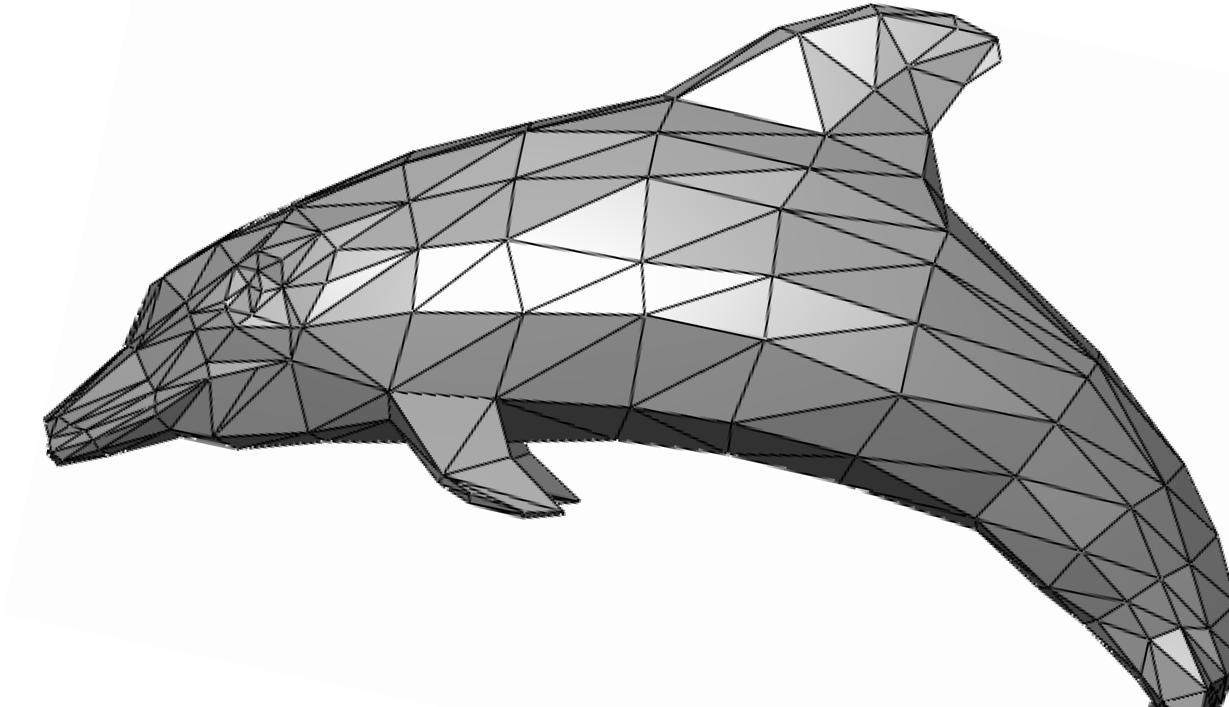
- Triangle mesh
- Triangle soup
- Graph
- Point cloud
- Pairwise distance matrix

Nearly anything with a notion of proximity/distance/curvature/…

Typical issue:

Euclidean? Riemannian?

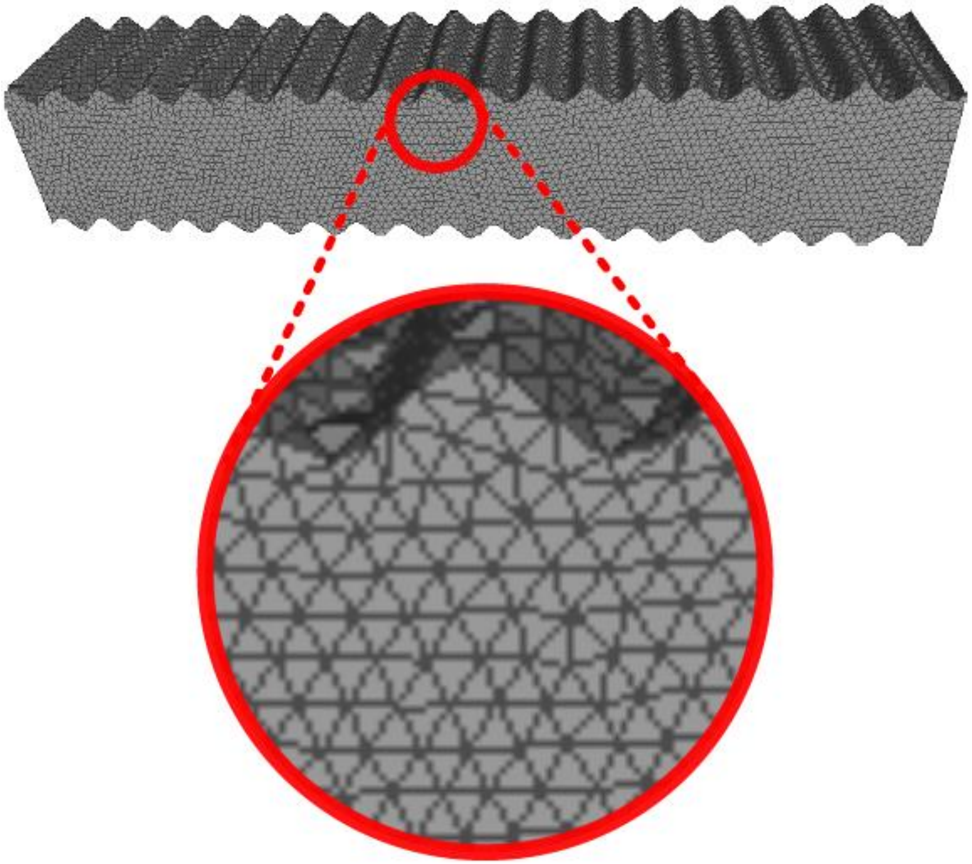
- Collection of flat triangles
- Approximates a smooth surface



Q:

Can a triangle mesh have curvature?

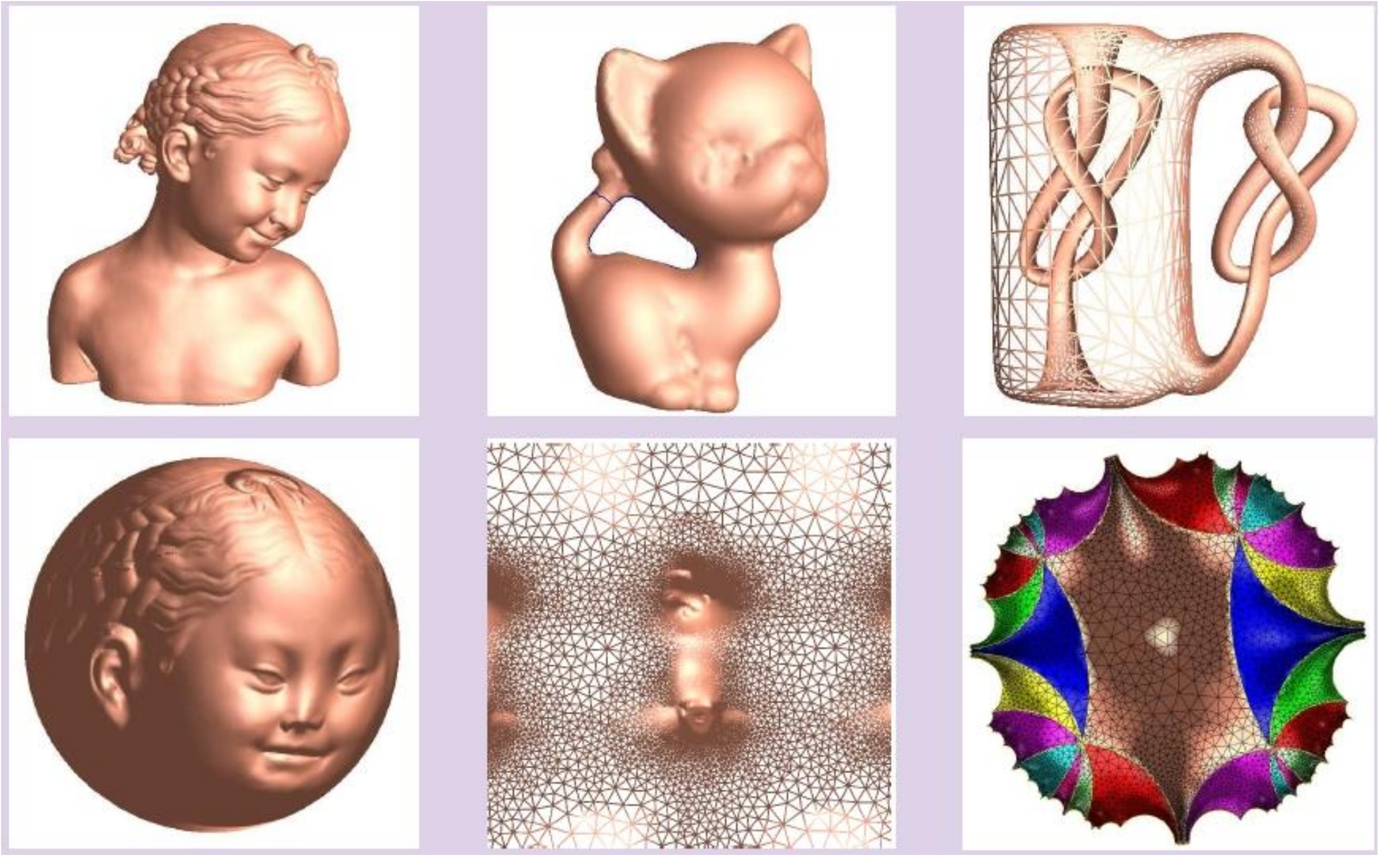
Jack of All Trades



Combine smooth and discrete

Example:

Discrete Differential Geometry



http://www.geomtop.org/teaching/conformal_2012Summer/title_image.jpg

Modern Approach

Discrete

vs.

Discretized

Discrete Differential Geometry

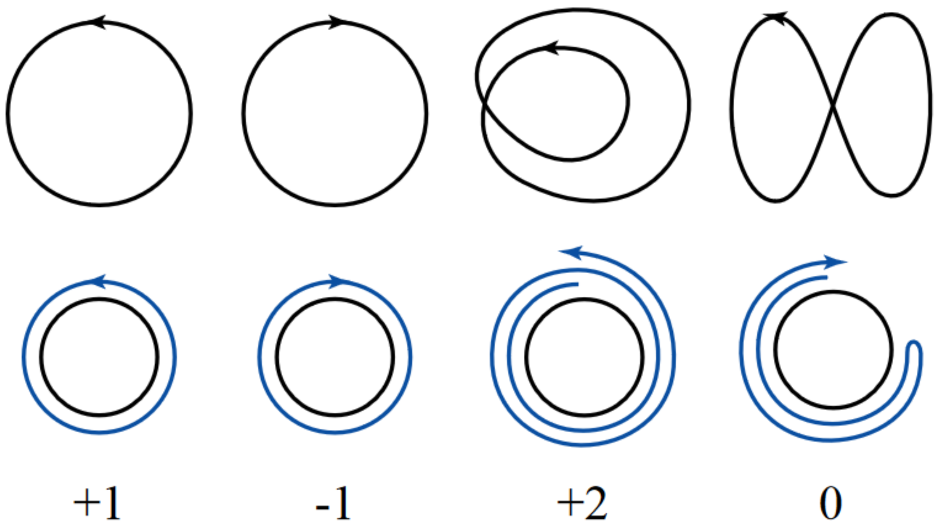
Discrete theory *paralleling* differential geometry.

Structure preservation

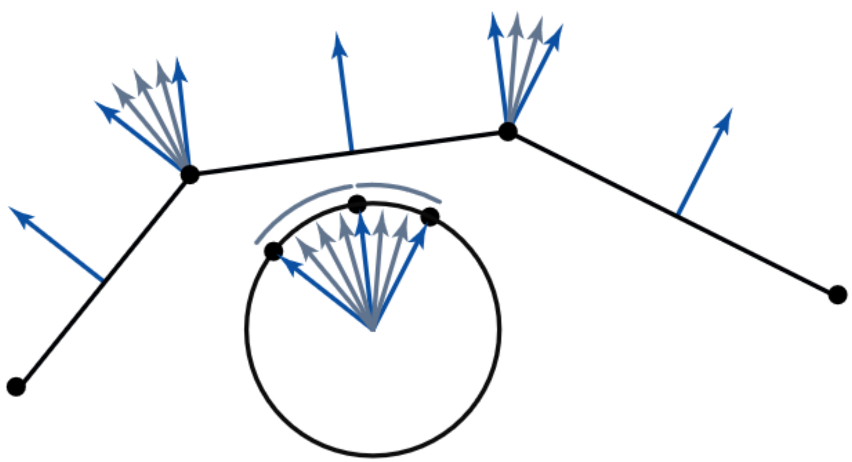
[**struhk**-cher pre-zur-**vey**-sh*uh* n]**:**

Keeping properties from the continuous abstraction exactly true in a discretization.

Example: Turning Numbers



$$\int_{\Omega} \kappa \, ds = 2\pi k$$



$$\sum_i \alpha_i = 2\pi k$$

Images from: Grinspun and Secord, "The Geometry of Plane Curves" (SIGGRAPH 2006)

Convergence

[k*uh* n-**vur**-j*uh* ns]**:**

Increasing approximation quality as a discretization is refined.

Convergence *and* Structure

Can you have it all?

Disappointing Result

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

Max Wardetzky[1] Saurabh Mathur[2] Felix Kälberer[1] Eitan Grinspun[2] †

[1]Freie Universität Berlin, Germany [2]Columbia University, USA

Abstract

Discrete Laplace operators are ubiquitous in applications spanning geometric modeling to simulation. For robustness and efficiency, many applications require discrete operators that retain key structural properties inherent to the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tau00, Zha04, FH05, Sor05].

In applications one often requires certain structural prop-

1.1. Properties of smooth Laplacians

Consider a smooth surface S, possibly with boundary, equipped with a Riemannian metric, *i.e.*, an intrinsic notion of distance. Let the intrinsic L^2 inner product of functions u and v on S be denoted by $(u,v)_{L^2} = \int_S uv\, dA$, and let $\Delta = -\operatorname{div} \operatorname{grad}$ denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

(NULL) $\Delta u = 0$ whenever u is constant

Disappointing Result

Eurographics Symposium on Geometry Processing (2007)
Alexander Belyaev, Michael Garland (Editors)

Discrete Laplace operators: No free lunch

Max Wardetzky[1] Saurabh Mathur[2] Felix Kälberer[1] Eitan Grinspun[2 †]

[1]Freie Universität Berlin, Germany [2]Columbia University, USA

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(NULL) $\Delta u = 0$ whenever u is constant

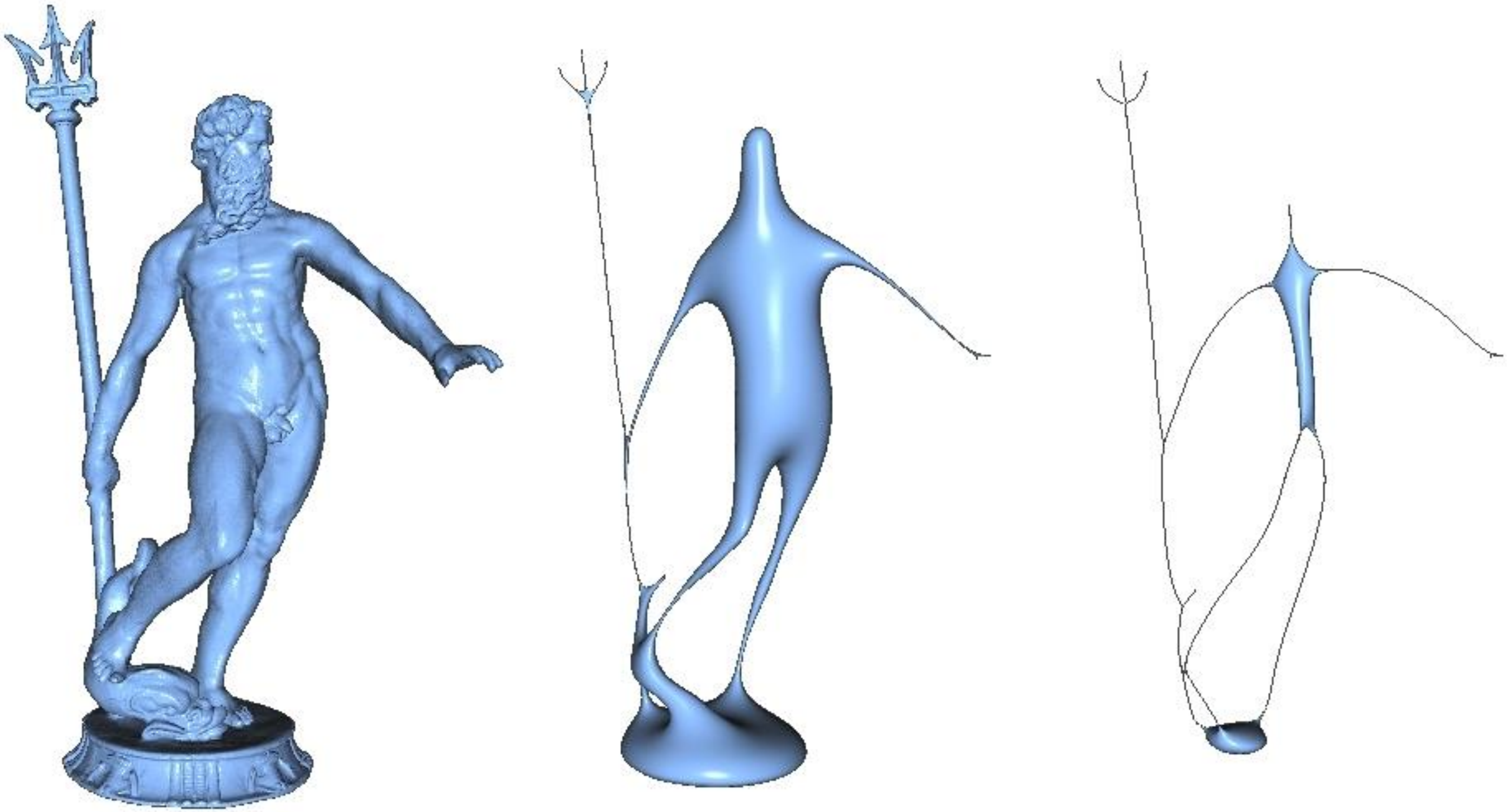
Theme

Pick and choose

which properties you need.

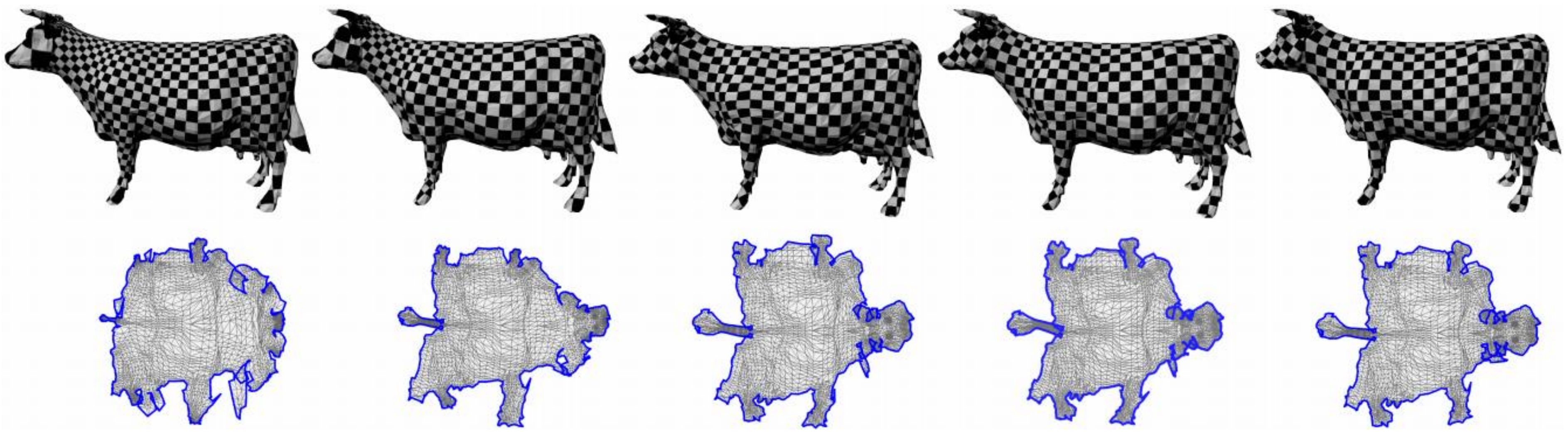
But there is a huge toolbox to draw from!

Numerical PDE



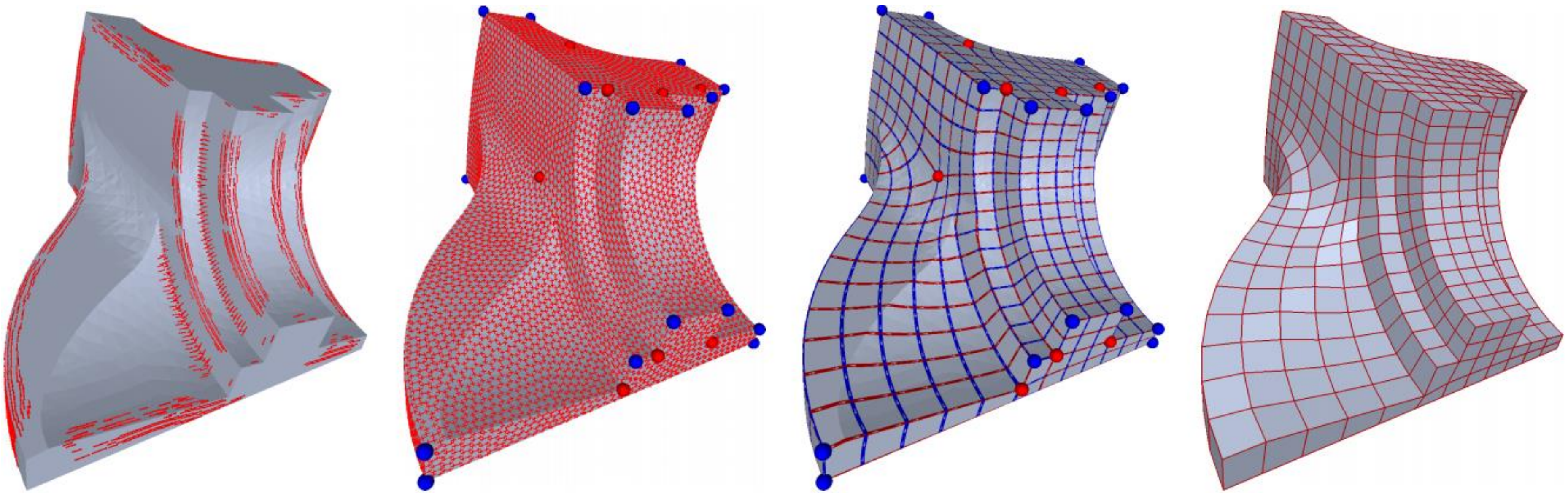
Chuang and Kazhdan.
Fast Mean-Curvature Flow via Finite-Elements Tracking.
CGF 2011.

Smooth Optimization



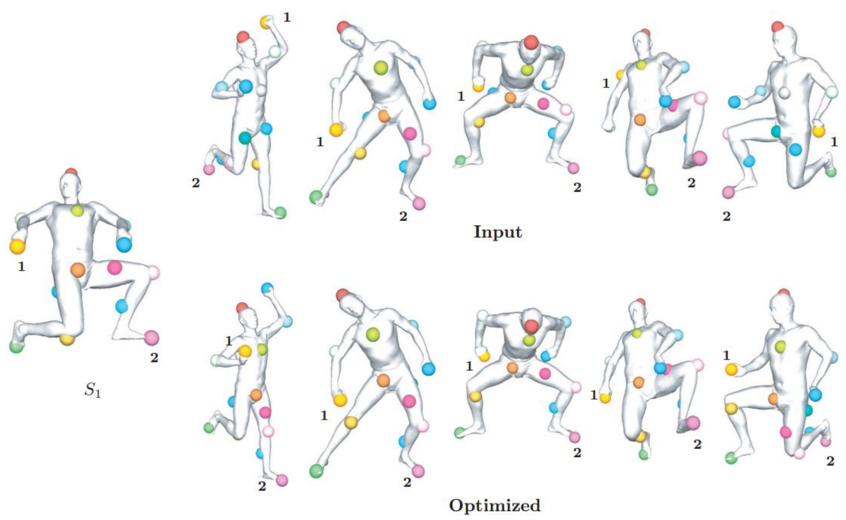
Smith and Schaefer. *Bijective parameterization with free boundaries*. SIGGRAPH 2015.

Discrete Optimization

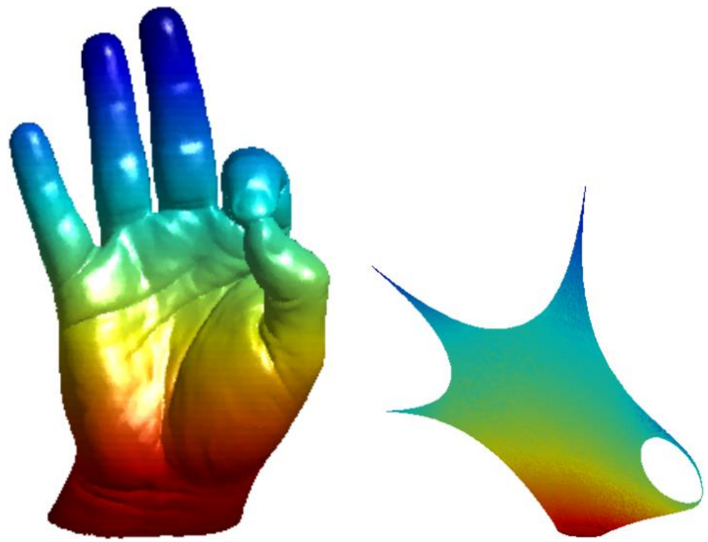


Bommes, Zimmer, Kobbelt. *Mixed-integer quadrangulation*. SIGGRAPH 2009.

Linear Algebra

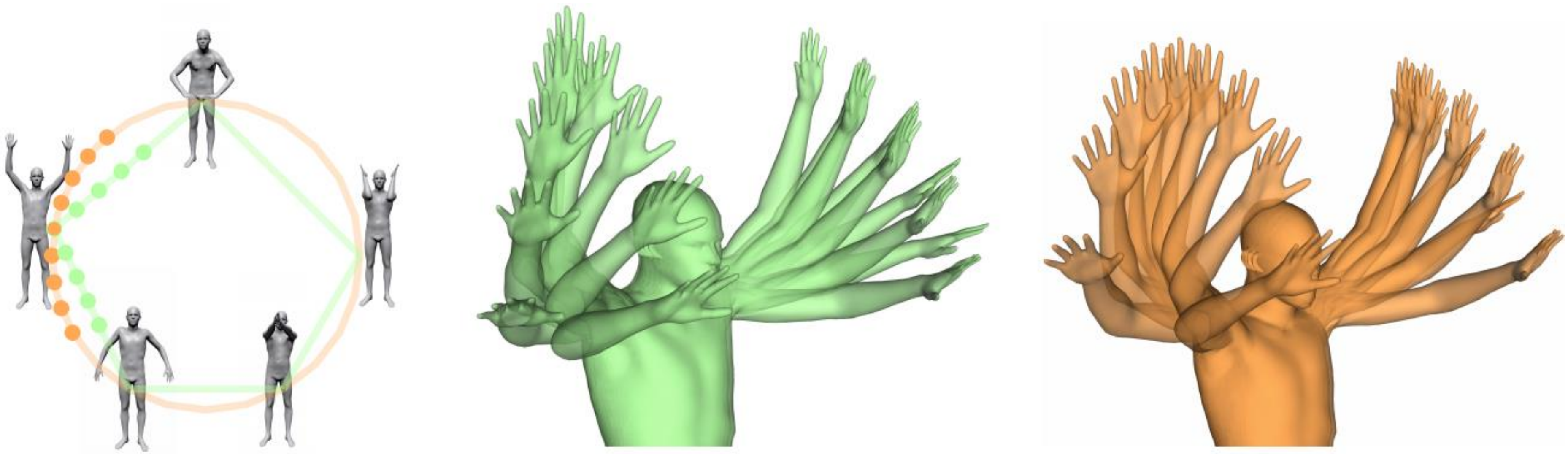


Huang, Guibas. *Consistent shape maps via semidefinite programming*. SGP 2013.



Krishnan, Fattal, Szeliski. *Efficient preconditioning of Laplacian matrices for computer graphics*. SIGGRAPH 2013.

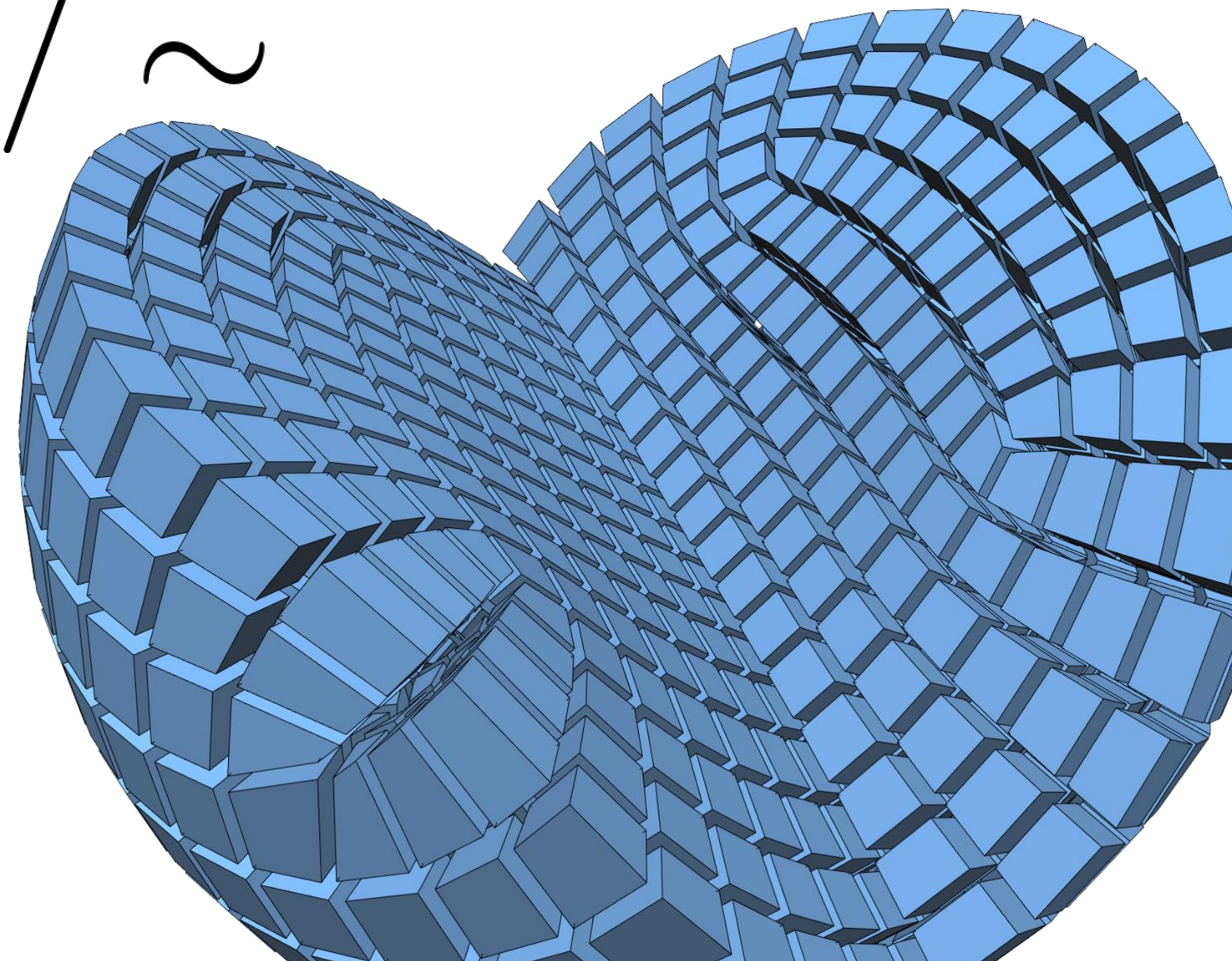
The "Geometry of Geometry"



Heeren et al. *Splines in the space of shells*. SGP 2016.

Algebra & Representation Theory

$$\mathrm{SO}(3)/\sim$$



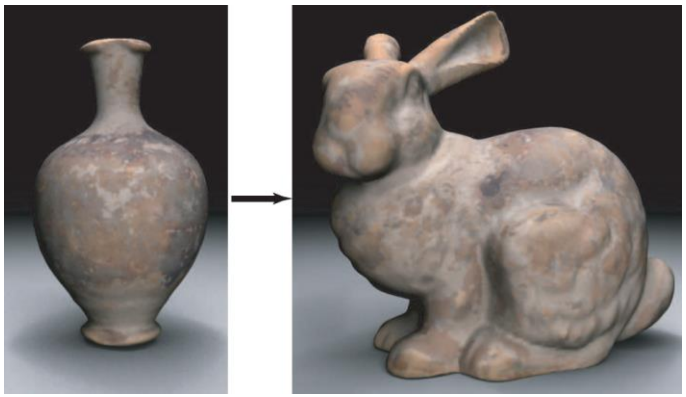
Plan for Today

I. Theoretical toolbox

II. Computational toolbox

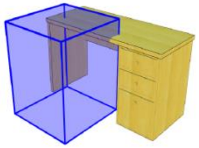
III. Application areas

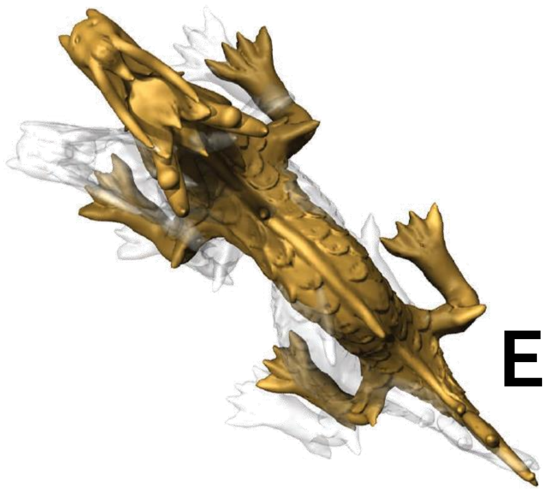
Applications



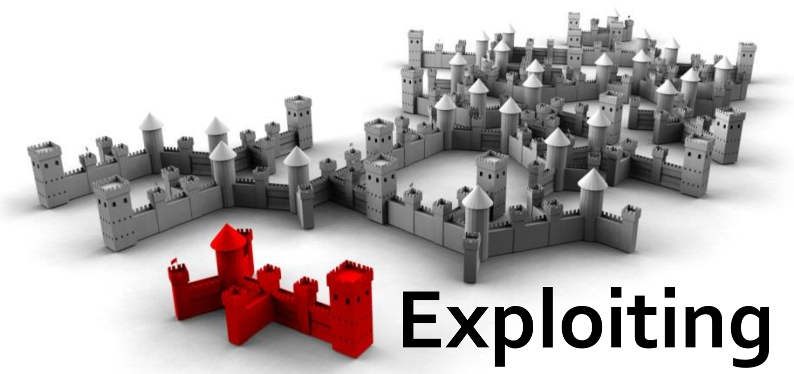
Transfer

Retrieval





Editing

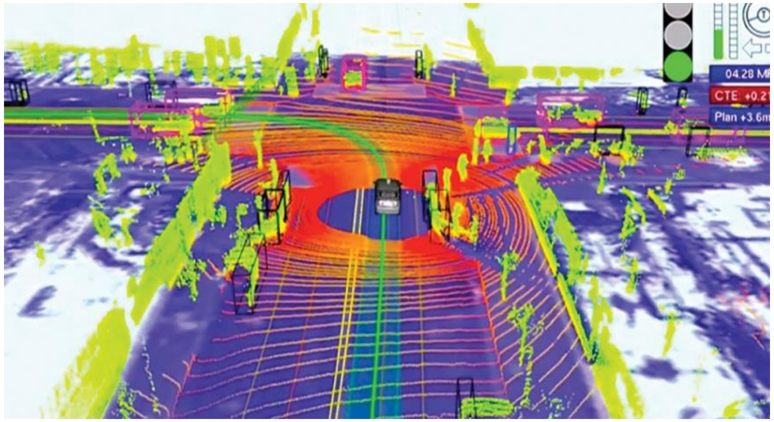


Exploiting patterns

http://people.csail.mit.edu/tmertens/papers/textransfer_electronic.pdf
http://graphics.stanford.edu/~mdfisher/Data/Context.pdf
http://graphics.stanford.edu/~niloy/research/symmetrization/paper_docs/symmetrization_sig_07.pdf
http://www.mpi-inf.mpg.de/~mbokeloh/project_dockingSites.html

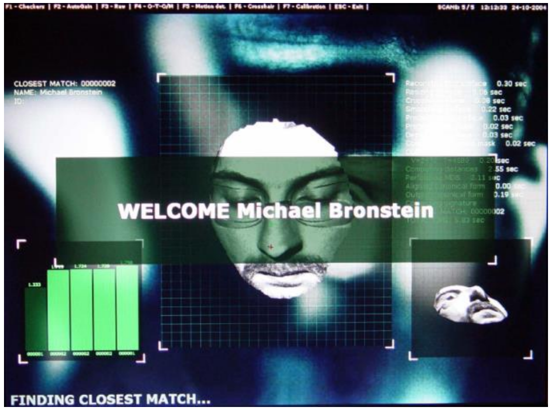
Graphics

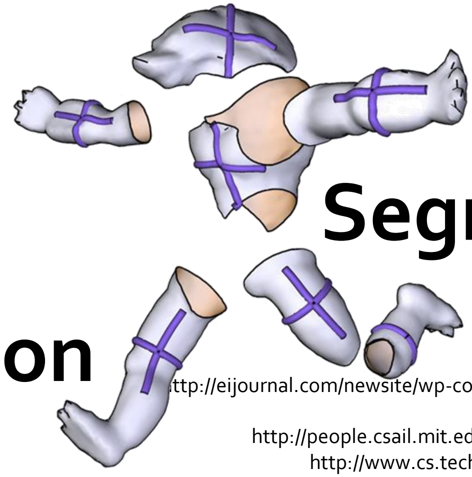
Applications



Recognition

Navigation





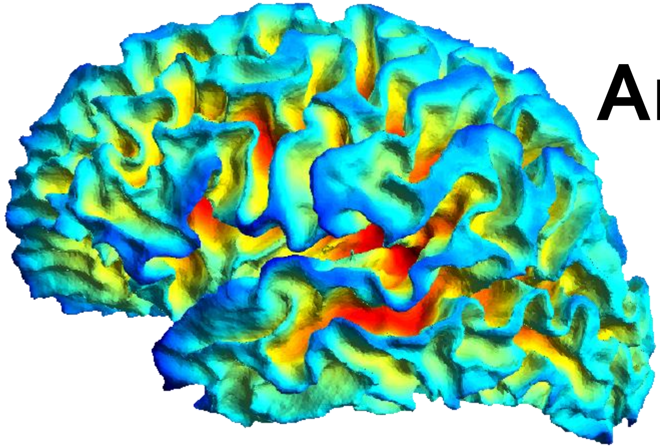
Segmentation

Reconstruction

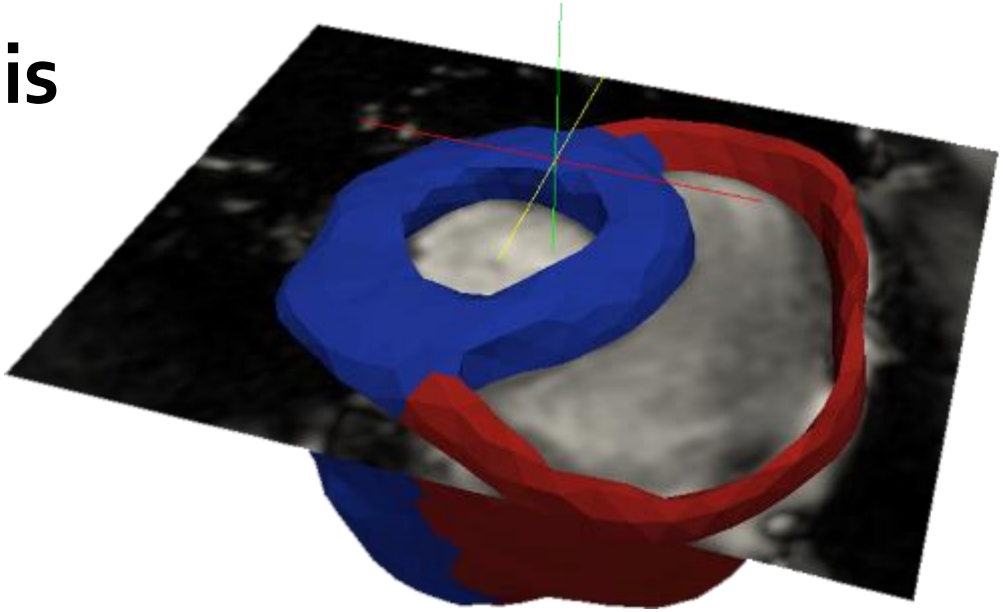
ttp://eijournal.com/newsite/wp-content/uploads/2012/01/VELODYNE-IMAGE.jpg
http://www.stanford.edu/~jinhae/iccv09/
http://people.csail.mit.edu/jsolomon/assets/intrinsic_part_discovery.pdf
http://www.cs.technion.ac.il/~ron/PAPERS/BroBroKimIJCV05.pdf

Vision

Applications



Analysis



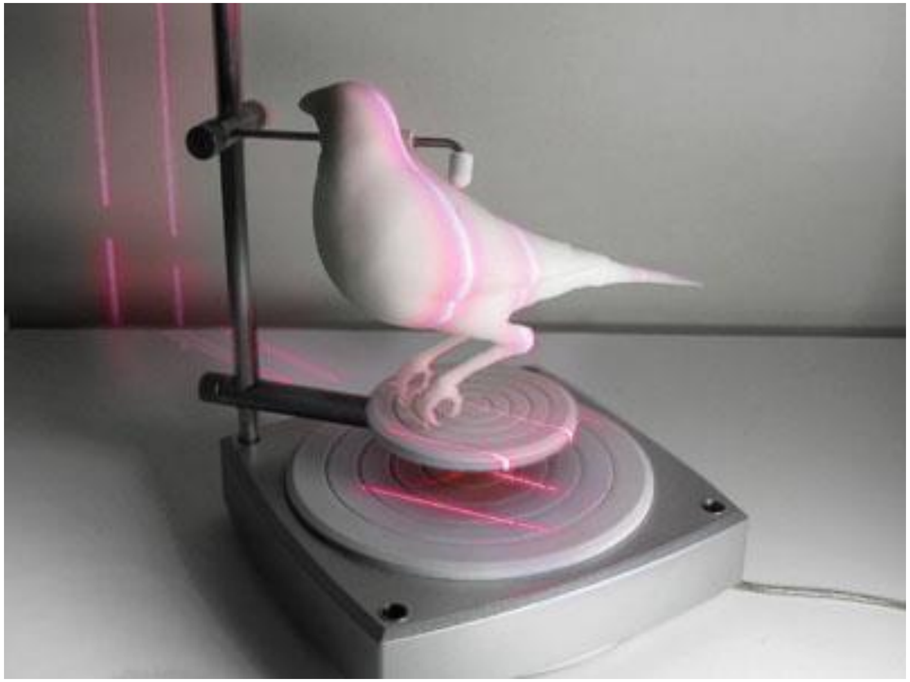
Segmentation

Registration

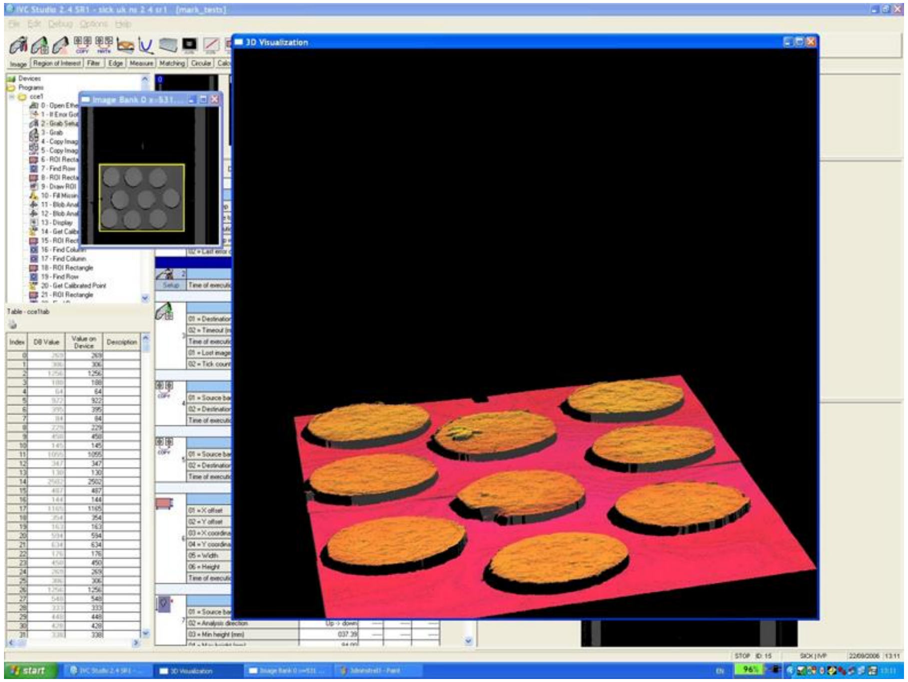
http://dmfr.birjournals.org/content/33/4/226/F3.large.jpg
http://www-sop.inria.fr/asclepios/software/inriaviz4d/SphericalImTransp.png
http://www.creatis.insa-lyon.fr/site/sites/default/files/segm2.png

Medical Imaging

Applications



Scanning

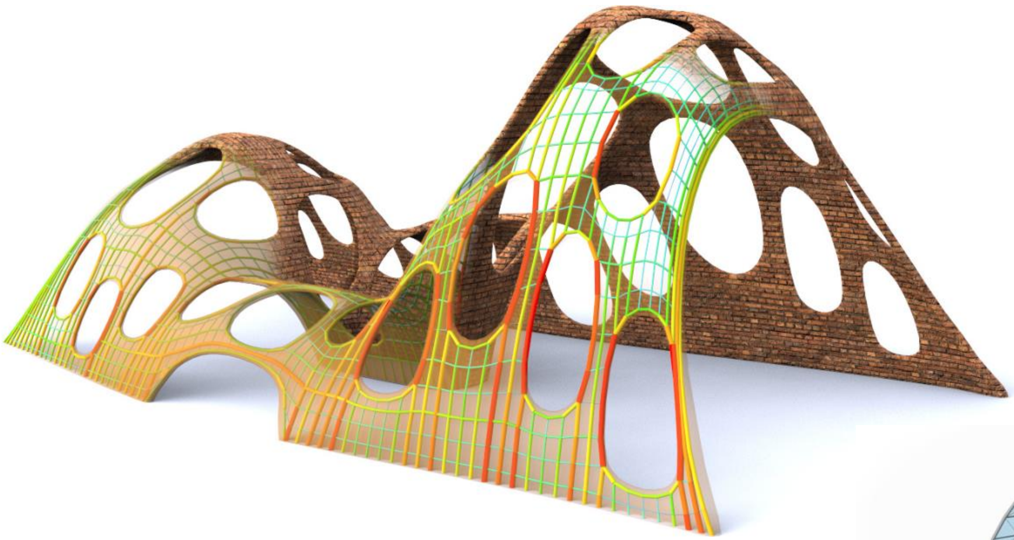


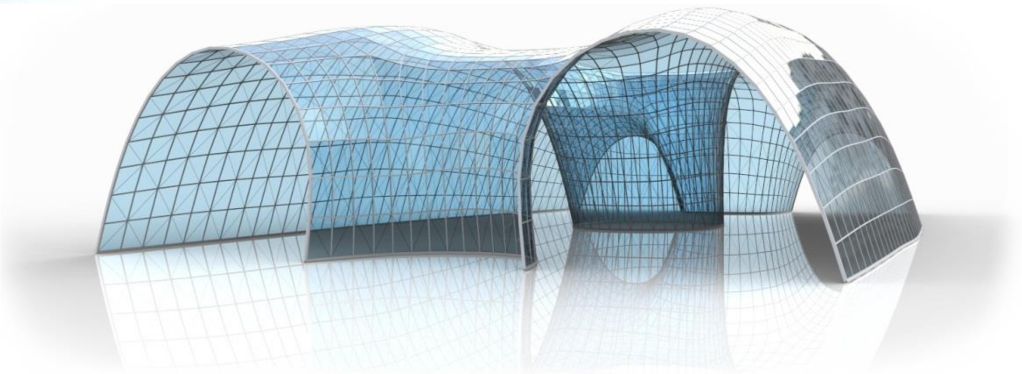
Defect detection

http://www.conduitprojects.com/php/images/scan.jpg
http://www.emeraldinsight.com/content_images/fig/0330290204005.png

Manufacturing and Fabrication

Applications



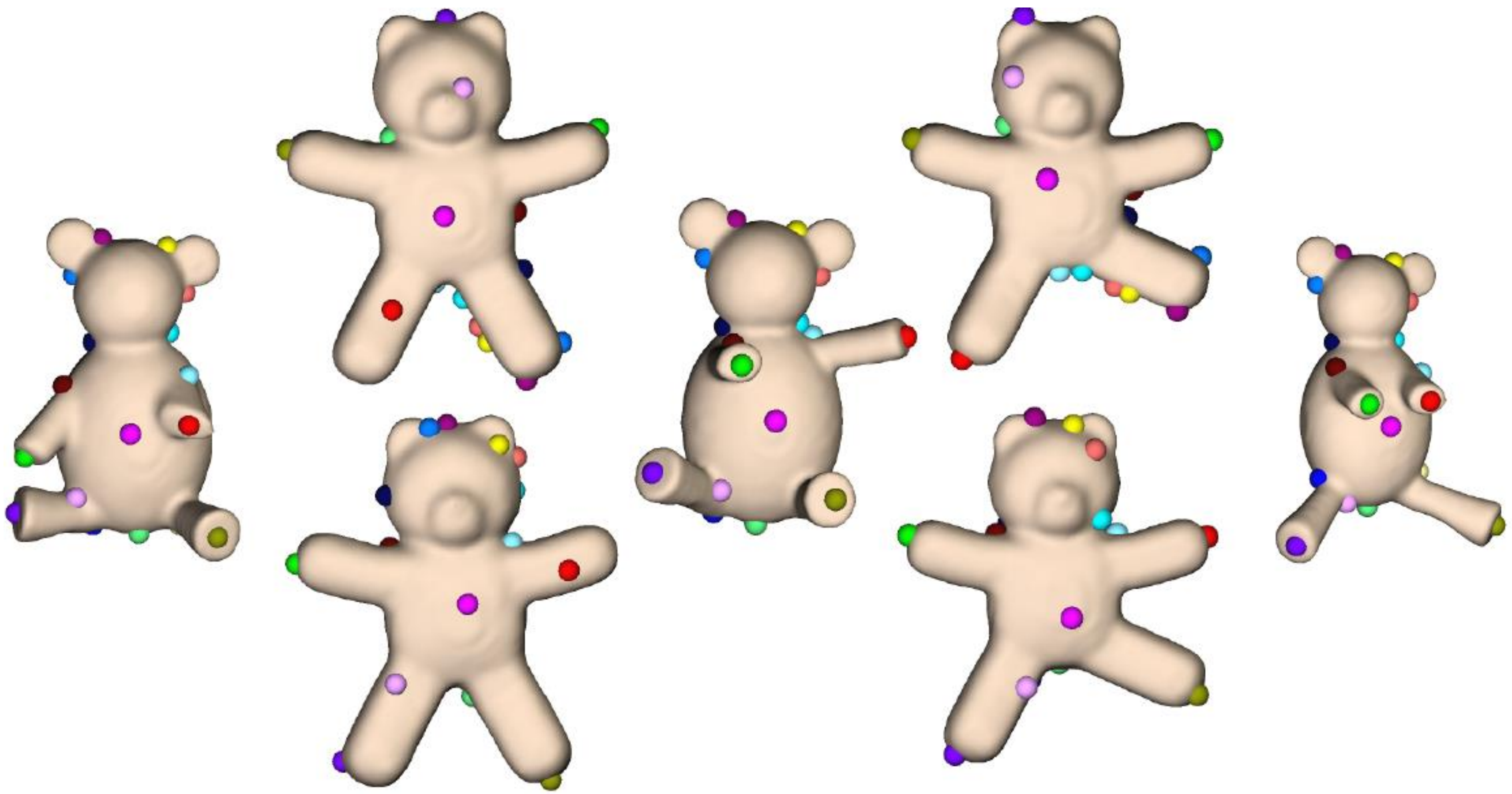


Design and analysis

http://gmsv.kaust.edu.sa/people/faculty/pottmann/pottmann_pdf/selfsupporting.pdf

Architecture

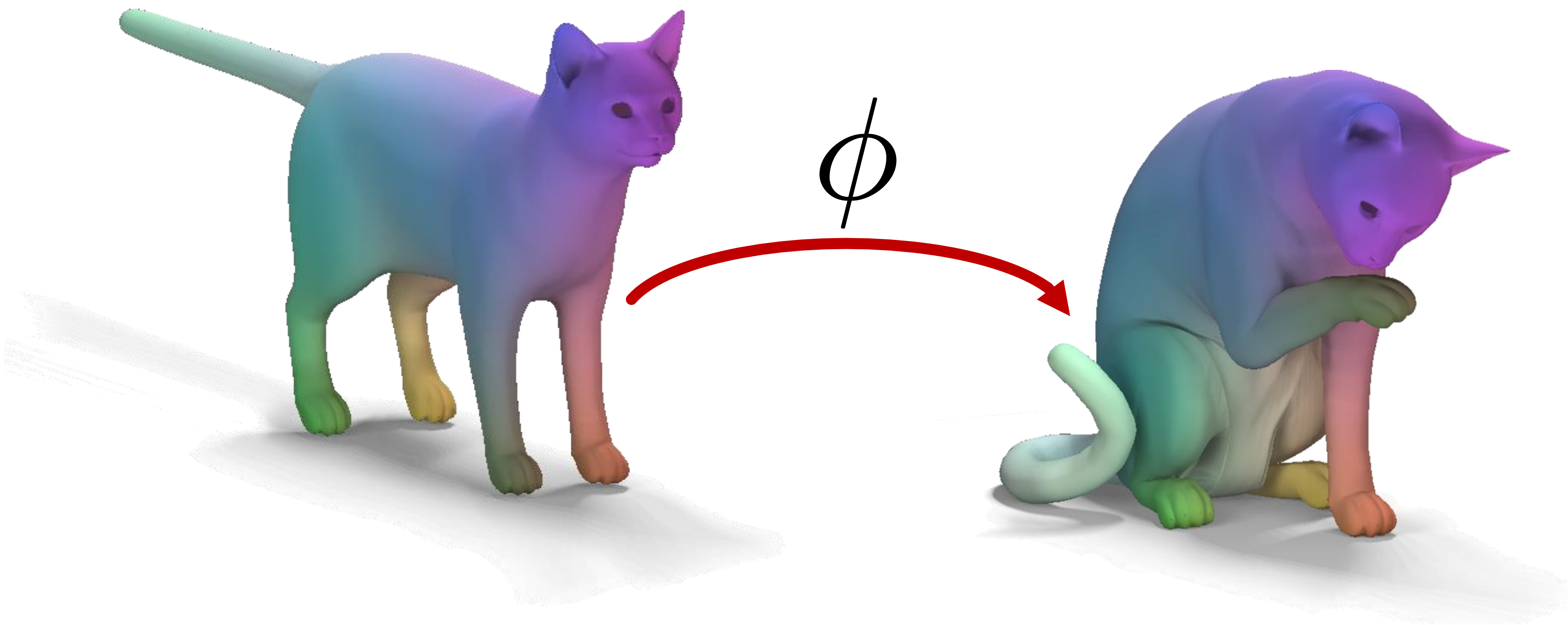
Applications



http://graphics.stanford.edu/projects/lgl/papers/nbwyg-oaicsm-11/nbwyg-oaicsm-11.pdf

Shape collection analysis

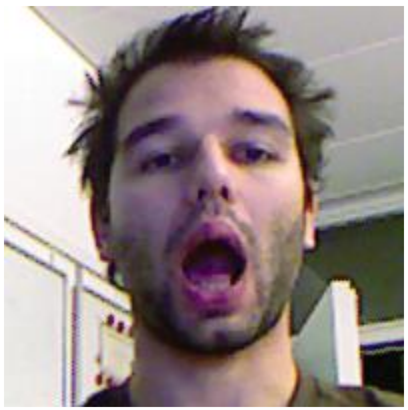
Applications



http://people.csail.mit.edu/jsolomon/assets/fmaps.pdf

Correspondence

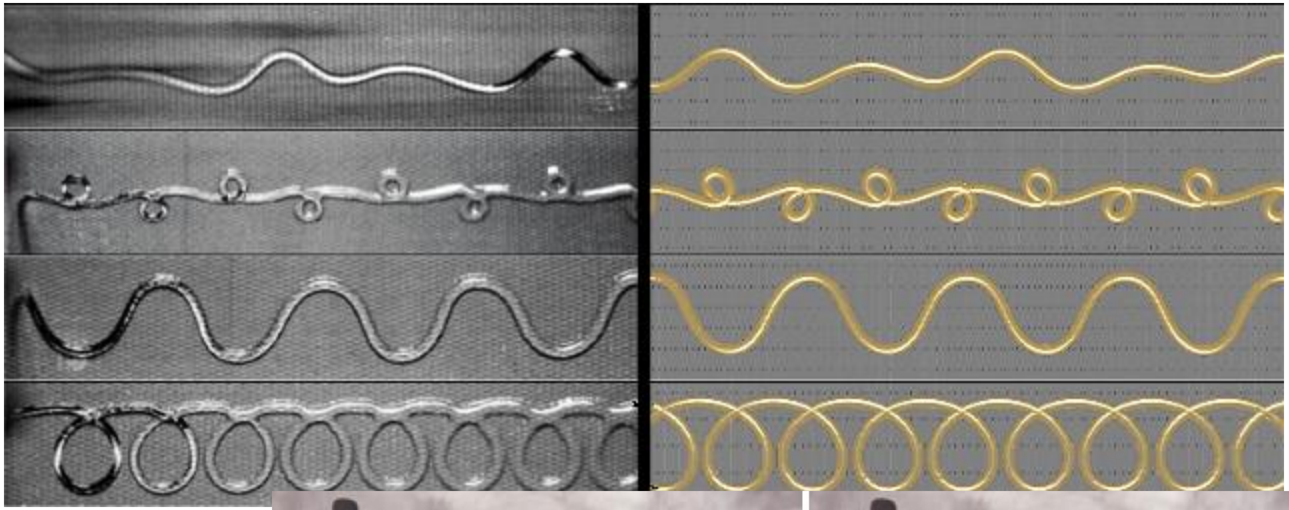
Applications



http://www.hao-li.com/publications/papers/siggraph2011RPBFA.pdf

Deformation transfer

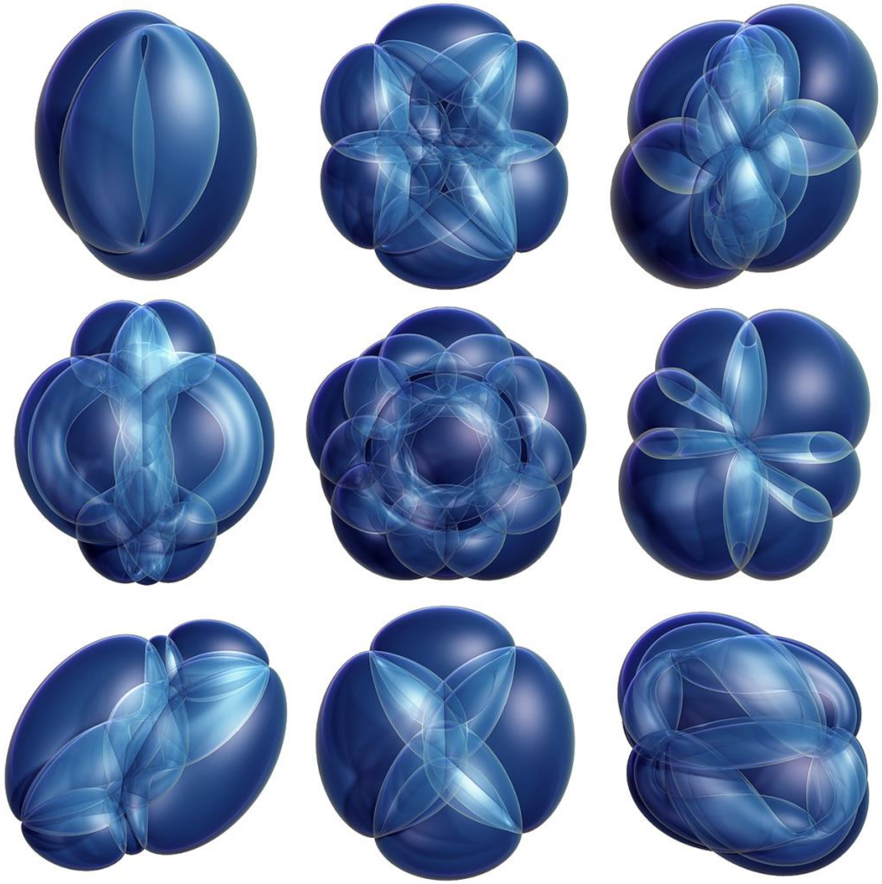
Applications



http://www.cs.columbia.edu/cg/threads/ http://mbergou.com/

Simulation

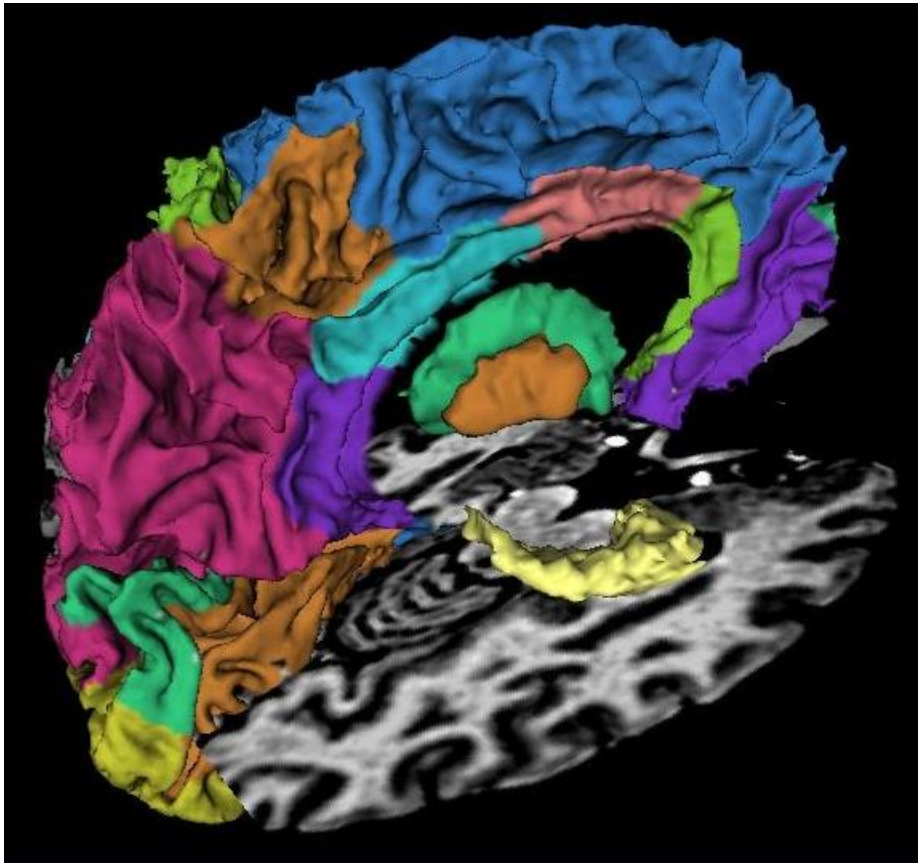
Applications

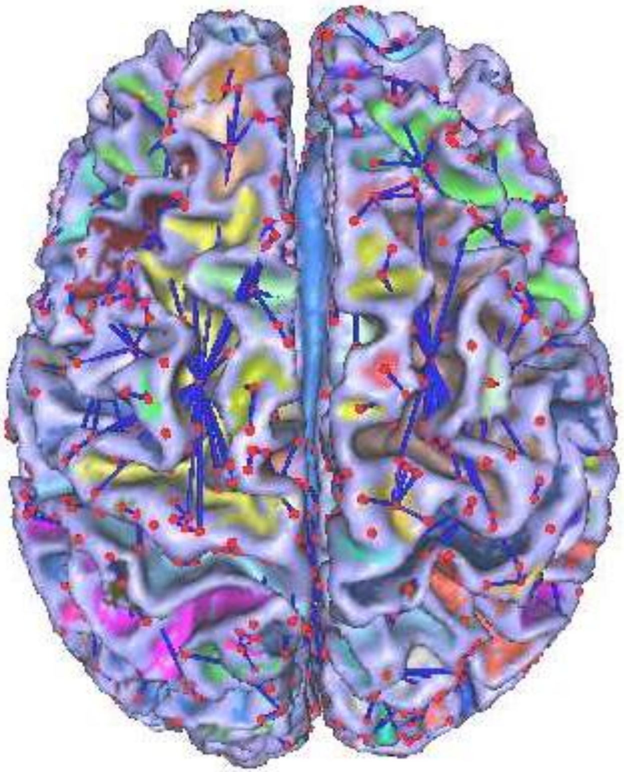


http://multires.caltech.edu/~keenan/pdf/spinxform.pdf

Scientific visualization

Applications





http://www.bioinformaticslaboratory.nl/twiki/pub/EBioScience/News/freesurfer-3d.jpg
http://hal.inria.fr/docs/00/40/21/30/IMG/vivodtzev_et_al-Dagstuhl03.jpg

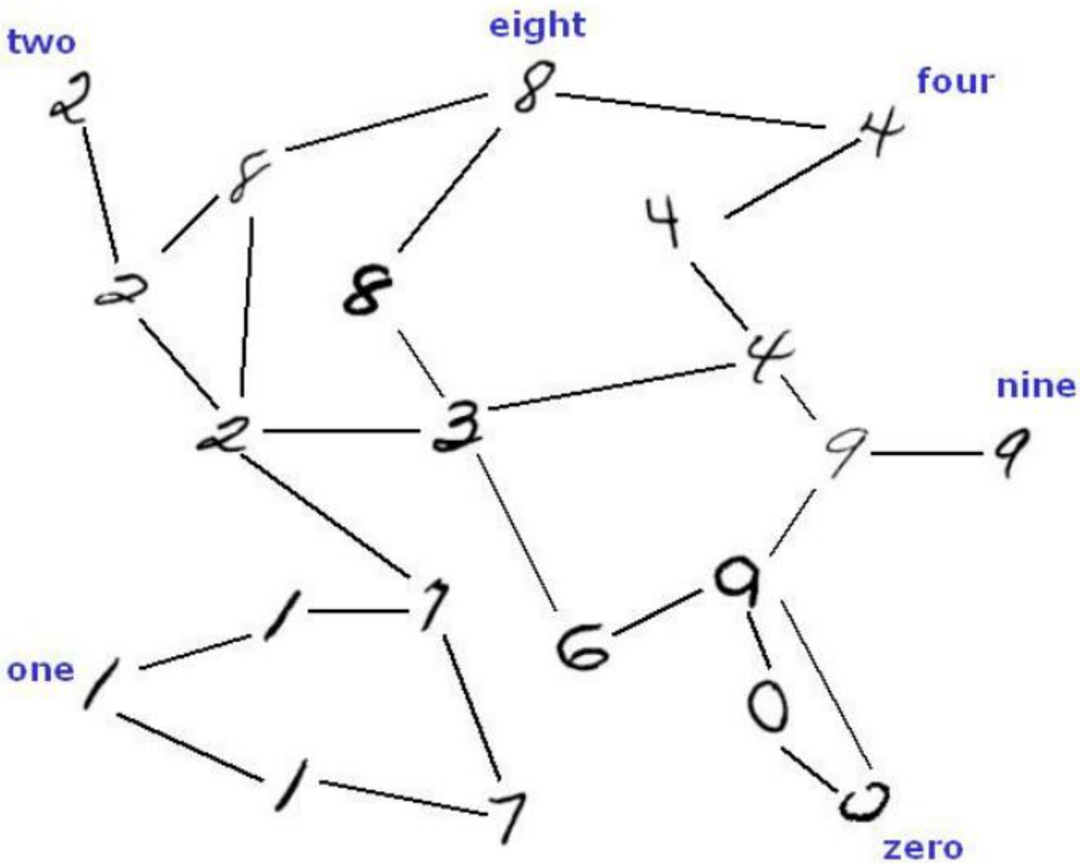
Segmentation

Applications

Su et al. *Estimating image depth using shape collections*. SIGGRAPH 2014.

Computer vision

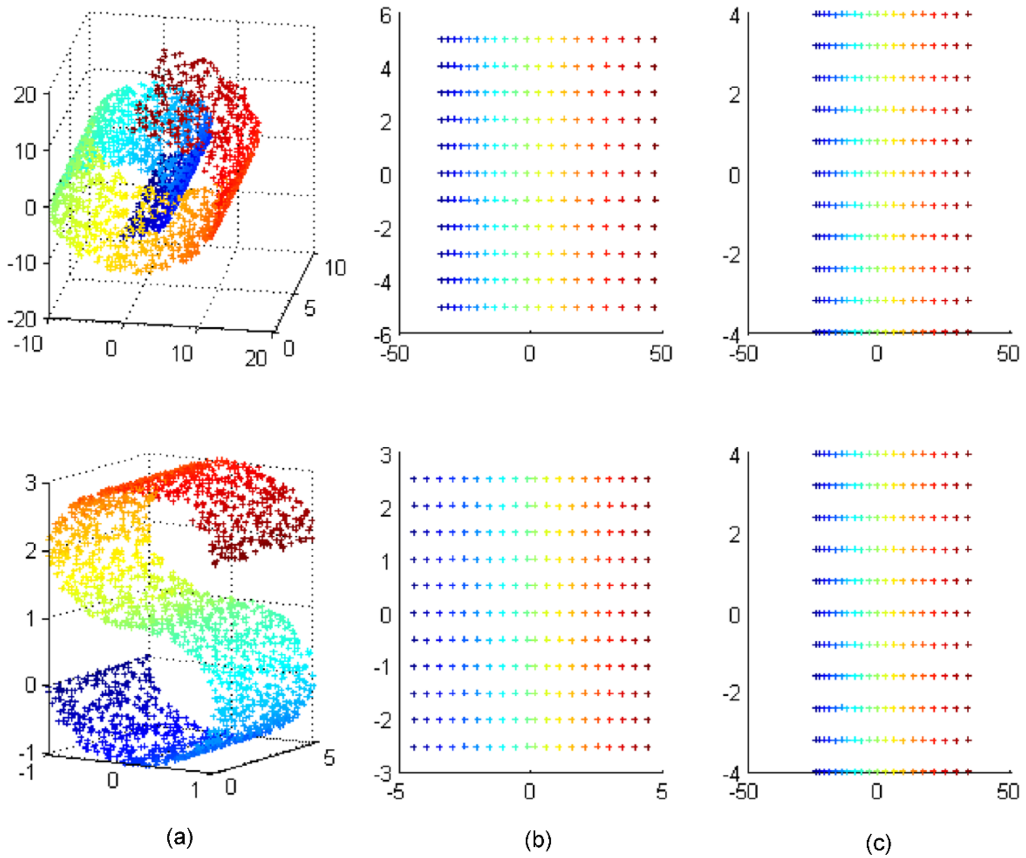
Applications



Zhu et al. *Semi-Supervised Learning Using Gaussian Fields and Harmonic Functions*. ICML 2003.

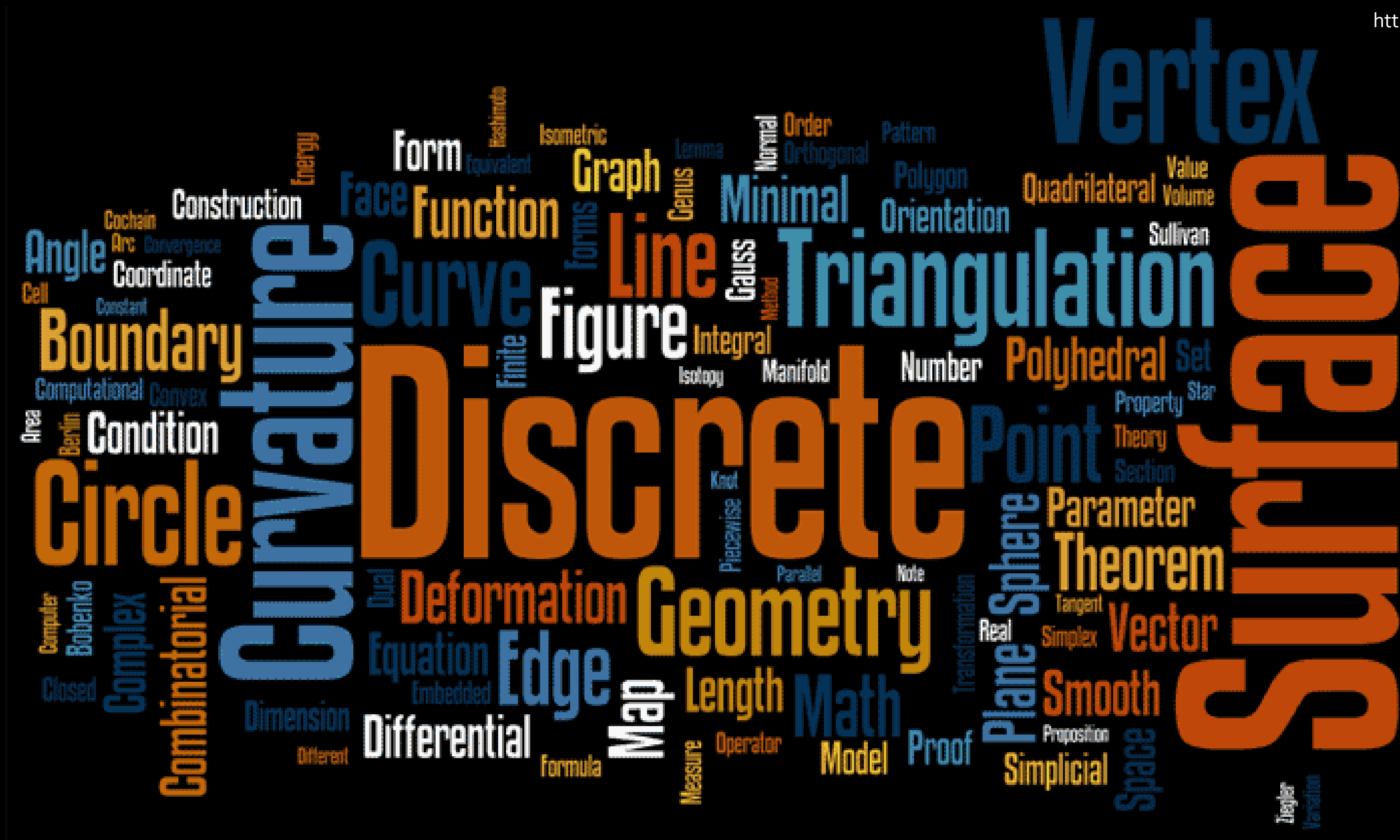
Machine learning

Applications



Hou et al. *Novel semisupervised high-dimensional correspondences learning method*. Opt. Eng. 2008.

Statistics



6.838: Shape Analysis

Justin Solomon

MIT, Spring 2017