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Course Instructor

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On the Web

gdp.csail.mit.edu/
6838_spring_2017.html

+ piazza

Register ASAP!
1. **Four homeworks (40%)**
   Written + coding

2. **One project (50%)**
   Instructions already online

3. **Biweekly nanoquizzes (10%)**
   Designed to be easy!
Prerequisites

- **Coding**
  - Python or Matlab preferred

- **Math**
  - *Fluency* in linear algebra and multivariable calculus

- **Not required (won’t hurt):**
  - Graphics, differential geometry, numerics
6.838, Shape Analysis: Homework 1

As an experiment, we will be doing our homework in Jupyter notebooks. These support \( \text{LaTeX} \) and Python, allowing us to share mathematical formulas and code easily. Please get started early to make sure that you are comfortable with this new tool.

The course staff will be extremely generous helping students figure out these problems if needed.

All homeworks will be graded out of 100 points.

Problem 1: Variational calculus (30 points)
Note: This problem may be tricky to think about for computer science students who are not used to these sorts of calculations. Leave yourself plenty of time, and get help from the instructional staff!

\( a \) (10 points). Suppose \( f : \mathbb{R}^n \to \mathbb{R} \) is a smooth function with \( \frac{\partial f}{\partial x_i} \) taking an arbitrary vector \( y \in \mathbb{R}^n \). Justify the relationship

\[
\frac{d}{dt} f(x^t + t y) = \frac{\partial f}{\partial x_i} \frac{d}{dt} x^t_i + \frac{\partial^2 f}{\partial x^2} \frac{d^2}{dt^2} x^t_i + \ldots
\]

**Try early!**

http://gryd1.csail.mit.edu/
New Course

- Schedule is **too ambitious!**

- **Contact Justin** with suggestions, must-cover topics, questions, etc.

- Experiment: **Video**
  *(unreliable!)*
I want you to take this course!

Assignments intended to be interesting
(may be unintentionally easy/hard!)

Will be generous with support/grading
Quick Survey

Degree

Undergraduate
M.Eng.
M.Sc./PhD
Quick Survey

Background

EECS
Math
Engineering
Elsewhere
</administrative>
1. **Geometric data analysis:** The analysis of geometric data
   - Modifier
   - Noun

2. **Geometric data analysis:** Data analysis using geometric techniques
   - Modifier
   - Noun
Applied Geometry

I. Theoretical toolbox
II. Computational toolbox
III. Application areas

Mostly a picture book!
I. Theoretical toolbox

II. Computational toolbox

III. Application areas
Euclidean Geometry
Euclidean Geometry
Euclidean Geometry
Differential Geometry

Spivak: A Comprehensive Introduction to Differential Geometry
Differential Geometry

Study of smooth manifolds

Manifold

\[ \phi(U) \subseteq \Sigma \subseteq \mathbb{R}^n \]

\[ \phi : U \rightarrow \mathbb{R}^n \]

\[ U \subseteq \mathbb{R}^k \]
Differential Geometry Toolbox

\[ K := \kappa_1 \kappa_2 = \det \mathbb{II} \]

\[ H := \frac{1}{2} (\kappa_1 + \kappa_2) = \frac{1}{2} \text{tr} \mathbb{II} \]

Curvature and shape properties

Differential Geometry Toolbox

Distances

Directional field synthesis, design, and processing.

**Differential Geometry Toolbox**

(a) [FSDH07]  
(b) [BCBSG10]  
(c) [KCPS13]  
(d) [ABCCO13]

Vaxman et al.

*Directional field synthesis, design, and processing.*
EG STAR 2016.

Flows and vector fields
Riemannian Viewpoint

Only need angles and distances
Riemannian Viewpoint

Only need angles and distances

http://www.phy.syr.edu/courses/modules/LIGHTCONE/pics/curved.jpg
Geometric Mechanics

Peyré, Cuturi, and Solomon.  
*Gromov-Wasserstein Averaging of Kernel and Distance Matrices.*  
ICML 2016.
Optimal Transport
Plan for Today

I. Theoretical toolbox

II. Computational toolbox

III. Application areas
Many Notions of Shape

- Triangle mesh
- Triangle soup
  - Graph
  - Point cloud
- Pairwise distance matrix

Nearly anything with a notion of proximity/distance/curvature/...
Typical issue:

Euclidean? Riemannian?

• Collection of **flat triangles**

• Approximates a **smooth surface**
Q: Can a triangle mesh have curvature?
Jack of All Trades

Combine smooth and discrete
Example:

Discrete Differential Geometry
Modern Approach

Discrete vs. Discretized
Discrete theory paralleling differential geometry.
Structure preservation

[\text{struhk-cher pre-zur-vey-shuh n}]:

Keeping properties from the continuous abstraction exactly true in a discretization.
Example: Turning Numbers

\[ \int_{\Omega} \kappa \, ds = 2\pi k \]

\[ \sum_{i} \alpha_i = 2\pi k \]

Convergence

[kuh n-vur-juh ns]: Increasing approximation quality as a discretization is refined.
Convergence and Structure

Can you have it all?
Discrete Laplace operators: No free lunch

Max Wardetzky\textsuperscript{1} Saurabh Mathur\textsuperscript{2} Felix Kälberer\textsuperscript{1} Eitan Grinspun\textsuperscript{2} \textdag

\textsuperscript{1}Freie Universität Berlin, Germany \textsuperscript{2}Columbia University, USA

Abstract
Discrete Laplace operators are ubiquitous in applications spanning geometric modeling to simulation. For robustness and efficiency, many applications require discrete operators that retain key structural properties inherent to the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators. Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction
Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tan00, Zha04, FH05, Sor05].

In applications one often requires certain structural properties of the Laplacian, such as positivity and discrete maximum principles. The need for such structural properties is often driven by a visualization, simulation, or optimization consideration. In this paper, we study a set of natural properties and explore to what extent they are satisfied by various discrete Laplace operators.

1.1. Properties of smooth Laplacians
Consider a smooth surface $S$, possibly with boundary, equipped with a Riemannian metric, \textit{i.e.}, an intrinsic notion of distance. Let the intrinsic $L^2$ inner product of functions $u$ and $v$ on $S$ be denoted by $\langle u, v \rangle_{L^2} = \int_S u v \, dA$, and let $\Delta = -\text{div} \text{grad}$ denote the intrinsic smooth Laplace-Beltrami operator [Ros97]. We list salient properties of this operator:

(Null) $\Delta u = 0$ whenever $u$ is constant.

...
Discrete Laplace operators: No free lunch

Max Wardetzky\textsuperscript{1} \hspace{1em} Saurabh Mathur\textsuperscript{2} \hspace{1em} Felix Kälberer\textsuperscript{1} \hspace{1em} Eitan Grinspun\textsuperscript{2} \textsuperscript{†}

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the continuous setting. Building on the smooth setting, we present a set of natural properties for discrete Laplace operators for triangular surface meshes. We prove an important theoretical limitation: discrete Laplacians cannot satisfy all natural properties; retroactively, this explains the diversity of existing discrete Laplace operators.

Finally, we present a family of operators that includes and extends well-known and widely-used operators.

1. Introduction

Discrete Laplace operators on triangular surface meshes span the entire spectrum of geometry processing applications, including mesh filtering, parameterization, pose transfer, segmentation, reconstruction, re-meshing, compression, simulation, and interpolation via barycentric coordinates [Tan00,Zha04,FF05,Sor05].

In applications one often requires certain structural properties of the Laplace operator. Among these, the preservation of certain differential invariants, such as geodesic curvature [FIS07], is a desirable property. However, such invariance is impossible to achieve simultaneously for all Laplace operators. This paper provides a formal framework for analyzing the invariance properties of a large family of discrete Laplace operators.

1.1. Properties of smooth Laplacians

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(Null) $\Delta u = 0$ whenever $u$ is constant.
Theme

Pick and choose which properties you need.

But there is a huge toolbox to draw from!
Numerical PDE

Chuang and Kazhdan.
Fast Mean-Curvature Flow via Finite-Elements Tracking.
CGF 2011.

$\text{SO}(3) / \sim$
Plan for Today

I. Theoretical toolbox

II. Computational toolbox

III. Application areas
Applications

Transfer

Retrieval

Editing

Exploiting patterns

Graphics

http://www.mpi-inf.mpg.de/~mbokeloh/project_dockingSites.html
Applications

Recognition

Navigation

Segmentation

Reconstruction

Vision

http://www.stanford.edu/~jinhae/iccv09/
Applications

Analysis

Segmentation

Registration
Applications

Scanning

Defect detection

Manufacturing and Fabrication

http://www.conduitprojects.com/php/images/scan.jpg
http://www.emeraldinsight.com/content_images/fig/0330290204005.png
Applications

Design and analysis

Architecture

Applications


Shape collection analysis
Applications

Correspondence

Applications

Deformation transfer

Applications

Simulation
Applications

Scientific visualization

http://multires.caltech.edu/~keenan/pdf/spinxform.pdf
Applications
Applications

Applications

Applications

6.838: Shape Analysis