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About Bond Graphs

1. Introduction

Bond graph is an explicit graphical tool for capturing the common energy structure of systems. It increases one's insight into systems behavior. In the vector form, they give concise description of complex systems. Moreover, the notations of causality provides a tool not only for formulation of system equations, but also for intuition based discussion of system behavior, viz. controllability, observability, fault diagnosis, etc.

In 1959, Prof. H.M.Paynter gave the revolutionary idea of portraying systems in terms of power bonds, connecting the elements of the physical system to the so called junction structures which were manifestations of the constraints. This power exchange portray of a system is called **Bond Graph** (some refer it as **Bondgraph**), which can be both power and information oriented. Later on, Bond Graph theory has been further developed by many researchers like Karnopp, Rosenberg, Thoma, Breedveld, etc. who have worked on extending this modeling technique to power hydraulics, mechatronics, general thermodynamic systems and recently to electronics and non-energetic systems like economics and queuing theory.

By this approach, a physical system can be represented by symbols and lines, identifying the power flow paths. The lumped parameter elements of resistance, capacitance and inertance are interconnected in an energy conserving way by bonds and junctions resulting in a network structure. From the pictorial representation of the bond graph, the derivation of system equations is so systematic that it can be algorithmized. The whole procedure of modeling and simulation of the system may be performed by some of the existing software e.g., **ENPORT, Camp-G, SYMBOLS, COSMO, LorSim** etc.

Bond graph bibliography by F.E.Cellier and Bondgraphs.com >> Bibliography give a through account of the work carried out in the particular field. The following sections will briefly explain the reader to understand the mnemonics of the bond graph. However, the explanation given here is not a complete one; it is only limited to the extent of initiating new bond graphers.

2. Power variables of Bond Graphs

The language of bond graphs aspires to express general class physical systems through power interactions. The factors of power i.e., **Effort** and **Flow**, have different interpretations in different physical domains. Yet, power can always be used as a generalized co-ordinate to model coupled systems residing in several energy domains. One such system may be an electrical motor driving a hydraulic pump or an thermal engine connected with a muffler; where the form of energy varies within the system. Power variables of bond graph may not be always realizable (viz. in bond graphs for economic systems); such factual power is encountered mostly in non-physical domains and pseudo bond graphs.

In the following table, effort and flow variables in some physical domains are listed.

Systems	Effort (e)	Flow (f)
Mechanical	Force (F)	Velocity (v)
	Torque (τ)	Angular velocity (ω)
Electrical	Voltage (V)	Current (i)
Hydraulic	Pressure (P)	Volume flow rate (dQ/dt)
Thermal	Temperature (T)	Entropy change rate (ds/dt)
	Pressure (P)	Volume change rate (dV/dt)
Chemical	Chemical potential (μ)	Mole flow rate (dN/dt)
	Enthalpy (h)	Mass flow rate (dm/dt)
Magnetic	Magneto-motive force (e_m)	Magnetic flux (ϕ)

3. Bond Graph Standard Elements

In bond graphs, one needs to recognize only four groups of basic symbols, i.e., three basic one port passive elements, two basic active elements, two basic two port elements and two basic junctions. The basic variables are effort (e), flow (f), time integral of effort (P) and the time integral of flow (Q).

Basic 1-Port elements
A 1-port element is addressed through a single power port, and at the port a single pair of effort and flow variables exists. Ports are classified as passive ports and active ports.

The passive ports are idealized elements because they contain no sources of power. The inertia or inductor, compliance or capacitor, and resistor or dashpot are classified as passive elements.

R-Elements :
The 1-port resistor is an element in which the effort and flow variables at the single port are related by a static function. Usually, resistors dissipate energy. This must be true for simple electrical resistors, mechanical dampers or dashpots, porous plugs in fluid lines, and other analogous passive elements. The bond graph symbol for the resistive element is shown below.

$$\xrightarrow[\quad f \quad]{\quad e \quad} \triangleright R$$
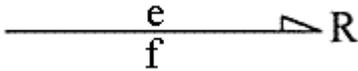
The half arrow pointing towards **R** means that the power i.e., product of *F* and *V* (or *e* * *f*) is positive and flowing into **R**, where *e*, represents effort or force, and *f*, represents flow or velocity. The constitutive relationship between *e*, *f* and *R* is given by :

$e = R * f$
Power $= e * f = R * f^2$

C-Elements :
Consider a 1-port device in which a static constitutive relation exists between an effort and a displacement. Such a device stores and gives up energy without loss. In bond graph terminology, an element that relates effort to the generalized displacement (or time integral of flow) is called a one port capacitor. In the physical terms, a capacitor is an idealization of devices like springs, torsion bars, electrical capacitors, gravity tanks, and accumulators, etc. The bondgraphic symbol, defining constitutive relation for a C-element are shown below.

$$\xrightarrow[\quad f \quad]{\quad e \quad} \triangleright C$$
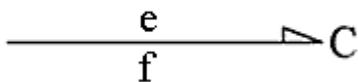
In a spring, the deformation (*Q*) and the effort (*e*) at any moment is given by,

$$Q = \int_{-\infty}^{t} f\, dt, \qquad e = K \int_{-\infty}^{t} f\, dt.$$

Here, flow is the cause and deformation (and hence effort) is the consequence. In a capacitor, the charge accumulated on the plates (*Q*) or voltage (*e*) is given by,

$$Q = \int_{-\infty}^{t} i\, dt, \qquad e = C^{-1} \int_{-\infty}^{t} i\, dt.$$

Here, the current is the cause and the total charge (and hence voltage) is the consequence.

I-Elements :
A second energy storing 1-port arises if the momentum, *P*, is related by a static constitutive law to the flow, *f*. Such an element is called an inertial element in bond graph terminology. The inertial element is used to model inductance effects in electrical systems and mass or

inertia effects in mechanical or fluid systems. The bond graph symbol for an inertial element is depicted in the figure given below.
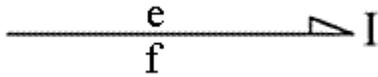
If the mechanics of mass point is examined by considering the impulse-momentum equation, then we have;

$$P = \int_{-\infty}^{t} e \, dt, \qquad f = m^{-1} \int_{-\infty}^{t} e \, dt.$$

Here, effort is the cause and velocity (and hence momentum) is the consequence. Similarly the current in an inductor is given by;

$$i = L^{-1} \int_{-\infty}^{t} e \, dt.$$

Effort and Flow Sources :
The active ports are those, which give reaction to the source. For, example if we step on a rigid body, our feet reacts with a force or source. For this reason, sources are called active ports. Force is considered as an effort source and the surface of a rigid body gives a velocity source. They are represented as an half arrow pointing away from the source symbol. The effort source is represented by
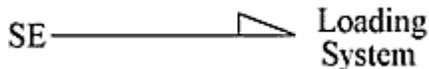
and the flow source is represented as shown below.
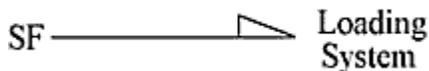
In electrical domain, an ideal shell would be represented as an effort source. Similarities can be drawn for source representations in other domains.

Basic 2-Port elements

There are only two kinds of two port elements, namely ``**Transformer**'' and ``**Gyrator**''. The bond graph symbols for these elements are **TF** and **GY**, respectively. As the name suggests, two bonds are attached to these elements.

The Transformer :
The bondgraphic transformer can represent an ideal electrical transformer, a mass less lever, etc. The transformer does not create, store or destroy energy. It conserves power and

transmits the factors of power with proper scaling as defined by the transformer modulus (discussed afterwards).

The meaning of a transformer may be better understood if we consider an example given here. In this example, a mass less ideal lever is considered. Standard and bondgraphic nomenclature of a lever are shown in the figure below. It is also assumed that the lever is rigid, which means a linear relationship can be established between power variables at both the ends of the lever.
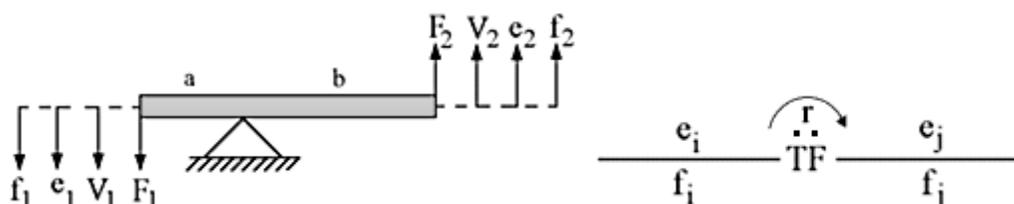
From the geometry, we have,

$$V_2 = (b/a)\ V_1$$

The power transmission implies

$$F_2 = (a/b)\ F_1 \quad , \quad \text{so that} \quad V_2 F_2 = V_1 F_1.$$

In bondgraphs, such a situation may be represented as shown in the above figure.

The '**r**' above the transformer denotes the modulus of the transformer, which may be a constant or any expression (like 'b/a'). The small arrow represents the sense in which this modulus is to be used.

$$f_j = r\ f_i \quad , \quad \text{and} \quad e_j = (1/r)\ e_i.$$

Thus the following expression establishes the conservation of power,

$$e_j f_j = e_i f_i.$$

The Gyrator :
A transformer relates flow-to-flow and effort-to-effort. Conversely, a gyrator establishes relationship between flow to effort and effort to flow, again keeping the power on the ports same. The simplest gyrator is a mechanical gyroscope, shown in the figure below.
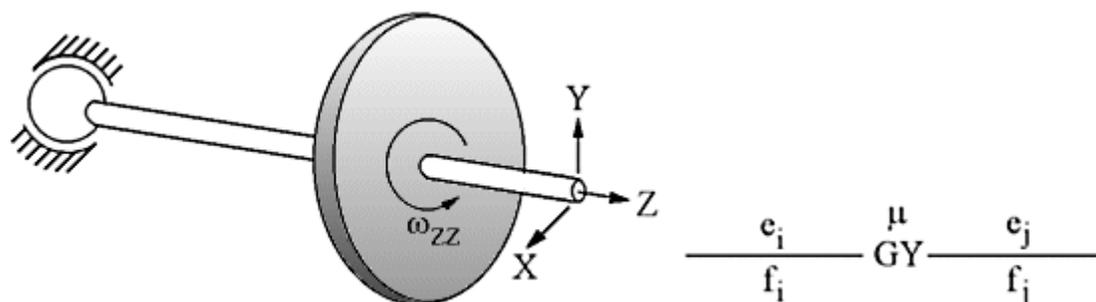
A vertical force creates additional motion in horizontal direction and to maintain a vertical motion, a horizontal force is needed. So the force is transformed into flow and flow is transformed into force with some constant of proportionality. In this example, I_{zz} stands for moment of inertia about z axis. w_x , w_y and w_z stand for angular velocities about respective axes; T_x , T_y and T_z represent torque acting about the corresponding axis.

$$T_x = I_{zz}\, w_z w_y\,.$$

The power transmission implies

$$T_y = I_{zz}\, w_z\, w_x\,, \qquad \text{so that} \qquad T_x w_x = T_y\, w_y\,.$$

Such relationship can be established by use of a **Gyrator** as shown in the figure above.

The μ above the gyrator denotes the gyrator modulus, where $\mu = I_{zz}\, w_z$. This modulus does not have a direction sense associated with it. This modulus is always defined from flow to effort.

$$e_j = \mu\, f_i\,, \qquad e_i = \mu\, f_j\,.$$

Thus the following expression establishes conservation of power, $e_i\, f_i = e_j\, f_j$.

In the electrical domain, an ideal DC motor is represented as an gyrator, where the output torque is proportional to the input current and the back emf is proportional to the motor angular speed. In general, gyrators are used in most of the cases where power from one energy domain is transferred to another, viz. electrical to rotational, electrical to magnetic, and hydraulic to rotational.

The 3-Port junction elements
The name 3-port used for junctions is a misnomer. In fact, junctions can connect two or more bonds. There are only two kinds of junctions, the **1** and the **0** junction. They conserve power and are reversible. They simply represent system topology and hence the underlying layer of junctions and two-port elements in a complete model (also termed the **Junction Structure**) is power conserving.

1 junctions have equality of flows and the efforts sum up to zero with the same power orientation. They are also designated by the letter **S** in some older literature. Such a junction represents a common mass point in a mechanical system, a series connection (with

same current flowing in all elements) in a electrical network and a hydraulic pipeline representing flow continuity, etc. Two such junctions with four bonds are shown in the figure below.
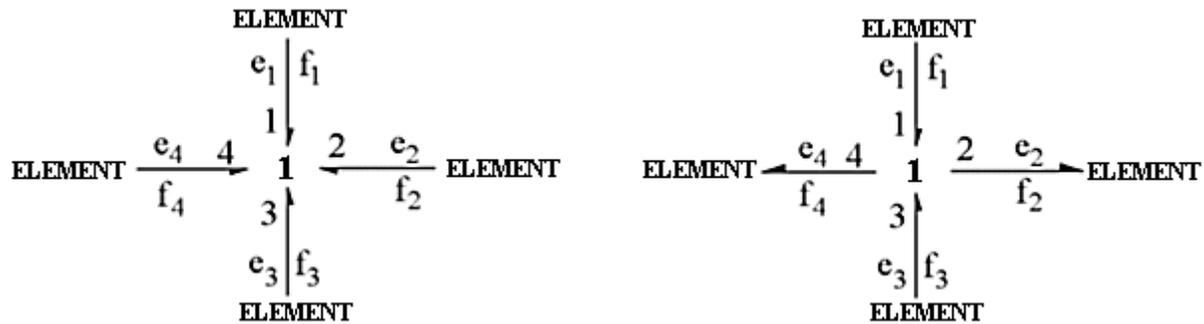
Using the inward power sign convention, the constitutive relation (for power conservation at the junctions) for the figure in the left may be written as follows;

$$e_1 f_1 + e_2 f_2 + e_3 f_3 + e_4 f_4 = 0.$$

As **1** junction is a flow equalizing junction,

$$f_1 = f_2 = f_3 = f_4 .$$

This leads to, $e_1 + e_2 + e_3 + e_4 = 0$.

Now consider the above bond graph shown on the right. In this case, the constitutive relation becomes,

$$e_1 f_1 - e_2 f_2 + e_3 f_3 - e_4 f_4 = 0 , \quad \text{and,} \quad f_1 = f_2 = f_3 = f_4 .$$

Thus, $e_1 - e_2 + e_3 - e_4 = 0$.

So, a **1** junction is governed by the following rules:

The flows on the bonds attached to a 1-junction are equal and the algebraic sum of the efforts is zero. The signs in the algebraic sum are determined by the half-arrow directions in a bond graph.

0 junctions have equality of efforts while the flows sum up to zero, if power orientations are taken positive toward the junction. The junction can also be designated by the letter **P**. This junction represents a mechanical series, electrical node point and hydraulic pressure distribution point or pascalian point.

In case of the model in the left, the constitutive relation becomes,

$$e_1 f_1 + e_2 f_2 + e_3 f_3 + e_4 f_4 = 0.$$

whereas, the model in the right is governed by the following relation,

$$e_1 f_1 - e_2 f_2 + e_3 f_3 - e_4 f_4 = 0.$$

As **0** junction is an effort equalizing junction,

$$e_1 = e_2 = e_3 = e_4.$$

This leads to, $f_1 + f_2 + f_3 + f_4 = 0$ and $f_1 - f_2 + f_3 - f_4 = 0$, for the left and the right models, respectively.

So, a **0** junction is governed by the following rules:

The efforts on the bonds attached to a 0-junction are equal and the algebraic sum of the flows is zero. The signs in the algebraic sum are determined by the half-arrow directions in a bond graph.

4. Power directions on the bonds

When one analyses a simple problem of mechanics, say, the problem of a single mass and spring system as shown in the figure below, one initially fixes a co-ordinate system.

One may take positive displacement, x, towards right and all its time derivatives are then taken positive towards right. The force acting on the mass may also be taken positive towards right. The system, however, in the course of motion may attain such a state that when it is displaced towards the right, the force on the mass happens to be towards the left.
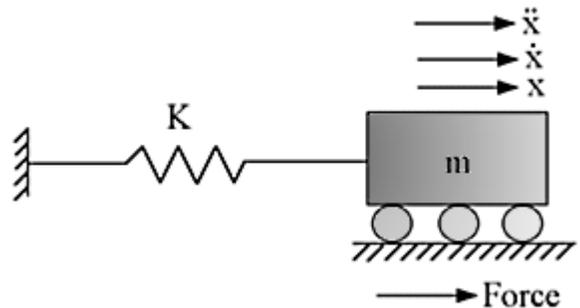
This phenomenon may be interpreted from the results obtained by solving the system of equation(s) when a positive value of displacement and a negative value of force are seen.

So, the initial fixing of a positivity is arbitrary. However, further analysis is related to this fixation. In practice, bond graphs are drawn for general systems. Thus left and right, up and down, clockwise or counter-clockwise, etc., may not be of general relevance. One has to then create a view point which is general and any particular system interpretation should be easily derivable. This is done by assigning the bonds with **Power directions**. This may be as arbitrary as fixing co-ordinate systems in classical analysis. Say in a bond graph, the power is directed as shown in figure below, where

J : junction,
E : element,
**half arrow** : direction of power.

This assignment means, such variables are chosen for effort and flow, so that whenever both these variables acquire positive values, then the power goes from J to E. However, for mixed signs of the variables, the power direction is reversed.

As has been discussed earlier, the interpretation of the relative orientation of positive effort and flow may be subjective depending on whether the analysis is carried out from the stand point of J or E. For instance, we can say the downward motion is positive for a particular mass and the upward motion is positive for another mass according to our co-ordinate system; but for a spring, upward and downward motion do not convey any meaning. In such cases, either the compression or the tension of the spring must be identified as a co-ordinate.

Consider, the following example which belongs to the field of mechanics. The mechanical system with its bond graph model are displayed in the figure below.

Here the variables are so chosen that for positive force and velocity, the power goes to the spring. This may be as per the instrumentation arranged as shown in the two illustrations given below.
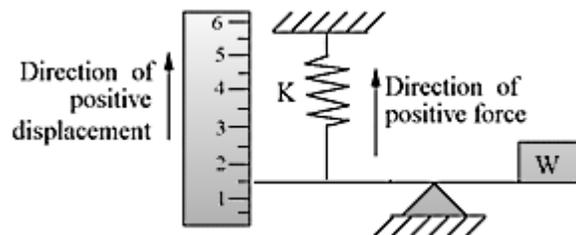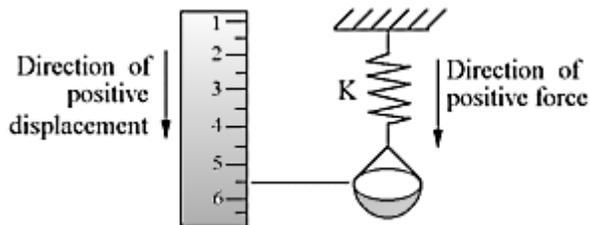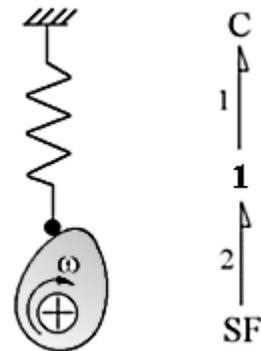
If one assumes the stand point of the junction **1**, then characteristic of the spring should be as determined by the arrangement as shown above, and may appear as plotted in the right.
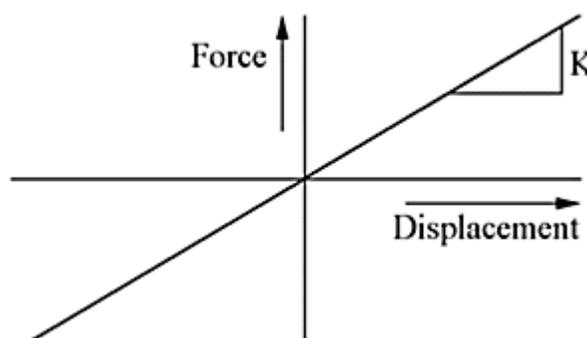
5. Assigning numbers to bonds

The bonds in a bond graph may be numbered sequentially using integers starting with 1. However, one need not follow any fixed rule. Assignment of bond number also fixes the name of the elements or junctions. This is the best bookkeeping technique adopted by most of the existing software products. Some software though follow numbering of elements according to their instance. However, in models using fields, where many bonds are connected to an element, such a nomenclatures cause difficulty in book-keeping.

For example, the two 1-junctions in the bond graph shown in the right can be uniquely identified as (S 1 2 3 4) and (S 5 8 9); similarly symbols like C3, C9 can be used to identify a particular element.
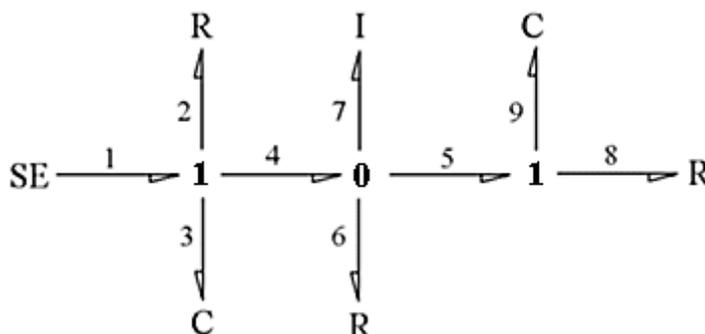
The use of characters **S** and **P** instead of numerals **1** and **0** allows to represent models in tele-type code in the list form, which can be exchanged across platforms. The power direction and causal information may also be appended to the list form to create a globally adoptable model code, which may read something like (S +-1 --2 --3 -+4)(P +-4 ....
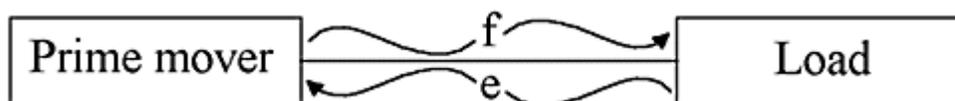
6. Causality

Causality establishes the cause and effect relationships between the factors of power. In bondgraphs, the inputs and the outputs are characterized by the causal stroke. The causal stroke indicates the direction in which the effort signal is directed (by implication, the end of the bond that does not have a causal stroke is the end towards which the flow signal is directed).

To illustrate the basic concept of the causality, let us assume that there is a prime mover with a smart speed governor as shown in the figure below.
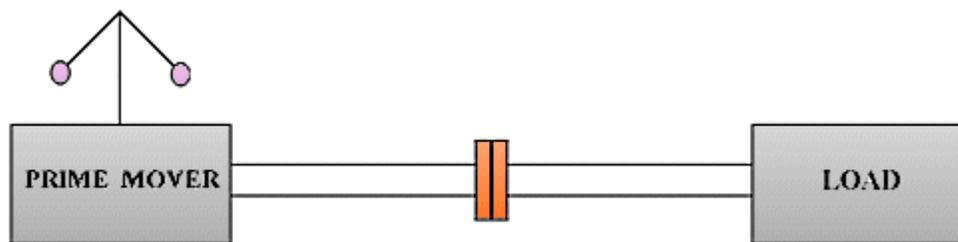
The prime mover is driving the load i.e., the power is going from the prime mover to the load. Apart from sending the power, the prime mover also decides that the load should run at a particular speed depending on the setting of the governor. So, it may be said that from the prime mover, the information of flow is generated which goes to the load. The load generates the information of torque (effort) which the prime mover receives and adjusts the inner mechanisms to compensate for it. The wavy lines in the following figure indicate the direction of flow of the particular information.

The selected causality is generally indicated by a cross bar or causal bar at the bond end to which the effort receiver is connected. In expressing causal relations between the effort and the flow, the choice of causality has an important effect. In the following section, the causality of the different elements are discussed.

1. For inertance **I** type storage elements, the flow (*f*) is proportional to the time integral of the effort.

$$f = m^{-1} \int_{-\infty}^{t} e\, dt$$

An example of this would be a mass subjected to a force, causing it to accelerate.

$$v = m^{-1} \int_{-\infty}^{t} F\, dt$$

In the above relationship effort history is integrated to generate contemporary flow. Therefore, an **I** element receives effort (cause) and generates flow (effect).

1. For capacitive **C** type storage elements, the effort (*e*) is proportional to the time integral of the flow (*f*).

$$e = k \int_{-\infty}^{t} f\, dt$$

Examples of this would be a spring being deformed by a force or a capcitor being charged.

$$F = K \int_{-\infty}^{t} v\, dt$$

$$V = C^{-1} \int_{-\infty}^{t} i \, dt$$

The above integral relationships show that a **C** element receives flow and generates effort.

1. The resistive or dissipative elements do not have time integral form of constitutive laws.

$$e = R f \qquad \text{or} \qquad f = e/R$$

The flow and the effort at this port are algebraically related and can thus have any type of causal structure, either with an open-ended bond (causal stroke is away from the element, i.e., at the junction end) indicating a resistive causality or a stroke ended bond indicating a conductive causality.

As per the above discussions, the causal strokes for **I**, **C** and **R** elements are shown in the figure below.

$$J \longrightarrow I \qquad J \vdash \longrightarrow C \qquad J \vdash \longrightarrow R \qquad J \longrightarrow R$$

1. Sources impose either an effort or a flow on a system, but not both.

$$SE \longrightarrow \qquad SF \vdash \longrightarrow$$

The bondgraphic sources are assumed to be robust, i.e., as providers of active and infinite energy. The effort source (**SE**), imposes an effort on the system which is independent of flow. An Example of this would be an electrical cell that decides the terminal voltage and the attached load decides the current that the cell has to take and adjust its chemical reactions to maintain the rated terminal voltage.

The flow source (**SF**), imposes a flow on the system independent of the effort. Examples of this would be cams, constant displacement hydraulic pumps, road excitation, etc. Thus the causality of the source elements are mandatory as shown in the figure below.

The elements **I, C, R, SF,** and **SE** are classified as single port, since they interact with the system through one bond only. However, the **I, C** and **R** elements can be connected to many bonds to represent tensorial nature, such as the spatial motion of a free body or the stress-strain relationships in a compressible material; in which case they are termed as field elements.

The transformer, by its elemental relation, receives either flow or effort information in one bond and generates the same in its other bond. Thus, one of its port is open-ended with the other end stroked as shown in the figure below.

$$\vdash \frac{e_i}{f_i} - \overset{\overset{\displaystyle r}{\displaystyle \frown}}{TF} \vdash \frac{e_j}{f_j} \qquad\qquad \vdash \frac{e_i}{f_i} - \overset{\overset{\displaystyle r}{\displaystyle \frown}}{TF} \vdash \frac{e_j}{f_j}$$

For the first case, the constitutive equations would be,

$f_j = r\, f_i$ and $e_i = r\, e_j$,

whereas; for the second, the relations are

$f_j = 1/r\, f_i$ and $e_i = 1/r\, e_j$.

As mentioned earlier, the gyrator relates flow to effort and effort to flow, therefore, both of its ports have either open-ended or stroke-ended causality as shown below.


For the first case, the constitutive equations would be

$e_j = r\, f_i$ and $e_i = r\, f_j$,

whereas; for the second, the relations are

$f_j = r\, e_i$ and $f_i = r\, e_j$.

At a **1** junction, only one bond should bring the information of flow; i.e., only one bond should be open ended and all others should be stroked as displayed in the figure on the right. This uniquely causalled bond at a junction is termed as **Strong bond**. In any case two bonds cannot be causalled away from the 1-junction, since this would lead to violation of rules of information exchange (the two bonds may not impart equal flows).
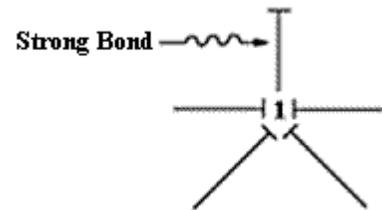
Similarly at a **0** junction, only one bond should be stroked nearer to the junction. This strong bond determines the effort at the junction, which the **weak bonds** (other bonds besides the strong bond) carry around.
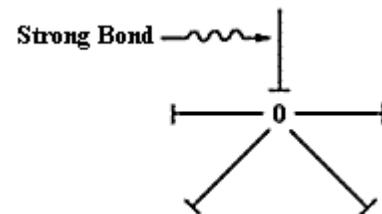
The proper causality, for a storage element (**I** or **C**), is called **Integral Causality**, where the cause is integrated to generate the effect. For example, in **C** element, a study of the constitutive equations reveal that flow is integrated and multiplied with the stiffness to generate effort.

Sometimes the causal strokes will have to be inverted, which means the constitutive relationship for the corresponding element is written as a differential equation. For example, flow in a spring is the time derivative of the ratio of effort and stiffness. Such causality pattern is called **Differential Causality**.

Implications of an Integral causality means the past data of cause or history function is integrated to arrive at the effect felt at present, whereas, differential causality needs differentiation of the cause at present (that cannot be found properly, since the future is not known) to arrive at the effect. Hence differential causality makes system dependent on future, as if the system is being dragged towards a predestined configuration or in other terms adds specific constrains on the dynamics of the system.

Genuine differential causality is not commonly encountered during system modeling except in certain cases of modeling mechanisms, robotics, etc., where link flexibilities or other aspects are neglected in the model. Such causality is a common occurrence owing to certain direct manipulations in the model. However, spurious occurrences are not ruled out in certain cases such as modeling of a superfluous element (try bond graph of a single degree freedom system, where another mass is placed directly in contact with the main mass), or leaving out certain important element from system (neglecting contact point stiffness etc.).

The occurrence of differential causalities in a system may indicate serious violations of principles of conservation of energy, as illustrated in the following example.

Let us consider the charging of a capacitor through a constant voltage battery source. The system and its bond graph model are shown below.
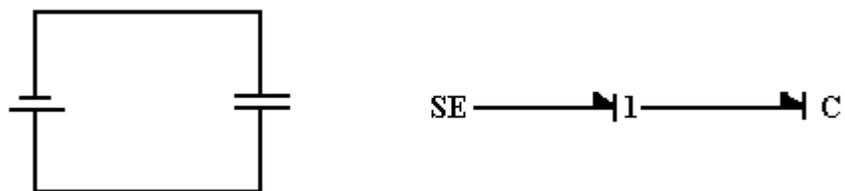
From the causalled bond graph model, we observe the differential causality in the C-element. If we consider the energy exchange, the energy (E) stored in the capacitor after complete charging to a voltage V is

$E = Q^2 / 2C$, where C is the capacitance and Q is the charge stored in the capacitor.

Also $Q = \int_{-\infty}^{t} i\, dt$ and $V = Q / C$.

The energy spent by the shell during charging is

$E = \int_{-\infty}^{t} V i\, dt = V \int_{-\infty}^{t} i\, dt = Q^2 / C$.

The later result is anomalous with the energy stored in the capacitor. The loss of half the energy is un accounted for. This can be attributed as one of the implications of the

differential causality. The half energy lost is always through dissipation in the system which wrongly has been neglected. If we introduce the resistance in the model, the causality problem is automatically corrected and proper energy conservation is obtained as shown in the figure on the right.
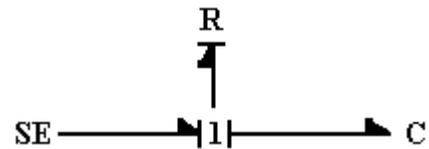

Causality Assignment Procedure:

1. Assign fixed causalities to sources.

2. Propagate the causality through junctions, if possible, i.e. if any bond has got a causality such that it has become the strong bond for a junction, the causality for all other bonds (week bonds) is determined by laws for causality of junctions and if all other bonds of junction are causalled, the last bond should be the strong bond. Similarly, if any port of a two port is causalled (of TF and GY), the causality of the other can be assigned.

3. Assign integral causality to one of the storage elements and propagate the causality through junctions. Continue the procedure with other storage elements. This should normally result in complete causalling of the graph.

4. If the graph is not completely causalled yet, start assigning a resistive causality to a R-element and propagate it. Continue till the entire graph is causalled. In cases, where the model is determined through causalities of R-elements, there may be several possible causal models. It is always advisable to maximize resistive causalities and minimize the conductive causalities in R-elements.

5. If the system develops differential causalities in some storage elements, try minimizing its number of occurrence through assigning initial integral causalities to other storage elements than those selected before.

6. Try to avoid differential causalities by suitable changes to the model, such as introducing some compliance or resistance or both.

7. Discard all models, which result in a causal structure, that violates junction causality rules.

Two animated examples of causalling a model in different ways are illustrated below.
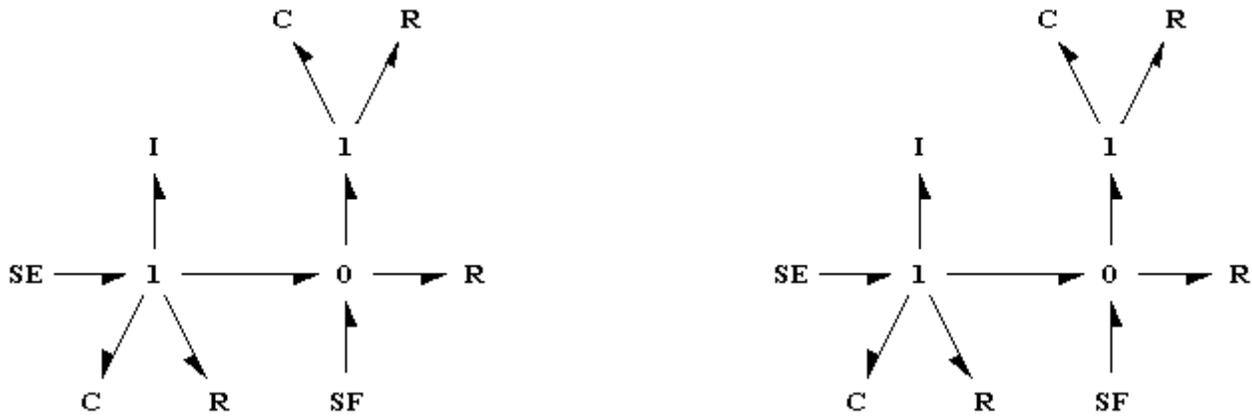
7. Generation of system equations

In this section the method of generation of system equations is discussed. From an augmented bond graph, using a step by step procedure, system equations may be generated. We take the model of a simple single degree of freedom mass-spring-damper system as the starting point.
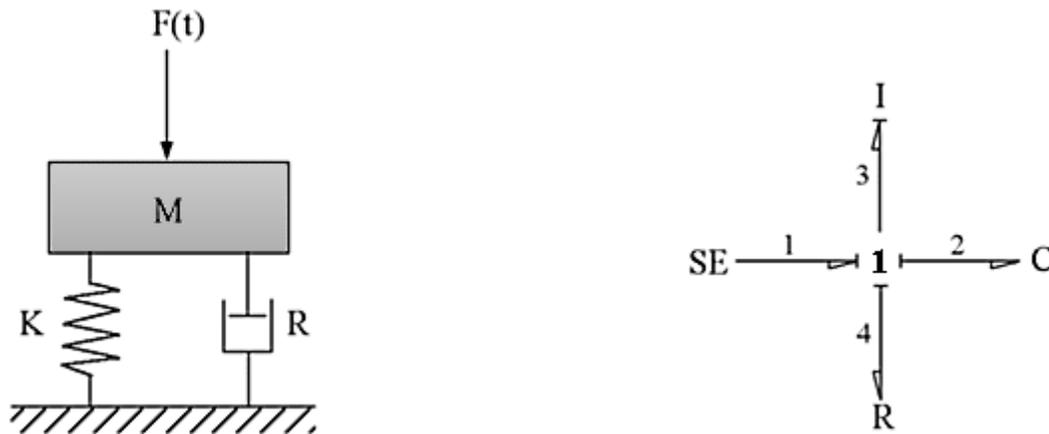
The differential equations describing the dynamics of the system are written in terms of the states of the system. All storage elements (I and C) correspond to stored state variables (P for momentum and Q for displacement, respectively) and equations are written for their time derivatives (i.e. effort and flow). These equations are derived in four steps as described below.

1. Observe what the elements (sources, I's, C's, and R's) are giving to the system and write down their equations looking at the causalities and using variables for strong bonds.

2. Write down equations for the junctions and the two-port elements for the variables for the strong bonds.

3. Replace the variables, which are expressed in terms of states in other equations. Continue sorting and replacement till the right side of the entire set of equations are expressed in terms of states and system parameters only.

4. If some equations are still not completely reduced, there is the existence of some kind of a loop (algebraic loop, causal loop or differential causality, as shall be discussed later). Try solving those as a set of linear equations either through substitution or matrix inversion.

     And finally, erase all trivial equations other than those for derivatives of state variables and write them in terms of state variables.

Thus using the following steps, the equations for the above system would be

---

Step 1 :
    e1=SE1
    f3=P3/M3
    e2=K2*Q2
    e4=R4*f4=R4*f3 *(by junction rule, f4=f3 and strong bond number is 3)*

Step 2 :
    e1-e2-e3-e4=0   or   e3=e1-e2-e4

Step 3 :
    e1=SE1
    f3=P3/M3
    e2=K2*Q2
    e4=R4*P3/M3
    e3=SE1-K2*Q2-R4*P3/M3

Step 4 :
    DQ2=f2=f3
    DP3=e3

where prefix 'D' stand for time derivative d/dt leads to

    DQ2=P3/M3
    DP3=SE1-K2*Q2-R4*P3/M3

The difference between equations derived from bond graphs and otherwise are that there will be `N' sets of first order differential equations, where `N' is the number of states. The term, number of states means the number of lumped parameter storage elements **I** and **C** with integral causality present in a system.

The equation of motion for the system discussed above by traditional method is :

m d^2x/dt^2 + r * dx/dt + k*x = F(t).

The two state equations derived from a bond graph model (dropping suffixes) are

dP/dt = -r/m * P - k*Q + SE,
dQ/dt = 1/m*P,

where, P is momentum of m*dx/dt, Q is displacement or x and SE is F(t)).
From the second equation, P=m*dQ/dt, which when replaced in the first leads to

$m*d^2Q/dt^2$ = -r*dQ/dt - k*Q + F(t).

This equation corresponds to that derived through traditional method after rearrangement.

However, manual derivation of equations for larger systems is not all that simple. For instance, derivation of system differential equations for the animated bond graph model discussed in the causality section after proper causalling would lead to formation of the so called **algebraic loops**. Similarly, complexities and errors of various types, like **Causal loops**, **Power loops**, and **Differential Causalities** may exist in the model of a system. The method for derivation of their equations is described in corresponding sections.

Example -2 :
Here we take another example of a system with two-ports, whose model is shown below.
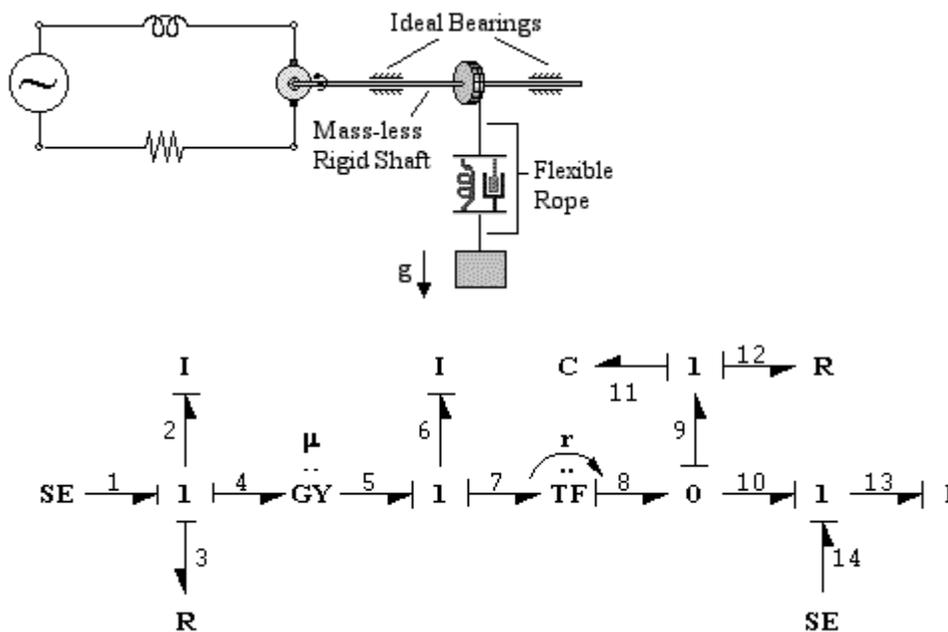
Then we follow the normal equation derivation steps.

Step 1 :
    e1 = SE1
    f2 = P2/M2
    e3 = R3*f3=R3*f2=R3*P2/M2
    f6 = P6/M6
    e11 = K11*Q11
    e12 = R12*f12 = R12*f9
    f13 = P13/M13

Step 2 :

$f1 = f2 = f4 = f3 = P3/M3$
$e2 = e1 - e3 - e4 = SE1 - R3*P2/M2 - e4$
$e4 = MU*f5 = MU*f6 = MU*P6/M6$
$e5 = MU*f4 = MU*f2 = MU*P2/M2$
$f5 = f7 = f6 = P6/M6$
$e6 = e5 - e7$
$f8 = r*f7 = r*P6/M6$
$e7 = r*e8 = r*e9$
$e8 = e10 = e9$
$f9 = f8 - f10 = r*P6/M6 - f13 = r*P6/M6 - P13/M13$
$f10 = f14 = f13 = P13/M13$
$e13 = e10 + e14 = e9 + SE14$
$f11 = f12 = f9$
$e9 = e11 + e12 = K11*Q11 + R12*f9$

Step 3 :
$e1 = SE1$
$f2 = P2/M2$
$e3 = R3*f3=R3*f2=R3*P2/M2$
$f6 = P6/M6$
$e11 = K11*Q11$
$e12 = R12*f9 = R12*(r*P6/M6 - P13/M13)$
$f13 = P13/M13$
$f1 = f2 = f4 = f3 = P3/M3$
$e2 = SE1 - R3*P2/M2 - e4 = SE1 - R3*P2/M2 - MU*P6/M6$
$e4 = MU*f5 = MU*f6 = MU*P6/M6$
$e5 = MU*f4 = MU*f2 = MU*P2/M2$
$f5 = f7 = f6 = P6/M6$
$e6 = e5 - e7 = MU*P2/M2 - r*(e11 + e12)$
$\quad = MU*P2/M2 - r*(K11*Q11 + R12*(r*P6/M6 - P13/M13))$
$f8 = r*f7 = r*P6/M6$
$e7 = r*e8 = r*e9 = r*(e11+e12) = r*(K11*Q11 + R12*(r*P6/M6 - P13/M13))$
$e8 = e10 = e9 = e11+e12 = K11*Q11 + R12*(r*P6/M6 - P13/M13)$
$f9 = f8 - f10 = r*P6/M6 - f13 = r*P6/M6 - P13/M13$
$f10 = f14 = f13 = P13/M13$
$e13 = e10 + e14 = e9 + SE14 = K11*Q11 + R12*(r*P6/M6 - P13/M13) + SE14$
$f11 = f12 = f9 = r*P6/M6 - P13/M13$
$e9 = e11 + e12 = K11*Q11 + R12*f9 = K11*Q11 + R12*(r*P6/M6 - P13/M13)$

Step 4 :
$DP2 = SE1 - R3*P2/M2 - MU*P6/M6$
$DP6 = MU*P2/M2 - r*(K11*Q11 + R12*(r*P6/M6 - P13/M13))$
$DP13 = K11*Q11 + R12*(r*P6/M6 - P13/M13) + SE14$
$DQ11 = r*P6/M6 - P13/M13$

In matrix form, these equations may be written as $d\{Y\}/dt = [A]\{Y\} + [B]\{u\}$,

where, $\{Y\}$ is a vector of states (P2,P6,P13 and Q11), [A] is square matrix, $\{u\}$ is the array of sources (SE1 and SE14} and [B] is a matrix of dimension N x M; N being the number of states and M being the number of sources. The matrices [A] and [B] are;

$$A = \begin{bmatrix} -R3/M2 & -MU/M6 & 0.0 & 0.0 \\ MU/M2 & -R12*r^2/M6 & R12*r^2/M13 & -r*K11 \\ 0.0 & R12*r/M6 & -R12/M13 & K11 \\ 0.0 & r/M6 & -1/M13 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$B = \begin{vmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{vmatrix}.$$

8. Activation

Some bonds in a bond graph may be only information carriers. These bonds are not power bonds. Such bonds, where one of the factors of the power is masked are called **Activated bonds**. As an example, let the system shown in the figure below be considered.
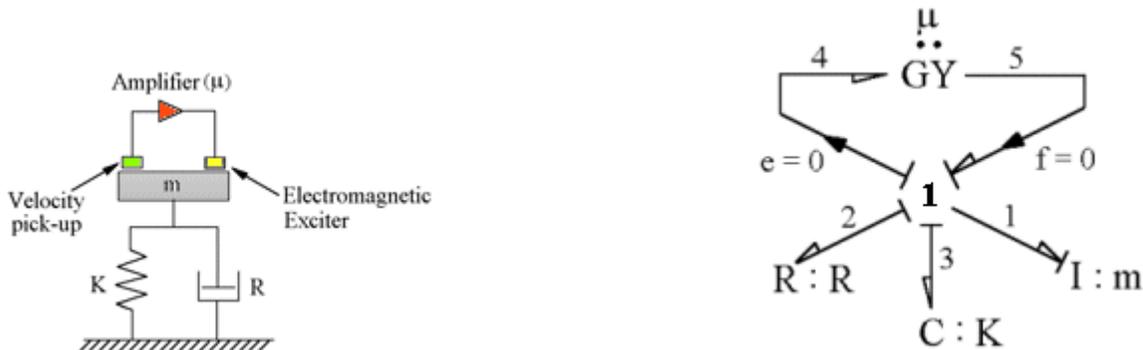

Here the velocity pick-up only carries the information of velocity to the amplifier through which an electro-magnetic exciter applies force proportional to velocity on the mass and the exciter does not carry the information of velocity and impose it back on the mass as an reactive force. So on the bond representing the velocity pick-up the information of force must be masked and on the bond representing the exciter the information of the flow must be masked. A full arrow somewhere on the bonds shows that some information is masked and what information is masked may be written near that full arrow. According to this convention the bond graph of the system is shown to the right of the system drawn above.

The bond number 4 in the figure is the pick-up bond where information of effort is masked and in bond number 5, which is the exciter bond, the flow is activated. The concept of activation is very significant to depict feedback control systems.

The term activation initially seems a misnomer. However, Paynter's idea was based on the fact that though the information of a factor of power is masked on one end, an activated bond on the other end can impart infinite power which is derived from a tank circuit used for both the measurement or actuation device (for instance, the pick-up, the amplifier and the exciter, all have external power sources).
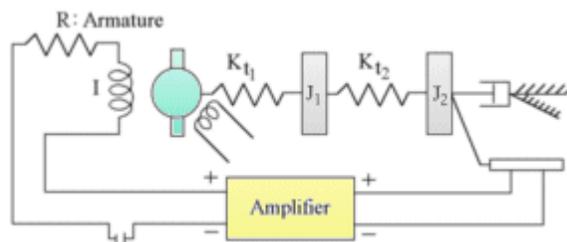
In the system shown above, a D.C. motor with externally energized field is driving an elastic shaft with two disks and with a fluctuating resistive load. The speed, picked up by a tachometer is fed-back to control the speed of the motor by adjusting the armature current (increasing voltage alternatively). The bond graph for this system is drawn as shown below.
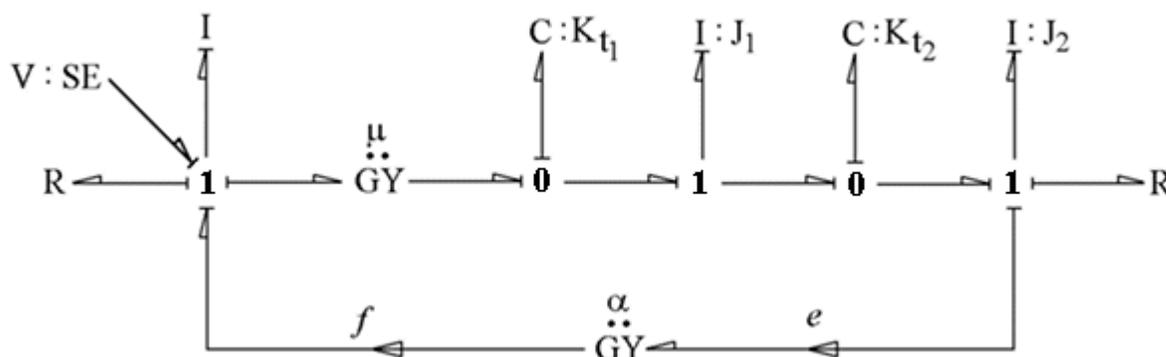

Observers : Additional states can be added for measurement of any factor of power on a bond graph model using the **Observer** storage elements. An effort activated C-element would observe the time integral of flow (and consequently flow), whereas a flow activated I-element would observe the generalized momentum (and consequently effot). Activated elements are percieved conceptual instrumentations on a model. They don't interfere in the dynamics of the system (i.e. their corresponding states never appear on the right-hand side of any state equation. A system with N states, M sources and L observers would have the state equations of the form

$d/dt\{Y\} = [A]\{Y\} + [B]\{u\}$,
$d/dy\{Z\} = [C]\{Y\} + [D]\{u\}$,

whre {Y} is a vector of true states, {Z} is the vector of observed states, {u} is a vector of sources, [A] is MxM matrix, [B] is MxN matrix, [C] is ZxM matrix and [D] is ZxN matrix.

Here is an example of a single-degree-of-freedom system with instrumentations, whose model with observer elements is shown below.
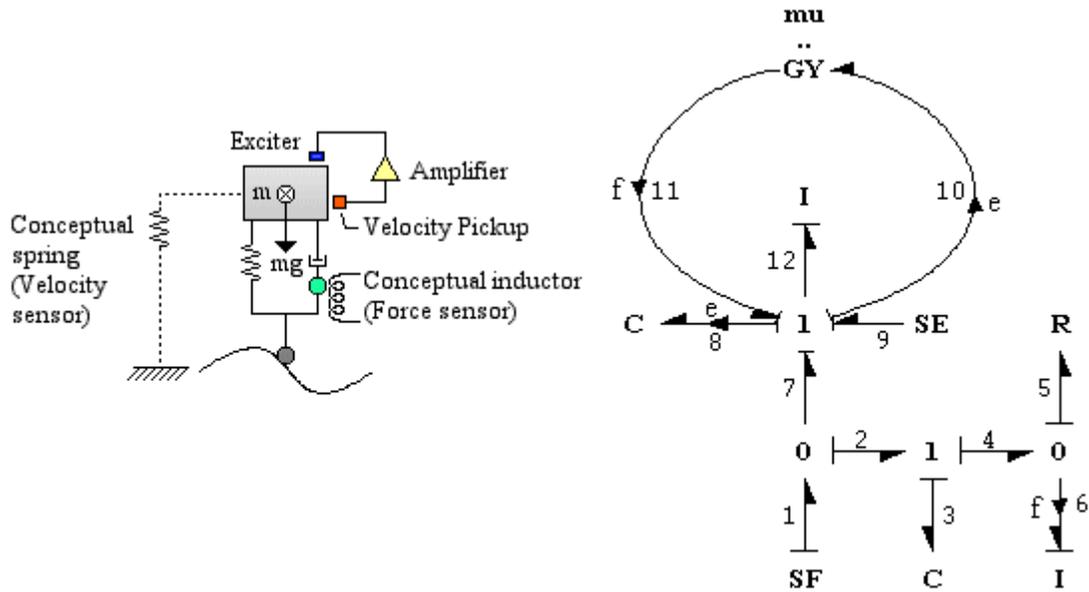
The conceptual instrumentations shown here, assumes a spring of zero stiffness (thus measuring displacement and not generating any reactive effort) and an inductive coil within which the segments of the damper move to induce emf (a measurement of force) without any reaction on the damper segment. The equations for this model can be derived as shown below.

Step 1 :
f1 = SF1
e3 = K3*Q3
e5 = R5*f5
f6 = 0
e9 = SE9
e8 = 0
e10 = 0
f12 = P12/M12
e11 = MU*f10 = MU*f12 = MU*P12/M12

Step 2 :
e12 = e7 + e9 - e8 + e11 - e10 = e2 + SE9 + MU*P12/M12
f2 = f1 - f7 = SF1 - P12/M12
f5 = f4 - f6 = f4 = f2 = SF1 - P12/M12
e2 = e3 + e4 = e3 + e5 = K3*Q3 + R5*f5 = K3*Q3 + SF1 - P12/M12

Step 3 :
e12 = K3*Q3 + R5*(SF1-P12/M12) + SE9 + MU*P12/M12
e5 = R5*(SF1 - P12/M12)

Step 4 :
The State equations are

DP12 = e12 = K3*Q3 + R5*(SF1-P12/M12) + SE9 + MU*P12/M12
DQ3 = f3 = f2 = SF1 - P12/M12

and the observer equations are

DP6 = e6 = e5 = R5*(SF1 - P12/M12)
DQ8 = f8 = f12 = P12/M12
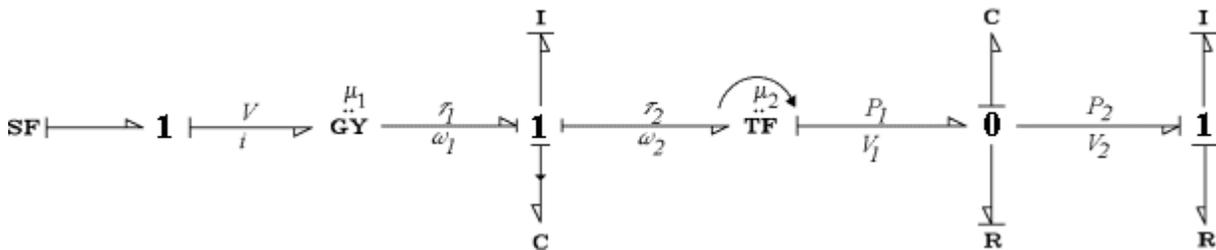
9. Bond graph modeling

A simple mechanical system shown in the figure below is considered here for explaining the modeling procedure.
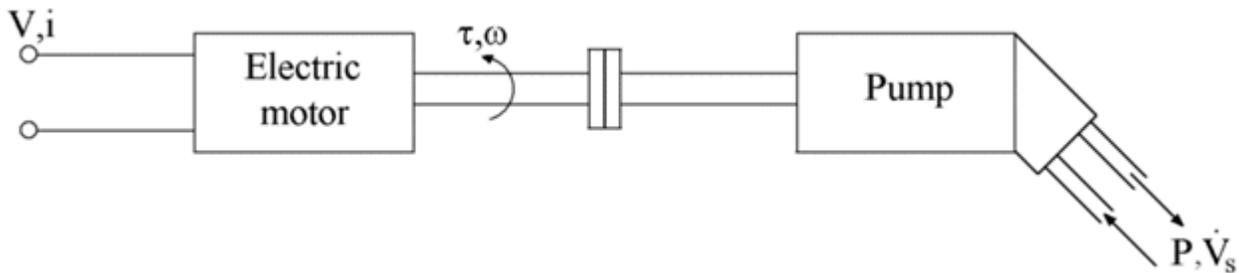

The output from the pump would be flow and pressure, while the input would be torque and angular velocity applied to the pump shaft. The input power is provided by an electric motor. The power flow diagram is shown in the figure below.

Two transformations are identified. First from electric power to mechanical power through the electric motor; followed by the mechanical power to the fluid power through the pump. In the next step to construct the model, the components are to be looked into more details. In this simple analysis of the pump it has its own moving parts (inertia); oil flow has got its own compressibility (capacitance) because of which it can store energy; pump has leakage due to its geometric tolerance and friction between the moving parts (resistance) through which it can dissipate energy. The bond graph of the system with power directions and causality is shown below.

A two degrees of freedom mechanical system and its corresponding Bond graph model shown below can be explained in the following manner.
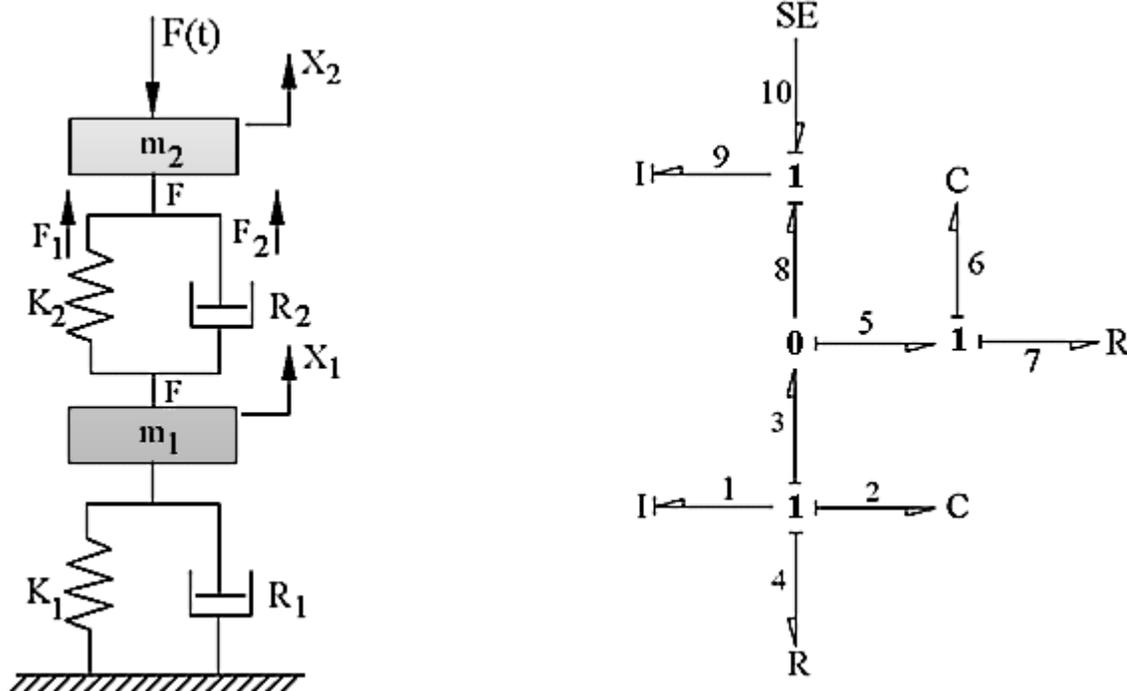
The bottom mass velocity is same as the rate of stretching of the spring and damper connected to the ground. So, these are modeled as being connected to the 1-junction. The middle spring and damper experience components of force, say F1 and F2, respectively such that F=F1+F2, i.e., the force felt by the bottom mass, the spring and damper in combination and the top mass is equal. Alternatively, the distortion of the spring and the damper is dependent on relative motion of the masses. Hence, one can easily smell the presence of a force pass or flow summation junction, in other words the 0-junction here. Further, this 0 junction has three branches corresponding to three components of distribution of effort.

Another 1-junction appears in the right, that signifies the equal relative velocity (i.e., rate of compression here considering the power directions) felt by the middle spring and the damper. The 1-junction at the top depicts the coherent motion of the forcing device with the top mass.

For the above system, alternative methods can be applied to arrive at a bond graph model, that may apparently look different. However, the final equations derived from different models lead to the same set. It may be noted that bond graphs for mechanical systems may be drawn using the method of flow balance (kinematics) or force transmission, or a combination of the both. Since the basic external elements used are the same and only the energy conserving junction structure may vary, considering one factor of power automatically takes care of the effect of the other factor. This is perhaps the greatest contribution of the bondgraph theory.

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