Decentralized, Adaptive Control for Coverage with Networked Robots

Mac Schwager*, Jean-Jacques Slotine†, and Daniela Rus*

*Computer Science and Artificial Intelligence Lab
MIT, Cambridge, MA, 02139
Email: schwager@mit.edu, rus@csail.mit.edu
†Nonlinear Systems Lab
MIT, Cambridge, MA, 02139
Email: jjs@mit.edu

Abstract—A decentralized, adaptive control law is presented to drive a network of mobile robots to a near-optimal sensing configuration. The control law is adaptive in that it integrates sensor measurements to provide a converging estimate of the distribution of sensory information in the environment. It is decentralized in that it requires only information local to each robot. A Lyapunov-type proof is used to show that the control law causes the network to converge to a near-optimal sensing configuration, and the controller is demonstrated in numerical simulations. This technique suggests a broader application of adaptive control methodologies to decentralized control problems in unknown dynamical environments.

I. INTRODUCTION

In this paper we present a control strategy which is both adaptive and decentralized, thereby combining two of the defining qualities of biological swarming, flocking, and herding systems. More importantly, the adaptive, decentralized control law has provable stability and convergence properties, which are summarized in the main result of this work. This work describes one example of the successful combination of these two disciplines, and, it is hoped, will provide a method that can be applied to other problems requiring control with local information in uncertain dynamical environments.

The specific problem we address is coverage control for mobile sensor networks. We consider controlling a group of mobile robots to monitor some quantity of interest over an area. Our solution to this problem would be useful in controlling teams of robots to carry out a number of tasks including search and rescue missions, environmental monitoring (e.g. for forest fires), automatic surveillance of rooms, buildings or towns, or simulating collaborative predatory behavior. Virtually any application in which a group of automated mobile agents is required to monitor an area could benefit from the proposed control law. We are constrained to use only local sensory and locational information available to each robot. Our control law causes the robots to spread over an area in such a way that their density in different regions of the area is directly related to the sensory importance of those regions. Thus areas of greater importance receive more concentrated coverage than areas that are less important. What is more, the robots do not know before hand what are the regions of sensor interest, but acquire this information dynamically.

A. Relation to Previous Work

There has been considerable activity in the controls and robotics community in studying decentralized control as a metaphor for natural and engineered swarming, flocking and herding systems. A far from exhaustive selection of such works can be found in [1]–[4], where consensus problems are treated, [5], which deals with the synchronization of oscillators, [6], [7], which deals with stability, and [8]–[10], which considers abstractions for representing decentralized dynamical systems. These works are formulated assuming local information to some degree, however, models of the dynamics of the robots and their surroundings are usually assumed to be exact. Conversely, the well-developed discipline of adaptive control [11], [12] focuses on controlling systems whose dynamics are not known to the controller. Controllers collect information as the system evolves, identifying system dynamics simultaneously while controlling the system. This work combines these two disciplines to produce a controller that is provably stable, uses only local information, and has limited prior knowledge of the sensing environment.

Various strategies have been introduced to address the specific problem of coverage control for mobile sensor networks, and our work builds on several important results in this category. In [13], mobile sensing agents are controlled using potential functions for inter-agent interactions. Stability results are derived, but the optimality of the network configuration is not addressed. Similarly, in [14] an algorithm is proposed that allows for agents to concentrate in areas of high event density while maintaining area coverage constraints. The algorithm is proved to maintain sensor coverage for a limited case without addressing optimality. Most relevant to this paper is a body of results reported in [15]–[17]. In this work, Cortés et al derived decentralized control laws for positioning mobile sensor networks optimally with respect to a known event probability density. This approach is advantageous because it guarantees that the network (locally) minimizes a cost function relevant to the coverage problem. However, the control strategy requires that each agent have a complete foreknowledge of the event probability density, thus it is not

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reactive to the sensed environment. In our previous work, [18], we removed the constraint on the global knowledge of the event density. We presented a control law based on a linear estimation of a sensory function and showed that the controller performed robustly in simulation and in hardware experiments. However, analytical stability guarantees were not obtained.

In this work, a new control law is proposed which implements an estimation of the sensory distribution by integrating measurements gathered throughout a robot’s trajectory into a parametric model of the sensory distribution. We use the locational optimization framework introduced in [16], and provide a control law that does not require foreknowledge of the distribution of sensory information in the environment, in the spirit of [18]. Using a Lyapunov-like proof, we show that the control law causes the network to converge to a near-optimal sensing configuration given that the actual sensory distribution is adequately represented by a parametric model. This assumption is analogous to the common “matching conditions” in adaptive control. Furthermore, we require that each agent can sense the value of the sensory distribution at its position—a relaxation of the conditions in [18], in which both the value and gradient were required. We provide some background on the results of locational optimization in Section II. We present the controller and prove its stability in Section III. The results of numerical simulations are described in IV.

II. LOCATIONAL OPTIMIZATION BACKGROUND

In this section, we build a function representing the sensing cost associated with a network configuration. A network is said to have optimal coverage if it minimizes this cost function over the region of interest. Following standard results in the field [16], we show that all configurations of a certain type, namely centroidal Voronoi configurations, correspond to local minima of the cost function.

The sensor network consists of a group of identical robots, each with some degree of mobility and the capacity for measuring a sensory function from the environment. The sensory function indicates the relative importance of different areas in the environment. It may be a quantity that is sensed directly, such as temperature, or it may demand more elaborate processing of sensory data, such as would be required to detect the concentration of a chemical, or the intensity of sound of a particular frequency. In addition, we assume that a robot can measure the positions of its Voronoi neighbors relative to itself, and that it can detect the boundaries of the region of interest. We review the formalism introduced in [16] to rigorously model the scenario described above.

Let there be \( n \) robots in a known, convex polytope \( Q \subset \mathbb{R}^N \). An arbitrary point in \( Q \) is denoted \( q \), the position of the \( i \)th robot is denoted \( p_i \), and the set of all robot positions is denoted \( P = \{ p_1, \ldots, p_n \} \). Let \( W = \{ W_1, \ldots, W_n \} \) be a partition of \( Q \) such that one robot at position \( p_i \) lies within each region \( W_i \). Define the sensory function, \( \phi(q) \), as a scalar function, \( \phi : Q \rightarrow \mathbb{R}_+ \). The function \( \phi(q) \) is not known by the robots in the network, but the robots are equipped with sensors from which a measurement of \( \phi(p_i) \) can be derived at the robot’s position \( p_i \).

Let the unreliability of the sensor measurement be defined by a quadratic function \( f(||q - p_i||) = \frac{1}{2} ||q - p_i||^2 \). Specifically, \( f(||q - p_i||) \) describes how unreliable is the measurement of the information at \( q \) by a sensor at \( p_i \) (henceforth, \( || \cdot || \) is used to denote the \( L^2 \)-norm). This form of \( f(||q - p_i||) \) is physically appealing since it is reasonable that sensing will become more unreliable farther from the sensor.

We can formulate the cost incurred by one robot sensing over one region \( W_i \) as

\[
h_i(p_i, W_i) = \int_{W_i} f(||q - p_i||)\phi(q) dq.\]

Notice that unreliable sensing is expensive and high values of \( \phi(q) \) are also expensive. Summing over all robots, a function representing the overall sensing cost of a given network configuration can be written

\[
\mathcal{H}(P, W) = \sum_{i=1}^{n} \int_{W_i} \frac{1}{2} ||q - p_i||^2 \phi(q) dq. \tag{1}
\]

An optimal network configuration corresponds to a particular pair \((P, W)\) which minimizes (1).

To solve this minimization problem, we must introduce the notion of a Voronoi partition. The Voronoi region, \( V_i \), of a given robot is the region of points that are closer to that robot than to any other, that is

\[
V_i = \{ q \in Q \mid ||q - p_i|| \leq ||q - p_j||, \forall j \neq i \}. \]

The division of an area into such regions is called a Voronoi partition, denoted \( \mathcal{V}(P) \), and is a function of the robot positions. We will use the shorthand \( \mathcal{H}_{\mathcal{V}}(P) = \mathcal{H}(P, \mathcal{V}(P)) \).

Because the function \( f(||q - p_i||) \) is strictly increasing, the Voronoi partition, \( \mathcal{V} \), minimizes the cost function, \( \mathcal{H} \), for any fixed configuration, \( P \), of robots. This is clear since, for an arbitrary point \( q \in Q \), \( q \in V_i \) gives the smallest value of \( f(||q - p_i||) \) over \( i \), and therefore the smallest contribution to \( \mathcal{H} \). Thus we have

\[
\min_{P, W} \mathcal{H} = \min_{P} \mathcal{H}_{\mathcal{V}}.
\]

To find local minima of \( \mathcal{H}_{\mathcal{V}} \), we examine solutions to the expression

\[
\nabla \mathcal{H}_{\mathcal{V}} = \left[ \cdots \frac{\partial \mathcal{H}_{\mathcal{V}}}{\partial p_i} \cdots \right]^T = 0.
\]

It is clear that each partial derivative must be zero for a local minimum. Applying a multi-variable generalization of Leibniz Rule, \(^2\) we can move the differentiation inside the

\(^1\)In contrast, Cortés et al [16] use a probability density function describing the likelihood of an event occurring in a particular area.

\(^2\)This procedure is known in fluid mechanics as the Reynolds Transport Theorem.


\[
\frac{\partial \mathcal{H}_V}{\partial p_i} = \int_{V_i} \frac{1}{2} \| q - p_i \|^2 \phi(q) dq + \sum_{j \in N_i} \int_{\partial V_j} \frac{1}{2} \| q - p_i \|^2 \phi(q) \frac{\partial \mathcal{V}_j}{\partial p_i} n_j dq + \int_{\partial V_i} \frac{1}{2} \| q - p_i \|^2 \phi(q) \frac{\partial \mathcal{V}_i}{\partial p_i} n_i dq,
\]

where \( \partial V_i \) denotes the boundary of the region \( V_i \), \( n_i(q) \) denotes the outward facing normal of \( \partial V_i \), and \( N_i \) is the set of indices of the neighbors of \( p_i \), excluding \( i \) itself. Note that all the integrals in (2) are \( i \) outside of the integral to give

\[
\int_{V_i} \phi(q) dq = \Lambda_i^2 \phi(q) dq.
\]

We can evaluate the partial derivative of \( \frac{1}{2} \| q - p_i \|^2 \phi(q) \frac{\partial \mathcal{V}_i}{\partial p_i} \phi(q) dq \)

and move \( p_i \) outside of the integral to give

\[
\frac{\partial \mathcal{H}_V}{\partial p_i} = -\int_{V_i} \phi(q) dq + p_i \int_{V_i} \phi(q) dq.
\]

Next we define three properties analogous to mass-moments of rigid bodies. The mass of \( V_i \) is defined as

\[
M_{V_i} = \int_{V_i} \phi(q) dq,
\]

the second mass-moment (not normalized) is defined as

\[
L_{V_i} = \int_{V_i} q \phi(q) dq
\]

and the centroid of \( V_i \) is defined as

\[
C_{V_i} = \frac{L_{V_i}}{M_{V_i}},
\]

Note that \( \phi(q) \) strictly positive imply both \( M_{V_i} > 0 \) \( \forall V_i \neq \emptyset \) and \( C_{V_i} \in V_i \cap \partial V_i \) (\( C_{V_i} \) is in the interior of \( V_i \)). Thus \( M_{V_i} \) and \( C_{V_i} \) have properties intrinsic to physical masses and centroids. Substituting these quantities into (3) gives

\[
\frac{\partial \mathcal{H}_V}{\partial p_i} = -M_{V_i} (C_{V_i} - p_i).
\]

Equation (7) implies that local minima of \( \mathcal{H}_V \), and therefore \( \mathcal{H}(P, W) \), correspond to the configurations, \( P \), such that \( p_i = C_{V_i}, \forall i \), that is, each agent is located at the centroid of its Voronoi region. We will denote the set of all such centroid Voronoi configurations as \( P_C \). Thus, the optimal coverage task is to drive the group of robots to a centroid Voronoi configuration, \( P \in P_C \).

### III. Decentralized Adaptive Control

We will design a control law with an intuitive interpretation and prove that it causes the network to converge to a near-centroidal Voronoi configuration. The control law will integrate sensory measurements available to each robot to form an on-line approximation of the centroids of its Voronoi regions.

Let the dynamics of each robot be modeled by the first-order equation

\[
\dot{p}_i = u_i,
\]

where \( u_i \) is the control input. This might mean that a low-level controller is in place to enforce first-order dynamics. Since this work is primarily concerned with the application of adaptive control to the decentralized coverage problem, simple dynamics were chosen so as not to obscure the result. More complicated dynamics can be accommodated.

Assume, furthermore, that the sensory function, \( \phi(q) \), can be parameterized as an unknown linear combination of a set of known basis functions. This requirement is formalized in the following

**Assumption 1 (Matching Conditions):** \( \exists a \in \mathbb{R}^m_+ \) and \( K : Q \mapsto \mathbb{R}_+^m \), such that

\[
\phi(q) = K(q)^T a,
\]

where the set of basis functions, \( K \), is available to each agent, but the parameter vector, \( a \), is unknown.

We now introduce a number of definitions which will be important in stating and proving the main stability result. Let \( \hat{a}_i(t) \) be robot \( i \)'s approximation of the parameter vector. Naturally, \( \hat{a}_i = K(q)^T \hat{a}_i \) is robot \( i \)'s approximation of \( \phi(q) \).

Define the mass moment approximations

\[
\hat{M}_{V_i} = \int_{V_i} \hat{\phi}_i dq,
\]

\[
\hat{L}_{V_i} = \int_{V_i} q \hat{\phi}_i dq \quad \text{and} \quad \hat{C}_{V_i} = \frac{\hat{L}_{V_i}}{\hat{M}_{V_i}}.
\]

which are analogous to (4), (5), and (6) respectively. Next, define the parameter error

\[
\hat{a}_i = \hat{a}_i - a_i,
\]

and the mass moment errors

\[
\hat{M}_{V_i} = \int_{V_i} K_T(q) \hat{a}_i dq = \hat{M}_{V_i} - M_{V_i},
\]

\[
\hat{L}_{V_i} = \int_{V_i} q K_T(q) \hat{a}_i dq = \hat{L}_{V_i} - L_{V_i}.
\]
However, notice that $\hat{C}_V = \hat{L}_V, \hat{M}_V \neq \hat{C}_V$. From (12), (14), and (15) we find that

$$L_V = M_V \hat{C}_V + \hat{M}_V (\hat{C}_V - \hat{C}_V).$$

(16)

This property will be useful in what follows. We also define two error vectors, the actual error, $e_i = \hat{C}_V - p_i$, and the estimated error $\hat{e}_i = \hat{C}_V - p_i$.

Finally, in order to compress the notation somewhat, we use the shorthand $K_i = K_i(p_i(t))$ for the value of the basis functions at the position of robot $i$ and $\phi_i = \phi(p_i(t))$ for the value of $\phi$ at the position of robot $i$. As stated previously, robot $i$ can measure $\phi_i$ with its sensors. We now propose to use the control law

$$u_i = k(\hat{C}_V - p_i),$$

(17)

with the adaptation laws

$$\dot{\hat{e}}_i = -\Gamma_i \int_{V_i} K(q)(q - \hat{C}_V)^T dq \hat{e}_i + \gamma_i (\Lambda_i \hat{a}_i - s_i),$$

(18)

$$\dot{\hat{a}}_i = -\alpha_i \hat{a}_i + K_i K_i^T,$$

(19)

and

$$\hat{s}_i = -\alpha_i s_i + K_i \phi_i,$$

(20)

where $k \in \mathbb{R}^+$ is a proportional control gain, $\Gamma_i \in \mathbb{R}^{m \times m}$ is a positive definite adaptation gain matrix, $\gamma_i \in \mathbb{R}^+$ is an adaptation gain scalar, and $\alpha_i \in \mathbb{R}^+$ is a time constant. Equation (19) provides the adaptation rule for the parameter $1$, for the system of robots with dynamics (8) and the control measurements $\hat{C}_V$ defined by (20), a “forgetting factor”. The main result of this work is now given.

Theorem 1 (Convergence Theorem): Under Assumption 1, for the system of robots with dynamics (8) and the control law (17),

$$i) \lim_{t \to \infty} \hat{e}_i(t) = 0 \quad \forall i = 1, \ldots, n$$

(21)

$$ii) \lim_{t \to \infty} K_i(\tau) \hat{a}_i(t) = 0$$

(22)

$$\forall 0 \leq \tau \leq t \quad \forall i = 1, \ldots, n$$

Proof: We will define a lower-bounded function and show that it is non-increasing along the trajectories of the system, and that its time derivative is uniformly continuous. Theorem 1 is then a direct implication of Barmalat’s lemma. Let

$$V = \mathcal{H}_V + \sum_{i=1}^n \frac{1}{2} \hat{a}_i^T K \Gamma^{-1} \hat{a}_i.$$ 

(23)

Taking the time derivative of $V$ along the trajectories of the system gives

$$\dot{V} = \sum_{i=1}^n \left[ \frac{\partial \mathcal{H}_V}{\partial \dot{p}_i} \dot{p}_i + \hat{a}_i^T K \Gamma^{-1} \hat{a}_i \right].$$

and substituting from (7) and noticing that $\dot{\hat{a}}_i = \dot{\hat{a}}_i$ yields

$$\dot{V} = \sum_{i=1}^n \left[ (M_V \dot{p}_i - M_V \hat{C}_V)^T \dot{p}_i + \hat{a}_i^T K \Gamma^{-1} \hat{a}_i \right].$$

Now we make use of the property in (16) to write

$$\dot{V} = \sum_{i=1}^n \left[ (-M_V \hat{C}_V + M_V \hat{C}_V)^T \dot{p}_i + \hat{a}_i^T K \Gamma^{-1} \hat{a}_i \right].$$

Substituting for $\hat{p}_i$ with (8) and (17), and simplifying with $L_V = M_V \hat{C}_V$ leads to

$$\dot{V} = \sum_{i=1}^n \left[ -M_V \hat{C}_V - p_i \right] k(\hat{C}_V - p_i) + \left( \hat{L}_V - \hat{M}_V \hat{C}_V \right) T \dot{p}_i + \hat{a}_i^T K \Gamma^{-1} \hat{a}_i.$$

Simplifying further with $\hat{e}_i = \hat{C}_V - p_i$ and expanding $\hat{L}$ and $\hat{M}$ with (15) and (14) respectively gives

$$\dot{V} = \sum_{i=1}^n \left[ -M_V \hat{e}_i^T k \hat{e}_i + \left( \int_{V_i} \hat{a}_i^T K \hat{C}_V T dq - \int_{V_i} \hat{a}_i^T K \hat{C}_V d \hat{e}_i \hat{e}_i + \hat{a}_i^T K \Gamma^{-1} \hat{a}_i \right).$$

Collecting terms under the integral, and noticing the crucial fact that $\hat{a}_i$ is not a function of $q$ we get

$$\dot{V} = \sum_{i=1}^n \left[ -M_V \hat{e}_i^T k \hat{e}_i + \hat{a}_i^T k \int_{V_i} K(q - \hat{C}_V) T dq \hat{e}_i + \hat{a}_i^T K \Gamma^{-1} \hat{a}_i \right].$$

Now we substitute for $\hat{a}_i$ with (18) to get

$$\dot{V} = -\sum_{i=1}^n \left[ M_V \hat{e}_i^T k \hat{e}_i + \hat{a}_i^T K \gamma_i \left( \Lambda_i \hat{a}_i - s_i \right) \right],$$

(24)

where the first term in the adaptive law cancels the integral term. Finally, notice from (19) and (20) that, with zero initial conditions,

$$\Lambda_i(t) = \int_0^t e^{-\alpha_i(t-\tau)} K_i(\tau) \hat{a}_i^T(\tau) d \tau$$

and

$$s_i(t) = \int_0^t e^{-\alpha_i(t-\tau)} K_i(\tau) \phi_i(\tau) d \tau.$$

Substituting these into (24), combining terms under the integral, and noting that $\hat{a}_i(t)$ is not a function of $\tau$ leads to

$$\dot{V} = -\sum_{i=1}^n \left[ M_V \hat{e}_i^T k \hat{e}_i + \hat{a}_i^T k \gamma_i \left( \Lambda_i \hat{a}_i(t) - \phi_i(t) \right) \right].$$
Using (9) and bringing the $\tilde{a}_i(t)$ inside the integral gives

$$V = -\sum_{i=1}^{n} \left[ M_i e_i^T k e_i + k\gamma_i \int_0^t e^{-\alpha_i(t-\tau)} (K_i^T(\tau)\tilde{a}_i(t))^2 d\tau \right].$$

Both terms inside the sum are non-negative, thus $V$ is negative semi-definite. Also, the three facts that $V$ has continuous first partial derivatives, $u_i$ is continuous $\forall i$, and the region $Q$ is bounded imply that $V$ is uniformly continuous, therefore, by Barbalat’s lemma $\lim_{t \to \infty} V = 0$, which directly implies (21) from Theorem 1, and

$$\lim_{t \to \infty} \int_0^t e^{-\alpha_i(t-\tau)} (K_i^T(\tau)\tilde{a}_i(t))^2 d\tau = 0$$

Now notice that the integrand in (26) is non-negative, therefore it must converge to zero for all $\tau$. Finally, $e^{-\alpha_i(t-\tau)} > 0 \forall 0 \leq \tau \leq t$ implies (22) from Theorem 1.

Remark 1: The term $K_i(\tau)^T\tilde{a}_i(t)$ in (22) can be interpreted as the difference between any previously measured value of $\phi(q)$ and the current estimate, $\hat{\phi}_i(q)$. Informally, this assertion from Theorem 1 states that the estimate $\hat{\phi}_i(q)$ will converge asymptotically to all previously measured values of $\phi(q)$. This does not, however, imply that $\hat{\phi}_i(q) \to \phi(q)$ $\forall q \in Q$.

A. Practical Algorithm

A practical method for implementing the proposed control law on a network of robots is detailed in Algorithm 1. Notice that the control law in (17) and adaptation law in (18) both require the computation of integrals over $V_i$, thus robot $i$ must be able to calculate continuously its Voronoi region. Several algorithms exist for computing $V_i$ in a distributed fashion, for example those given in [16], [19].

Algorithm 1: Adaptive Coverage Control Algorithm

Require: Each robot can compute its Voronoi region
Require: $\phi(q)$ can be parameterized as in (9)

Initialize $\Lambda_i$, $s_i$, and $\dot{a}_i$ to zero
loop
Compute the robot’s Voronoi region
Compute $\dot{C}_{V_i}$ according to (12)
Update $\dot{a}_i$ according to (18)
Update $\Lambda_i$ and $s_i$ according to (19) and (20)
Apply control input $u_i = -k(\dot{C}_{V_i} - p_i)$
end loop

Algorithm 1 is decentralized, fully distributed, and requires minimal communication between neighboring robots. It can be used on teams of large robots, on teams of small robots such as [20], or on mobile sensor network nodes with limited computation and storage capabilities such as the mobile Mica Motes described by [21].

IV. Numerical Simulations

A. Implementation

Simulations were carried out in a Matlab environment. The dynamics in (8) with the control law in (17), and the adaptation laws in (18), (19), and (20) for a group of $n = 20$ robots were modeled as a system of coupled differential equations. A custom, fixed-time-step numerical solver was used to integrate the equations of motion of the group of robots. The region $Q$ was taken to be the unit square. The sensory function, $\phi(q)$, was parameterized as a Gaussian network. In particular, for $K = [K_1 \cdots K_m]^T$, each component, $K_i$, was implemented as

$$K_i = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(q - \mu_i)^2}{2\sigma_i^2}},$$

where $\sigma_i = 0.2 \forall i$. The unit square was divided into an even $5 \times 5$ grid and each $\mu_i$ was chosen so that one Gaussian was centered at the middle of each grid square, resulting in $m = 25$ Gaussians. The parameters were chosen as $a = [100 \ 0 \cdots 0 100]^T$ so that only the lower left and upper right Gaussians contributed to the value of $\phi(q)$, producing a bimodal distribution.

The robots in the network were started from random initial positions. Each robot used a copy of the Gaussian network described above for $K_i(q)$. The estimated parameters $\dot{a}_i$ for each robot were started at some very small positive value, and $\Lambda_i$ and $s_i$ were each started at zero. The gains used by the robots were $k = 3$, $\Gamma_i = 10^{-5}$, $\gamma_i = 10^2$ and $\alpha_i = 1 \forall i$. In practice, the first integral term in the adaptive law (18) seems to have very little effect on the performance of the controller. Choosing $\Gamma_i$ small and $\gamma_i$ correspondingly large puts more weight on the second term, which is responsible for integrating measurements of $\phi(p_i)$ into the parameters. The spatial integrals in (10), (11), and (18) required for the control law were computed by summing contributions of the integrand over a discretized grid. Voronoi regions were computed in a centralized fashion using the built-in Matlab function, although equivalent performance is observed with a custom decentralized Voronoi algorithm. The Matlab Voronoi command was used only for computational speed.

B. Results

Figure 1 shows the positions of the robots in the network over the course of a simulation run. The initial configuration of the network is shown in Figure 1(a), the trajectories of the agents (dashed lines) in Figure 1(b), and the final configuration in Figure 1(c). The centers of the two contributing Gaussian functions are marked with $\times$’s. The performance of the control scheme is clearly demonstrated in the simulation.

In figure 2 the first assertion (21) of Theorem 1 is demonstrated for the same network of robots. The norm of the estimated error averaged over all the robots is shown to converge asymptotically to zero. This implies that the robots move to the estimated centroid of their Voronoi regions. We call such a configuration near-optimal.

The second assertion (22) of Theorem 1 is demonstrated in figure 3. The plot shows the integral in (26) as a function of
time averaged over all the robots in the network converging asymptotically to zero. This implies that the parameters adjust in such a way that the estimate, $\hat{a}_i$, matches all previously measured values of $\phi(q)$. As stated previously, this does not imply that $\hat{a}_i \to a$.

V. CONCLUSION

In this work we proposed an adaptive, decentralized controller to drive a network of robots to a near-optimal sensing configuration. The controller was proven to cause the robots to move to the estimated centroids of their Voronoi regions, while also causing their estimation of the sensory distribution to improve over time until all previous sensor measurements fit the estimated sensory distribution. The control law was demonstrated in numerical simulations of a group of 20 robots sensing over an area with a bi-modal Gaussian distribution of sensory information.

We expect that the technique used in this paper will find broader application beyond the problem chosen here. A similar approach could yield fruitful combinations of adaptive control and decentralized control to produce engineered agents that can cooperate with one another while gathering information from their environment to proceed toward a common goal.

REFERENCES


