Invited Workshop on Compiler Techniques for Sparse Tensor Algebra

Cambridge MA, January 26th

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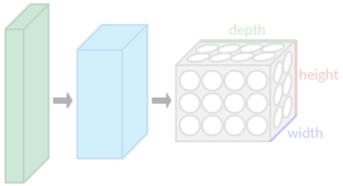
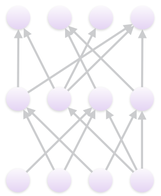
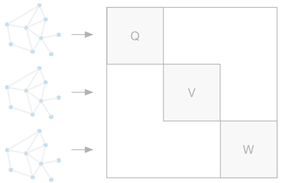


Convolutional Networks

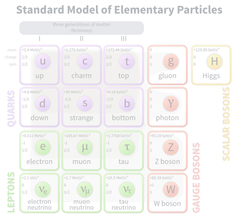


Sparse Networks [GRK 2017]

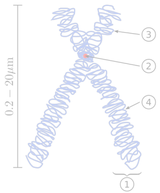


Graph Convolutional Network [KW 2017]

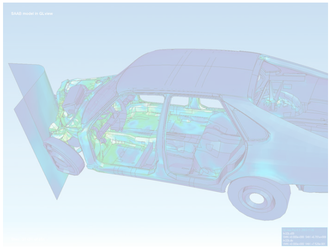
Science and Engineering



QCD



Quantitative Trait Loci



Finite Element Method

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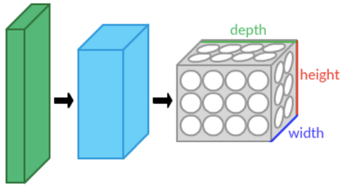

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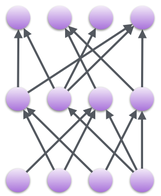

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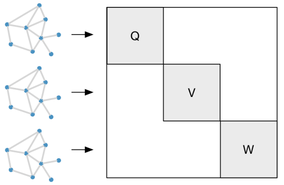

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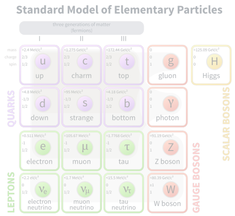

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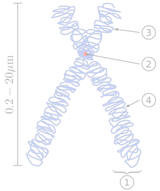

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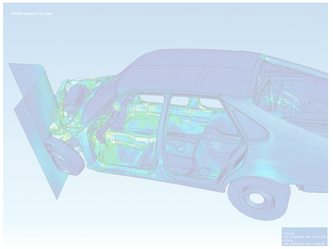

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Sparse Networks [GRK 2017]

Graph Convolutional Network [KW 2017]

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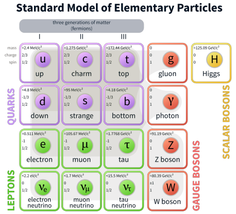

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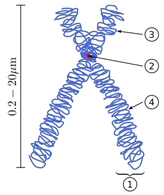

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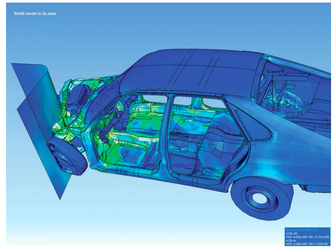

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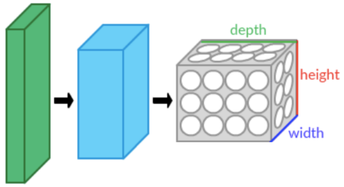
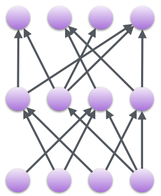


Convolutional Networks



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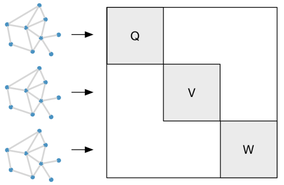
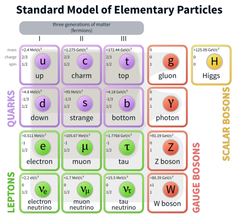


Graph Convolutional Network [KW 2017]

Science and Engineering



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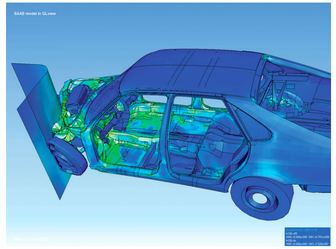
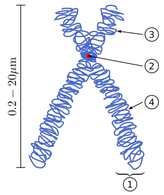


Finite Element Method



Quantitative Trait Loci

Scalars have 0 dimensions
Vectors have 1 dimension
Matrices have 2 dimensions
An n-tensor has n dimensions

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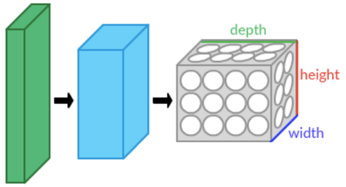
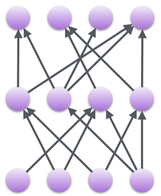


Convolutional Networks



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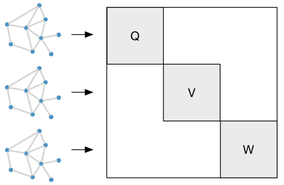

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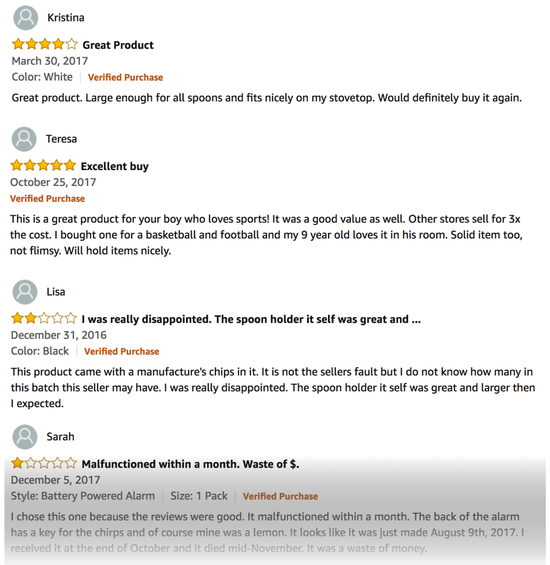

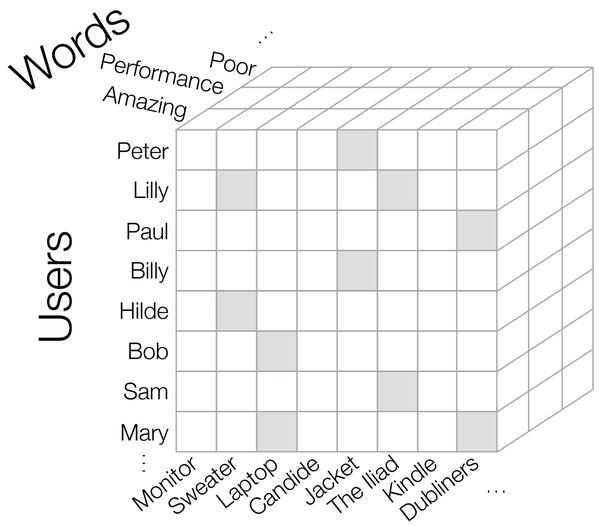

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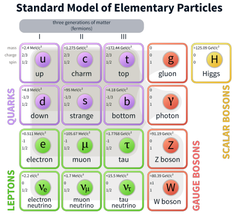

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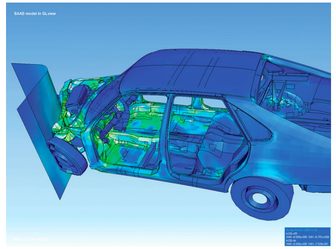

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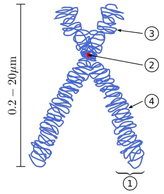

Quantitative Trait Loci

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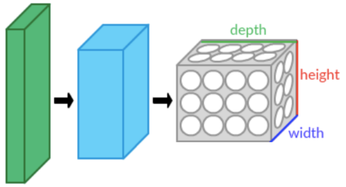

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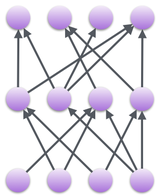

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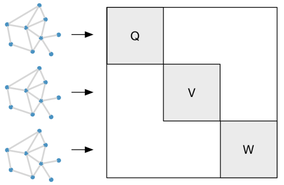

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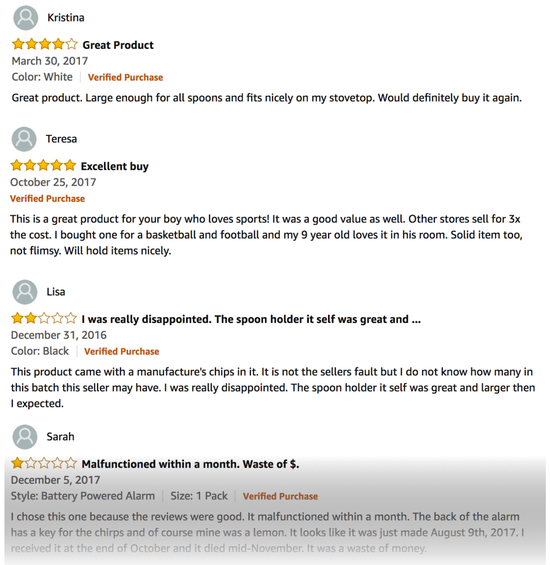

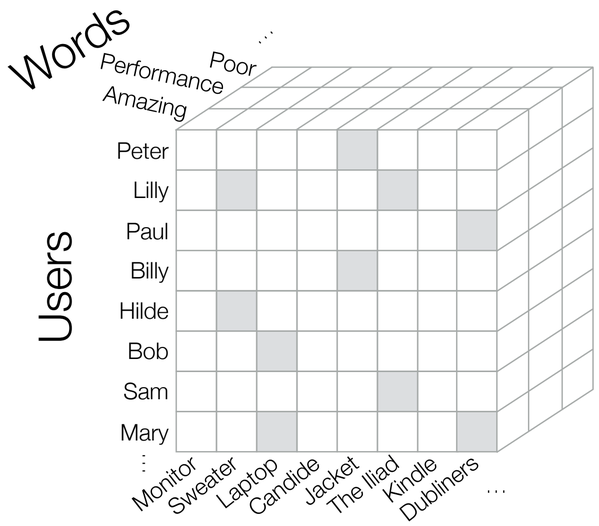

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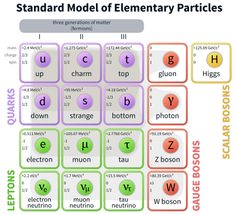

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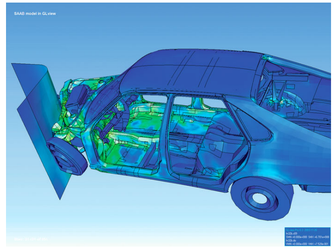

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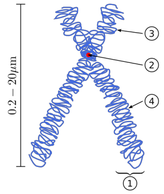

Quantitative Trait Loci

Extremely sparse
Dense storage: 107 Exabytes
Sparse storage: 13 Gigabytes

Too many tensor kernels for a fixed-function library

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

$$a = B^T c + d \quad A = B + C + D \quad A = BC$$

$$A = B \odot C \quad a = b \odot c \quad A = 0 \quad A = B \odot (CD)$$

$$A = BCd \quad A = B^T \quad a = B^T Bc$$

$$a = b + c \quad A = B \quad K = A^T C A$$

$$A_{ij} = \sum_{kl} B_{ikl} C_{lj} D_{kj} \quad A_{kj} = \sum_{il} B_{ikl} C_{lj} D_{ij}$$

$$A_{lj} = \sum_{ik} B_{ikl} C_{ij} D_{kj} \quad A_{ij} = \sum_{k} B_{ijk} c_k$$

$$A_{ijk} = \sum_{l} B_{ikl} C_{lj} \quad A_{ik} = \sum_{j} B_{ijk} c_j$$

$$A_{jk} = \sum_{i} B_{ijk} c_i \quad A_{ijl} = \sum_{k} B_{ikl} C_{kj}$$

$$\tau = \sum_{i} z_i (\sum_{j} z_j \theta_{ij})(\sum_{k} z_k \theta_{ik})$$

$$C = \sum_{ijkl} M_{ij} P_{jk} \overline{M_{lk}}\, \overline{P_{il}}$$

$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

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Linear Algebra

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Data analytics
(tensor factorization)

$$\tau = \sum_i z_i (\sum_j z_j \theta_{ij})(\sum_k z_k \theta_{ik})$$

$$C = \sum_{ijkl} M_{ij} P_{jk} \overline{M_{lk}}\, \overline{P_{il}}$$

$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

Quantum Chromodynamics

Too many tensor kernels for a fixed-function library

Eigen (SpMV)

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

Too many tensor kernels for a fixed-function library

CSparse

$a = Bc + a$

Eigen (SpMV)

$a = Bc$

OSKI

$a = \alpha Bc + \beta a$

$a = Bc + b$

$A = B + C$

PETSc

$a = B^T c$

$A = \alpha B$

$a = B(c + d)$

$a = B^T c + d$

$A = B + C + D$

$A = BC$

$A = B \odot C$

$a = b \odot c$

$A = 0$

$A = B \odot (CD)$

$A = BCd$

$A = B^T$

$a = B^T Bc$

$a = b + c$

$A = B$

$K = A^T C A$

$$A_{ij} = \sum_{kl} B_{ikl} C_{lj} D_{kj} \qquad A_{kj} = \sum_{il} B_{ikl} C_{lj} D_{ij}$$

$$A_{lj} = \sum_{ik} B_{ikl} C_{ij} D_{kj} \qquad A_{ij} = \sum_{k} B_{ijk} c_k$$

$$A_{ijk} = \sum_{l} B_{ikl} C_{lj} \qquad A_{ik} = \sum_{j} B_{ijk} c_j$$

$$A_{jk} = \sum_{i} B_{ijk} c_i \qquad A_{ijl} = \sum_{k} B_{ikl} C_{kj}$$

$$\tau = \sum_i z_i (\sum_j z_j \theta_{ij})(\sum_k z_k \theta_{ik})$$

$$C = \sum_{ijkl} M_{ij} P_{jk} \overline{M_{lk}}\, \overline{P_{il}}$$

$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

Too many tensor kernels for a fixed-function library

CSparse

$a = Bc + a$

Eigen (SpMV)

$a = Bc$

$a = Bc + b$

$A = B + C$

OSKI

$a = \alpha Bc + \beta a$

OSKI has 282 specialized variants of this expression

PETSc

$a = B^T c$

$A = \alpha B$

$a = B(c + d)$

$a = B^T c + d$

$A = B + C + D$

$A = BC$

$A = B \odot C$

$a = b \odot c$

$A = 0$

$A = B \odot (CD)$

$A = BCd$

$A = B^T$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

$$a = B^T c + d \quad A = B + C + D \quad A = BC$$

$$A = B \odot C \quad a = b \odot c \quad A = 0 \quad A = B \odot (CD)$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

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$$A_{jk} = \sum_{i} B_{ijk} c_i \quad A_{ijl} = \sum_{k} B_{ikl} C_{kj}$$

$$\tau = \sum_{i} z_i (\sum_{j} z_j \theta_{ij})(\sum_{k} z_k \theta_{ik})$$

$$C = \sum_{ijkl} M_{ij} P_{jk} \overline{M_{lk}}\, \overline{P_{il}}$$

$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR DCSR BCSR Thermal Simulation

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

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$$C = \sum_{ijkl} M_{ij} P_{jk} \overline{M_{lk}}\, \overline{P_{il}}$$

$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR | DCSR | BCSR | Web matrix [BG 2008]

COO | ELLPACK | CSB

Blocked COO | CSC

DIA | Blocked DIA | DCSC

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Finite Elements Method,
Block-Sparse NN Weights [GRK 2017]

Too many tensor kernels for a fixed-function library

$$a = Bc$$

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$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB Data Analytics

Blocked COO CSC

DIA Blocked DIA DCSC

Too many tensor kernels for a fixed-function library

$$a = Bc$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB Mesh Simulations on GPUs [BG 2009]

Blocked COO CSC

DIA Blocked DIA DCSC

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

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$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC Convolutions, Image Processing

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

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$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC Eulerian Simulations

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

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$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Sparse vector Hash Maps

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$

$$a = B^T c + d \quad A = B + C + D \quad A = BC$$

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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Sparse vector Hash Maps

Coordinates

CSF Dense Tensors

Blocked Tensors

Too many tensor kernels for a fixed-function library

$$a = Bc$$

$$a = Bc + a$$

$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$

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$\times$

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Sparse vector Hash Maps

Coordinates

CSF Dense Tensors

Blocked Tensors

Linked Lists Database

Compression Schemes

Cloud Storage

Too many tensor kernels for a fixed-function library

$$a = Bc$$
$$a = Bc + a$$
$$a = Bc + b \quad A = B + C \quad a = \alpha Bc + \beta a$$
$$a = B^T c \quad A = \alpha B \quad a = B(c + d)$$
$$a = B^T c + d \quad A = B + C + D \quad A = BC$$
$$A = B \odot C \quad a = b \odot c \quad A = 0 \quad A = B \odot (CD)$$
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$$a = \sum_{ijklmnop} M_{ij} P_{jk} M_{kl} P_{lm} \overline{M_{nm}} P_{no} \overline{M_{po}}\, \overline{P_{ip}}$$

×

Dense Matrix

CSR DCSR BCSR

COO ELLPACK CSB

Blocked COO CSC

DIA Blocked DIA DCSC

Sparse vector Hash Maps

Coordinates

CSF Dense Tensors

Blocked Tensors

Linked Lists Database

Compression Schemes

Cloud Storage

×

CPU

GPUs TPUs

FPGA

Sparse Tensor Hardware

Cloud Computers

Supercomputers

Logistics

Sessions

Sparse Algorithms and Data Structures I

Sparse Algorithms and Data Structures II

Sparse Polyhedral Optimization

Tensor Compilers I

Tensor Compilers II

Tensors and Graphs

Hardware for Sparse Tensor Algebra

Applications in Science and Engineering

Session Format

10 min	Speaker
10 min	Speaker
10 min	Speaker
20 min	Discussion
10/15 min	Break

Dinner and Posters

Dinner (Sri Lankan)

Posters

Eddie Davis

Suzy Mueller

Mahdi Mohammadi

Ryan Senanayake

Srinivas Eswar

Stephen Chou

Peter Ahrens

Patricio Noyola

Matt Fishman

Yunming Zhang