Domain Specific Optimizations

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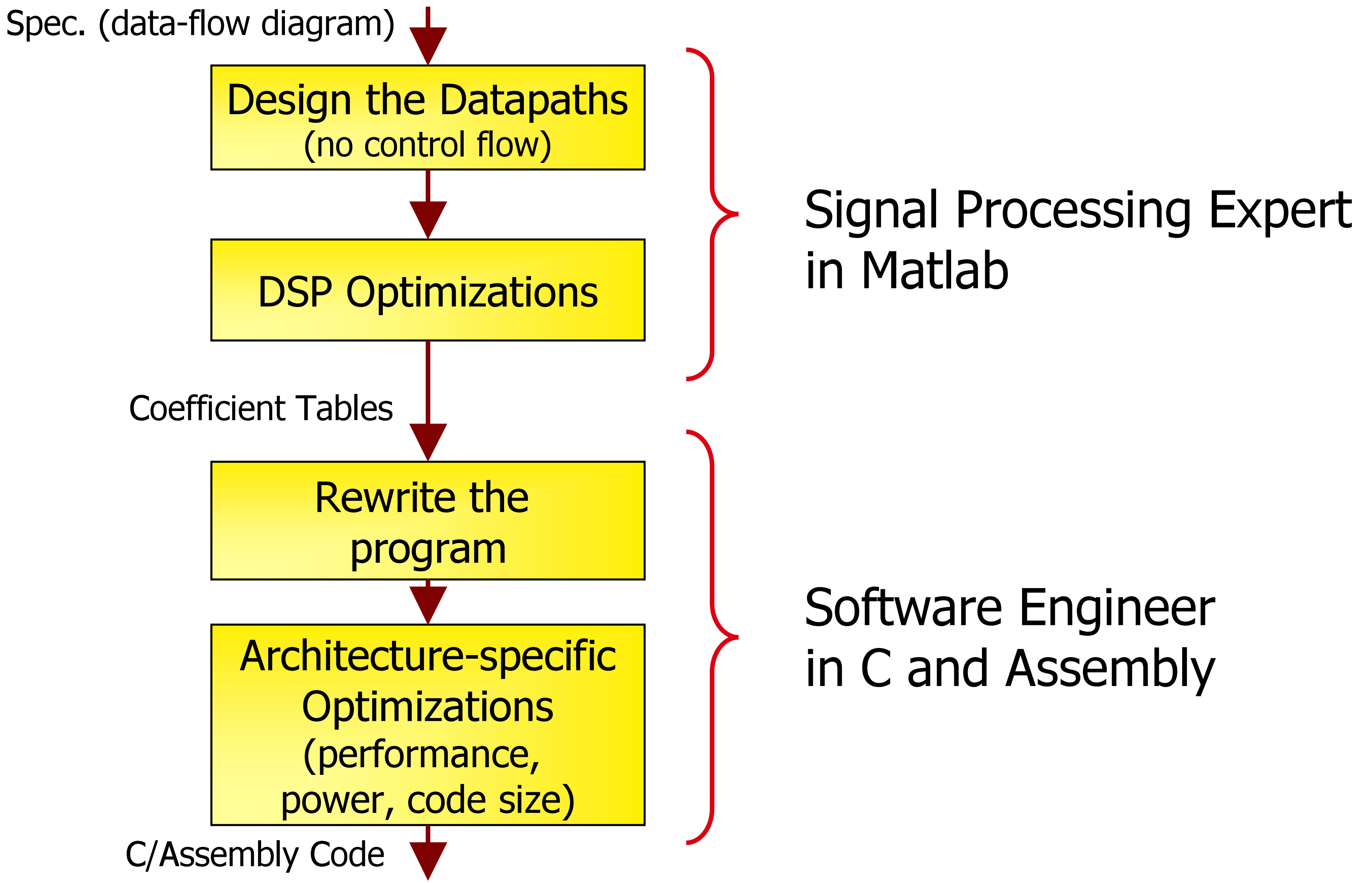
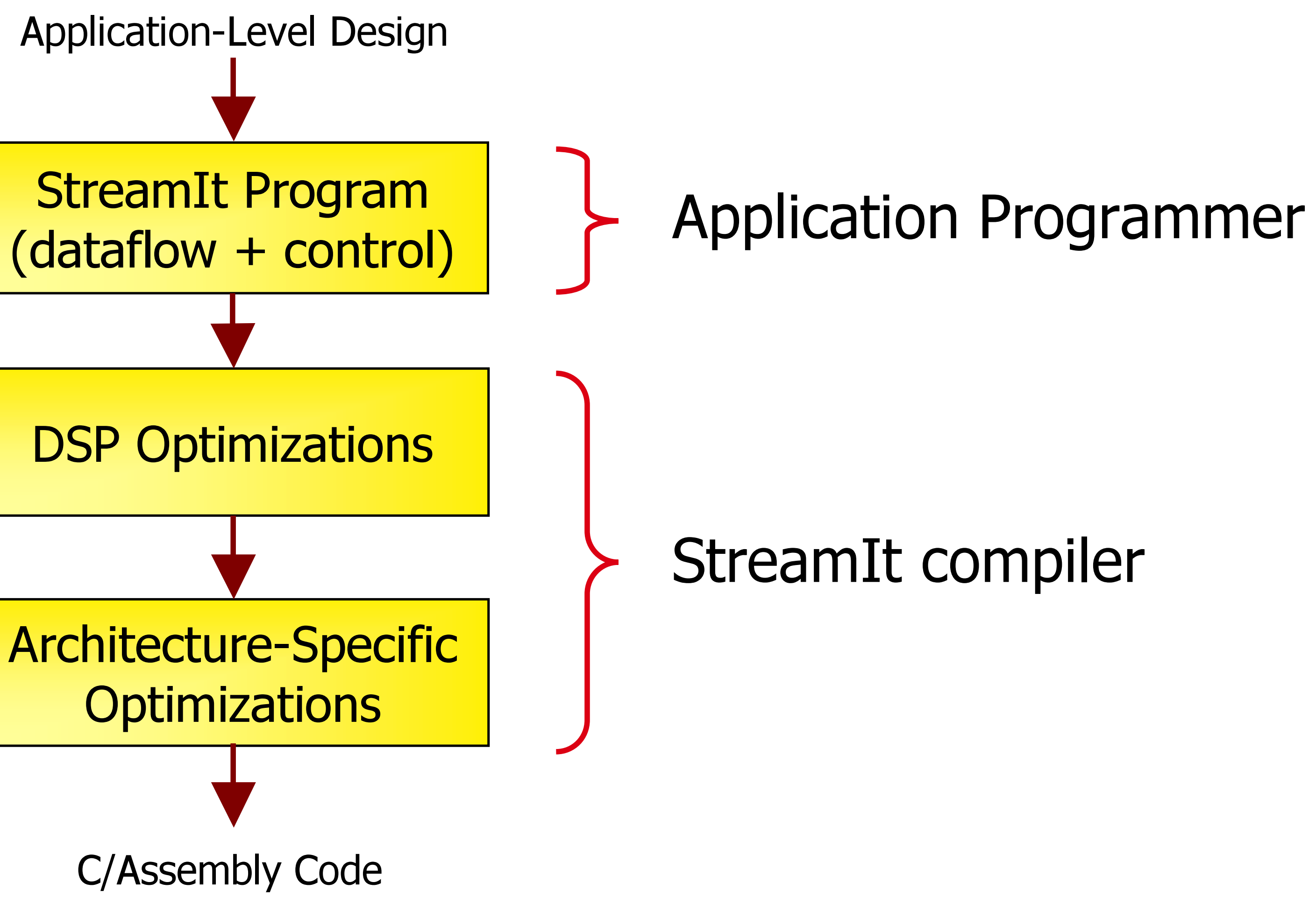
Schedule

1:30-1:40	Overview (Saman)
1:40-2:20	Stream Architectures (Saman)
2:20-3:00	Stream Languages (Bill)
3:00-3:30	Break
3:30-3:55	Stream Compilers (Saman)
3:55-4:20	Domain-specific Optimizations (Saman)
4:20-5:00	Scheduling Algorithms (Bill)

Conventional DSP Design Flow



Design Flow with StreamIt



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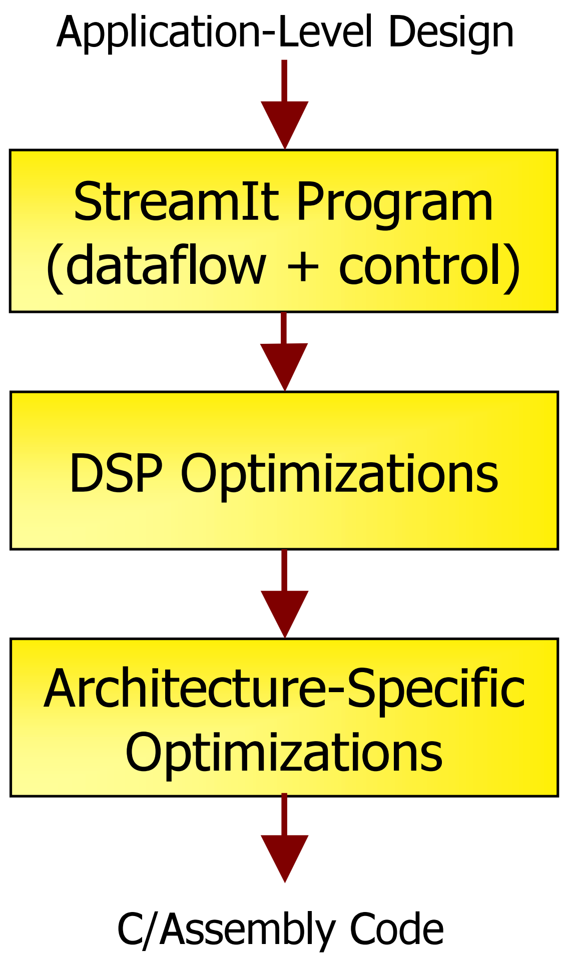


- Benefits of programming in a single, high-level abstraction
  - Modular
  - Composable
  - Portable
  - Malleable
- The Challenge: Maintaining Performance
  - Replacing Expert DSP Engineer
  - Replacing Expert Assembly Hacker

Our Focus: Linear Filters

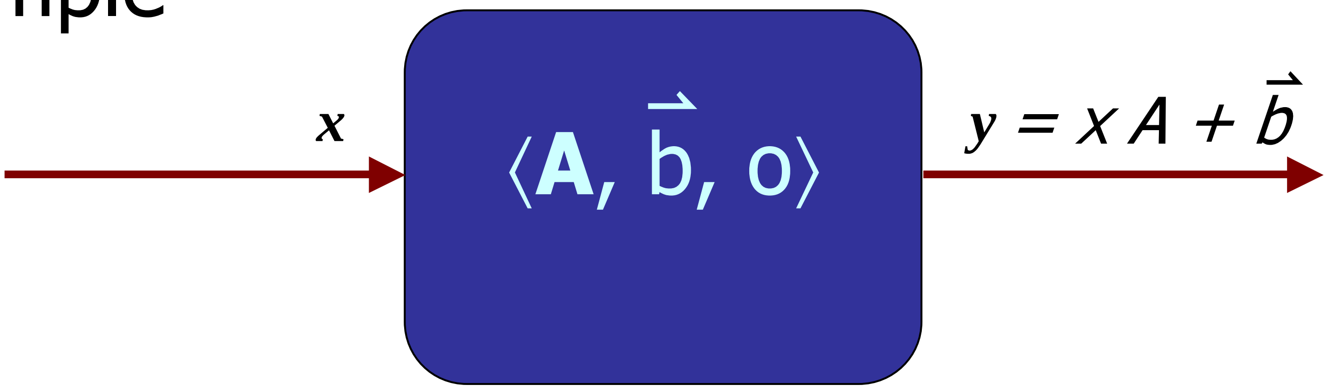
- Most common target of DSP optimizations
  - FIR filters
  - Compressors
  - Expanders
  - DFT/DCT

Output is weighted sum of inputs

- Example optimizations:
  - Combining Adjacent Nodes
  - Translating to Frequency Domain

Representing Linear Filters

- A linear filter is a tuple $\langle \mathbf{A}, \vec{b}, o \rangle$
  - **A**: matrix of coefficients
  - $\vec{b}$: vector of constants
  - o: number of items popped
- Example



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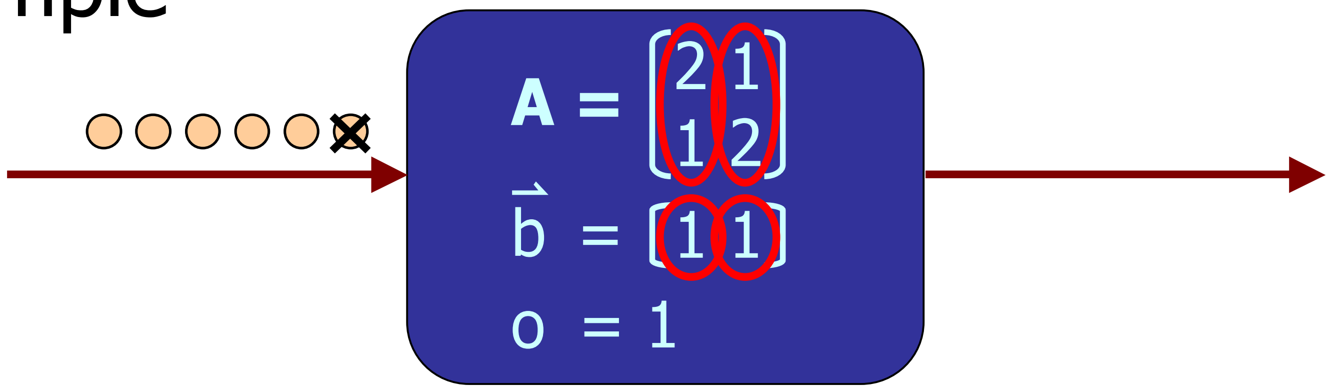
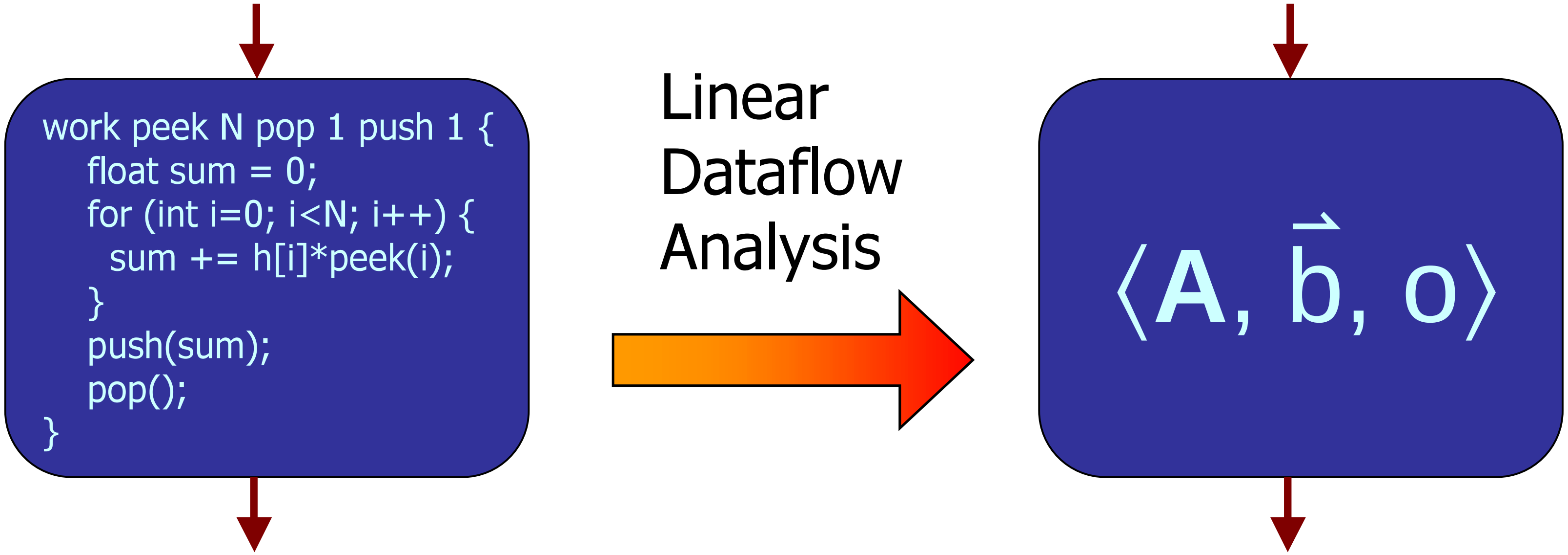


Extracting Linear Representation



- Resembles constant propagation
- Maintains linear form $\langle \vec{v}, \vec{b} \rangle$ for each variable
  - Peek expression: generate fresh $\vec{v}$
  - Push expression: copy $\vec{v}$ into **A**
  - Pop expression: increment o

Optimizations using Linear Analysis

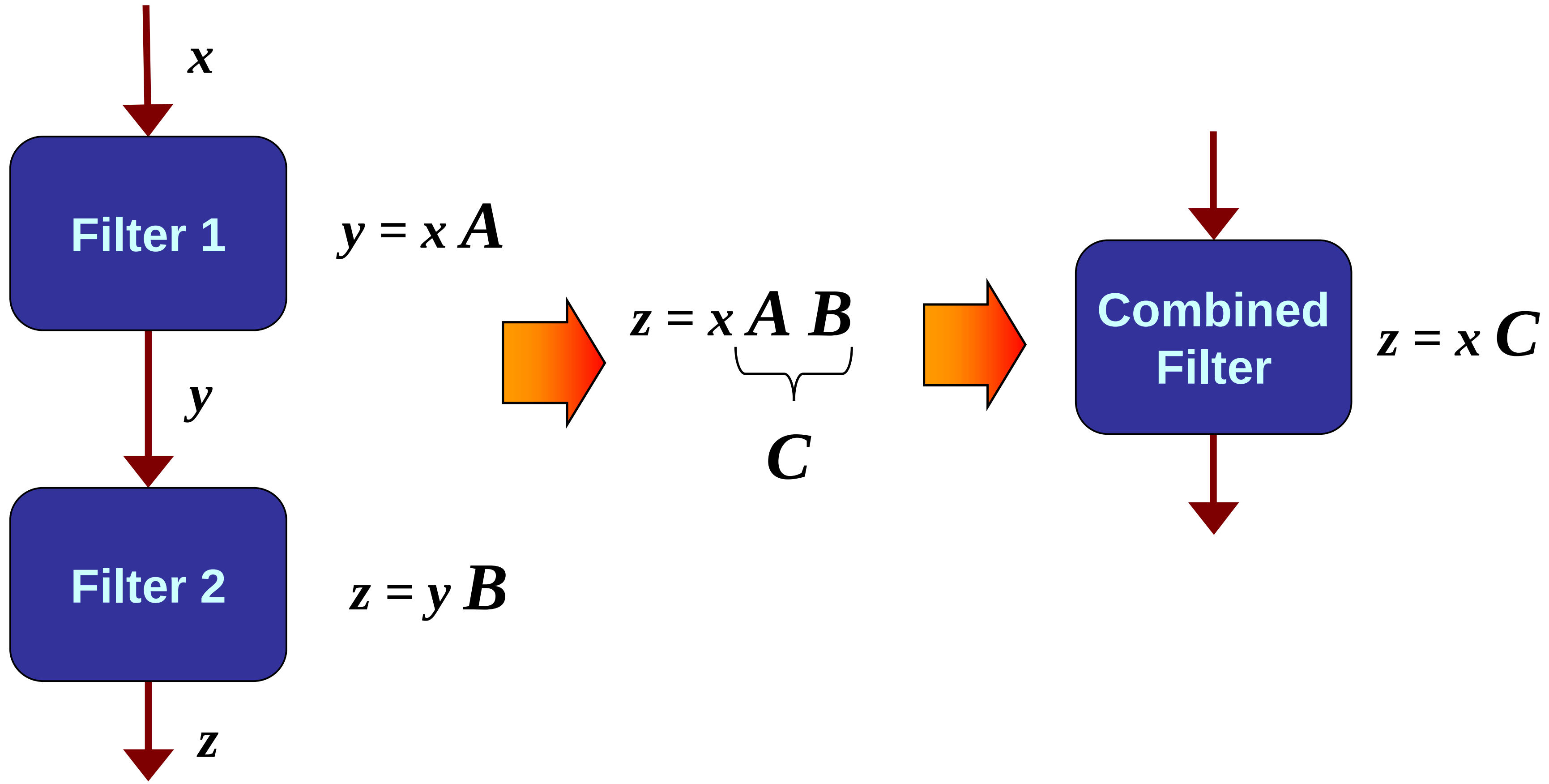
1) Combining adjacent linear structures

2) Shifting from time to the frequency domain

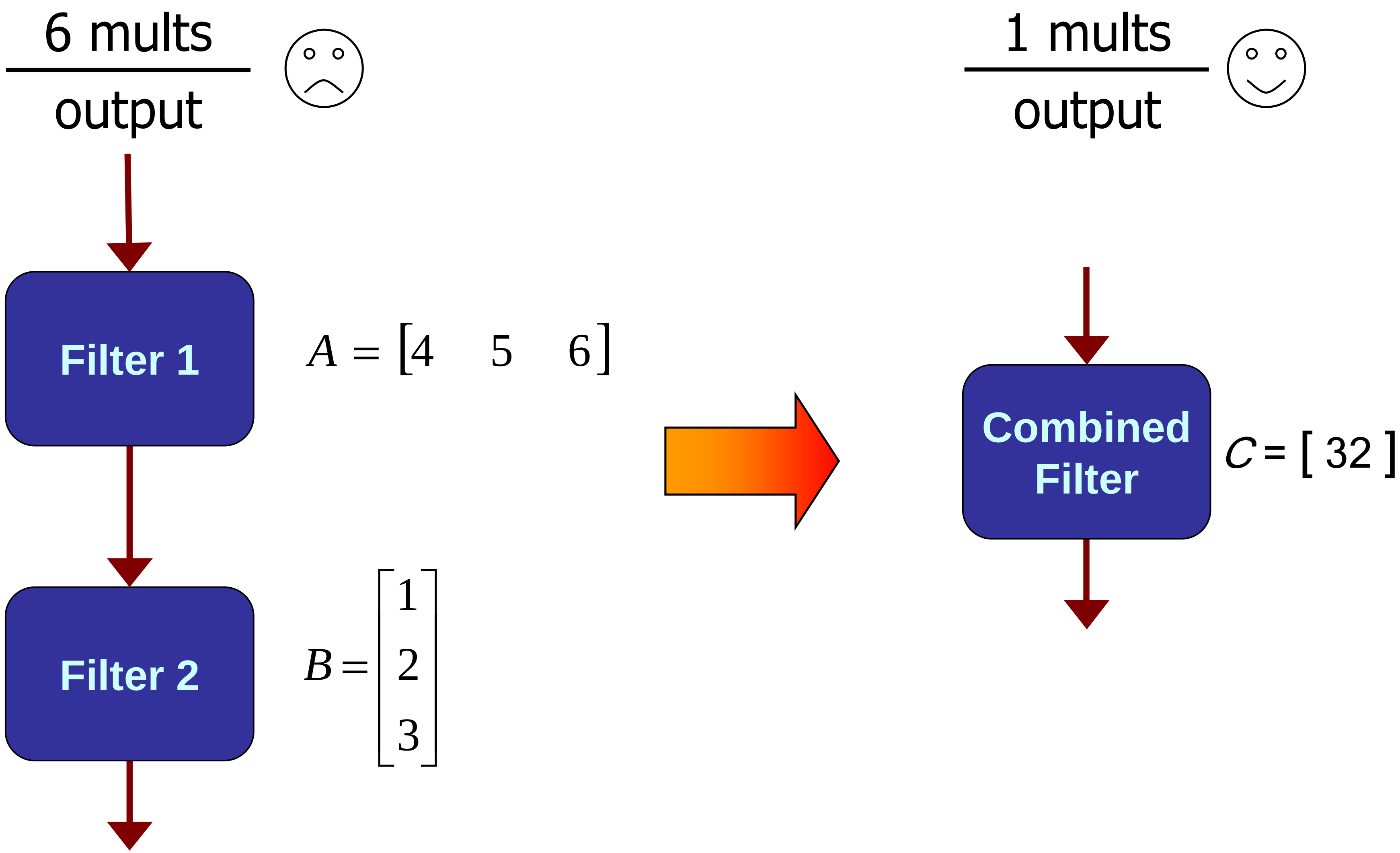
3) Selection of 'optimal' set of transformations

1) Combining Linear Filters

- Pipelines and splitjoins can be collapsed
- Example: pipeline

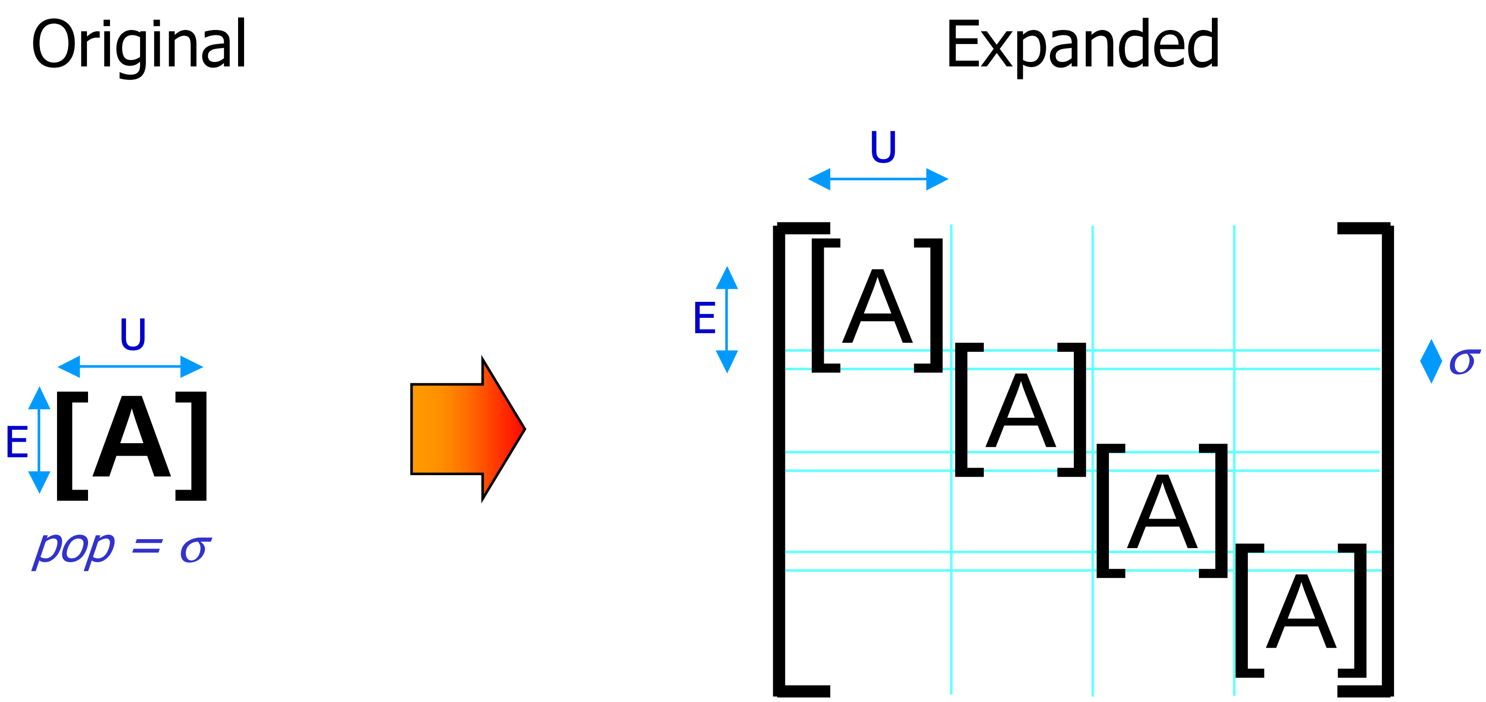


Combination Example



AB for any A and B??

- Linear Expansion



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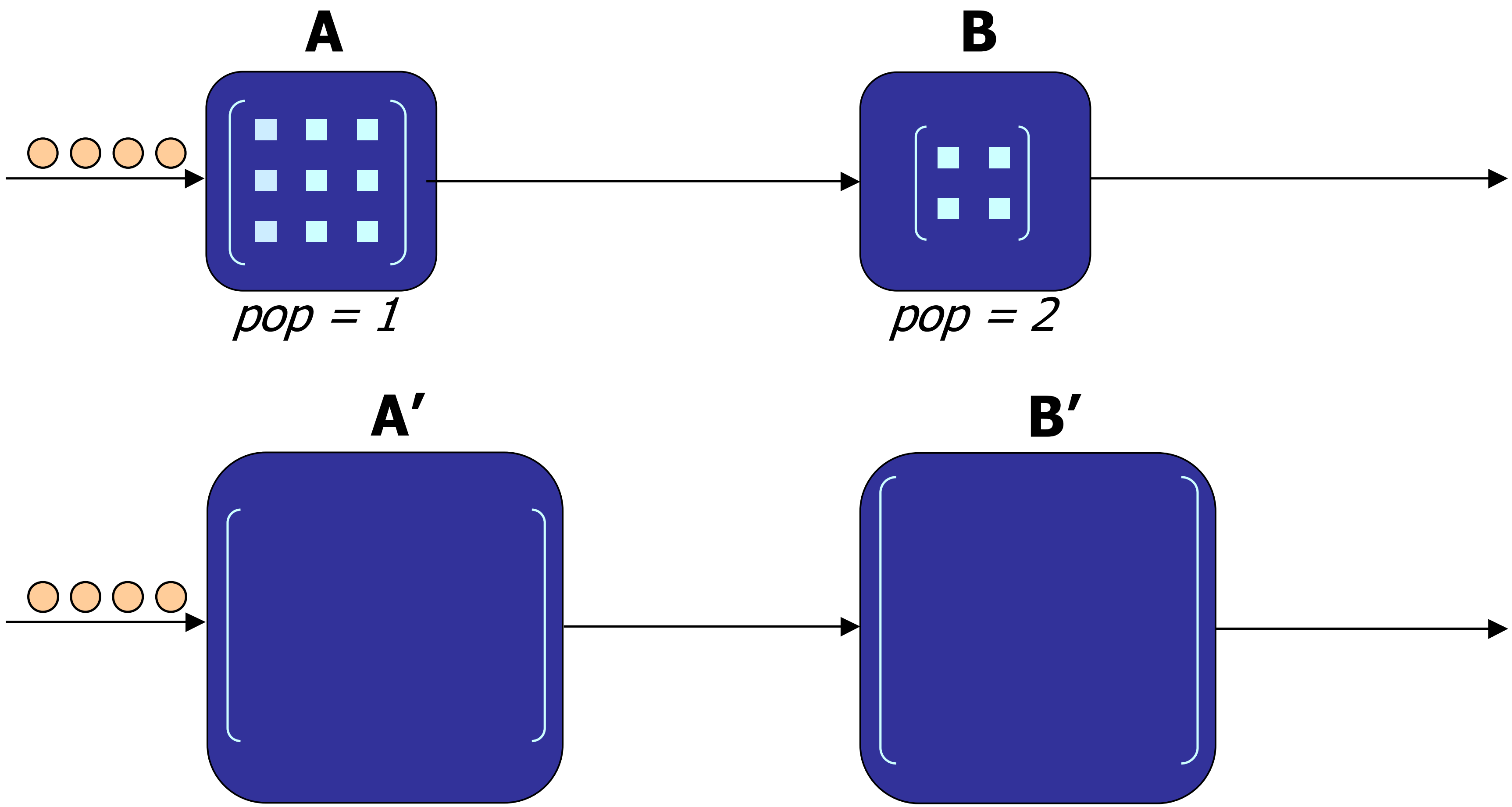
Need to "expand" matrices to a steady-state cycle:



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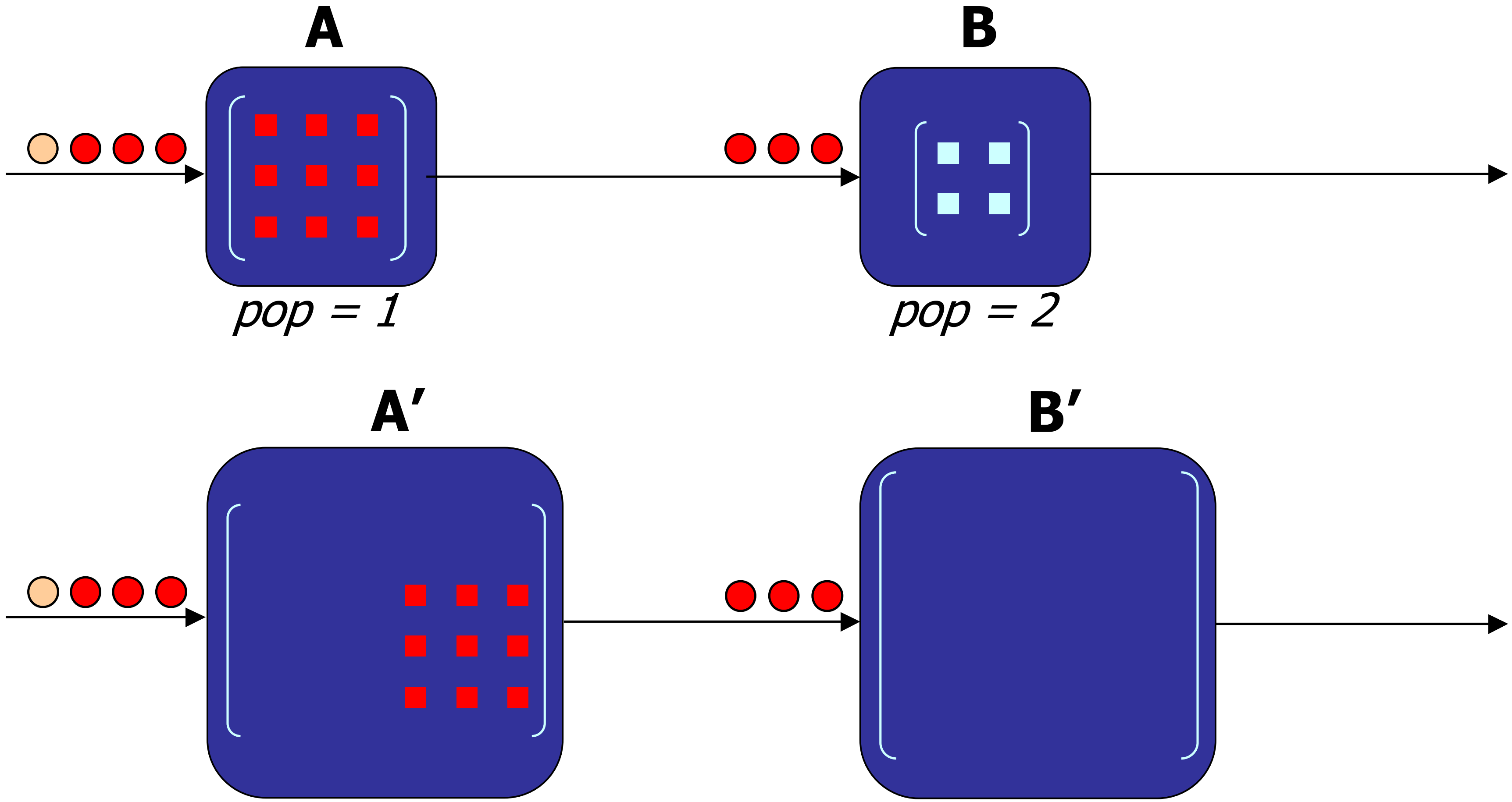

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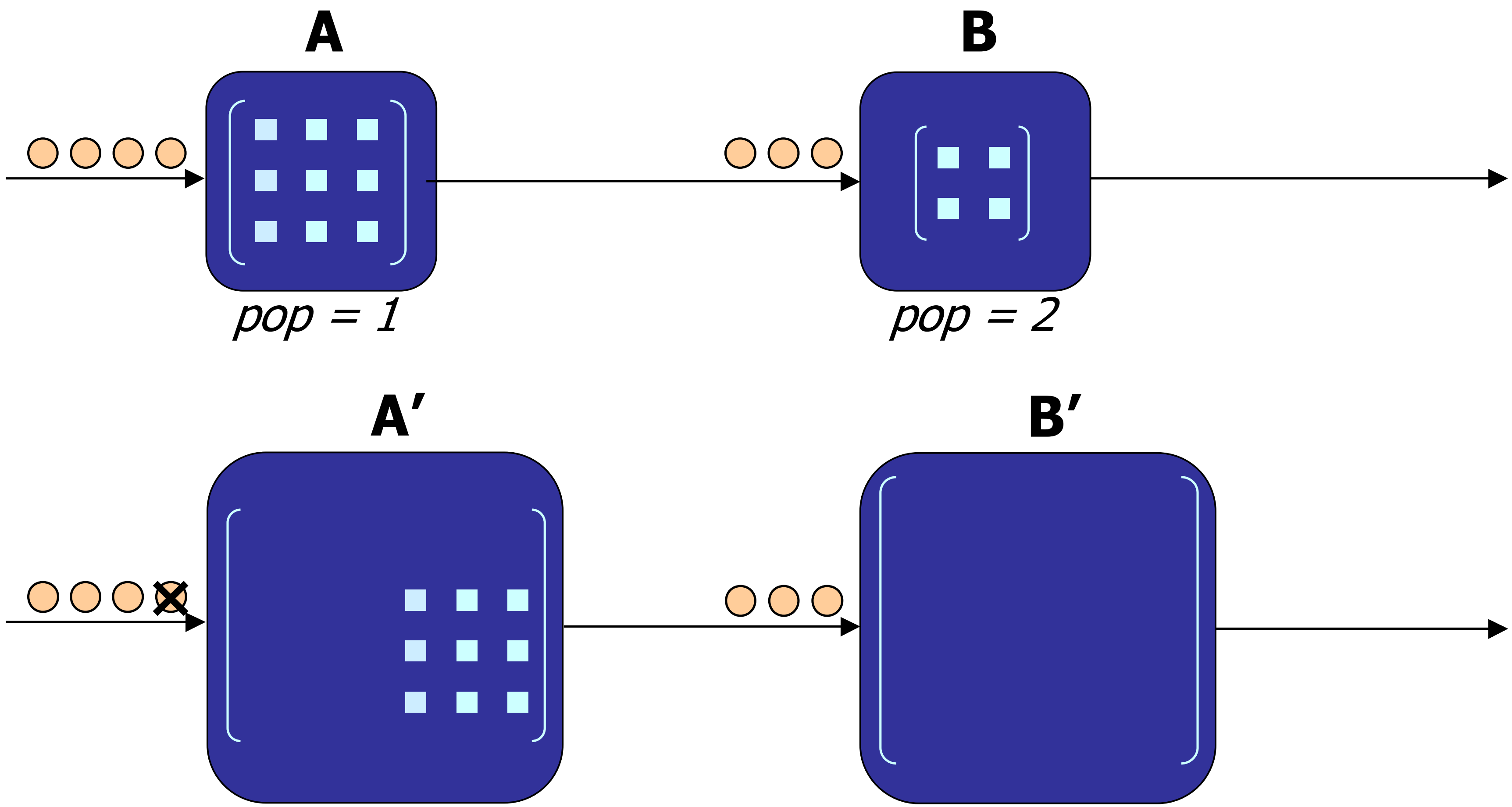

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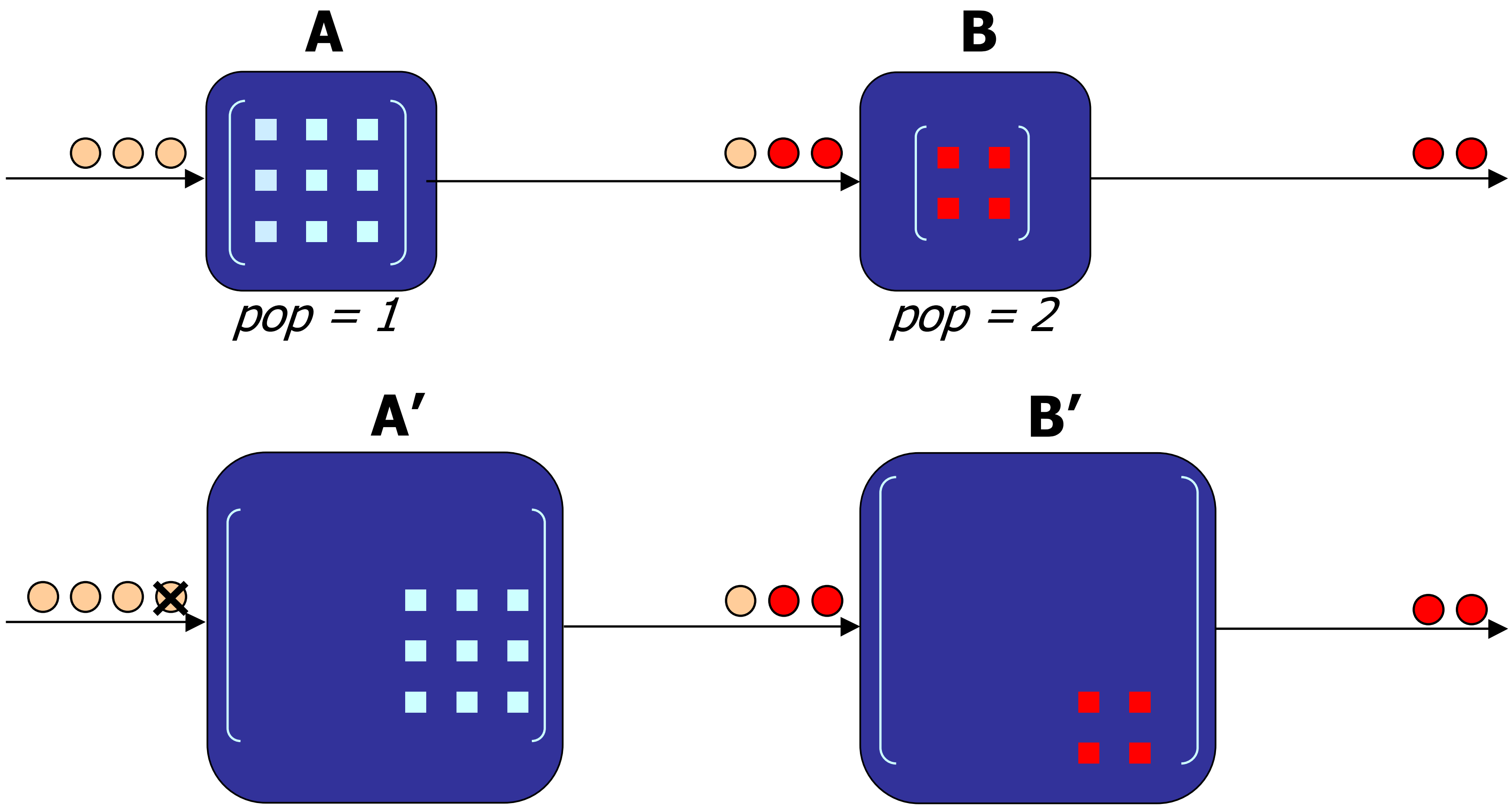

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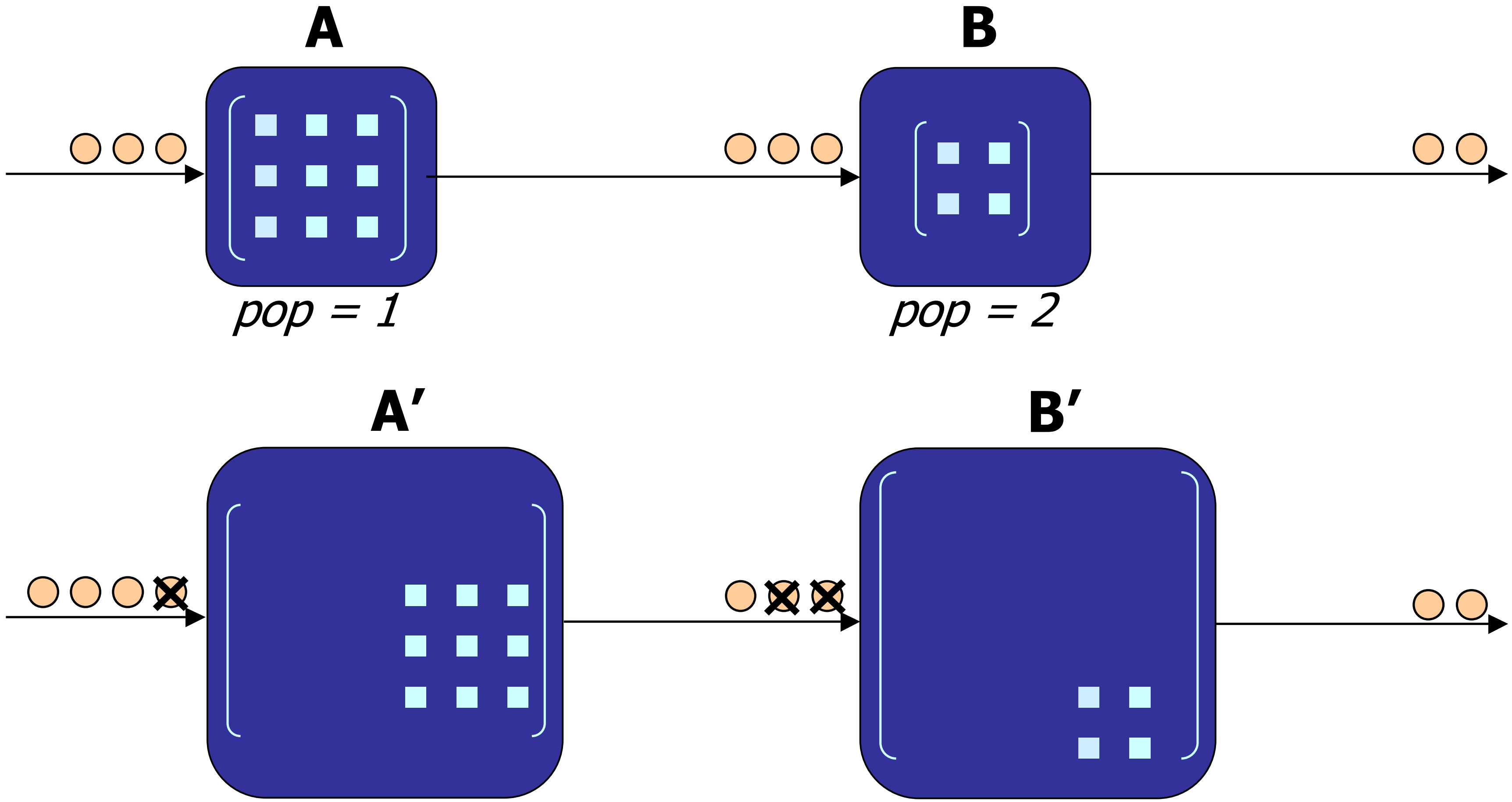

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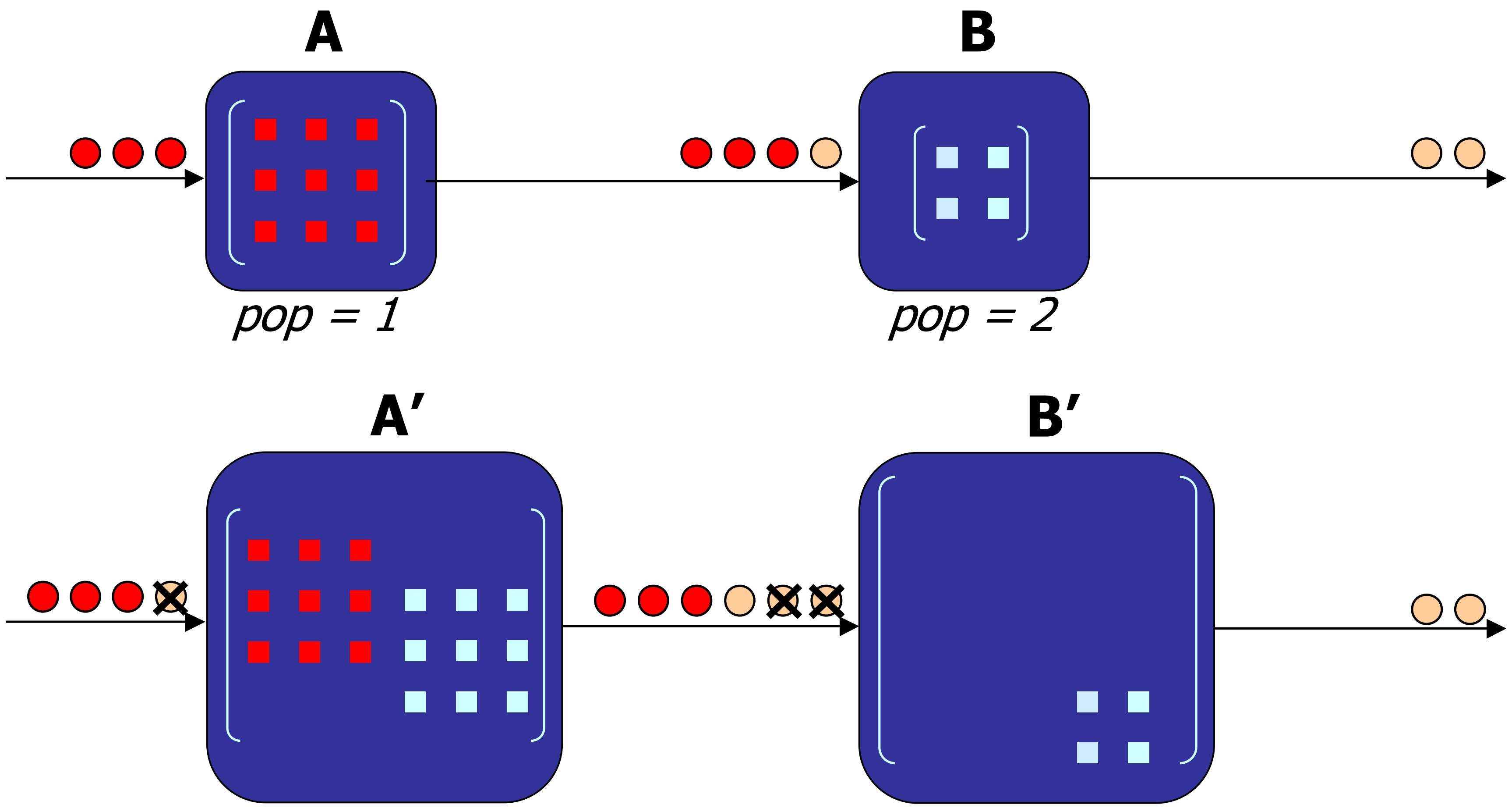

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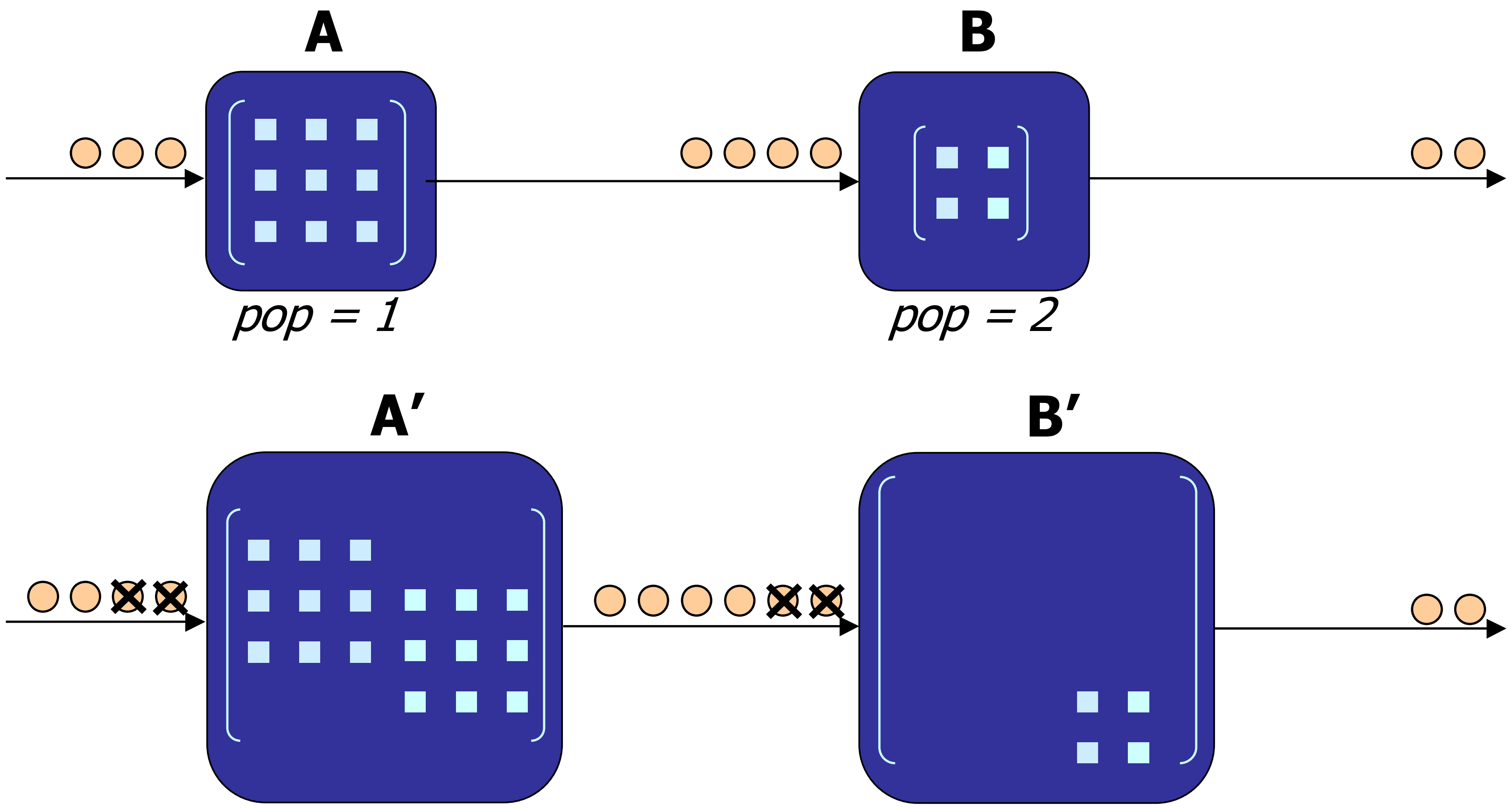

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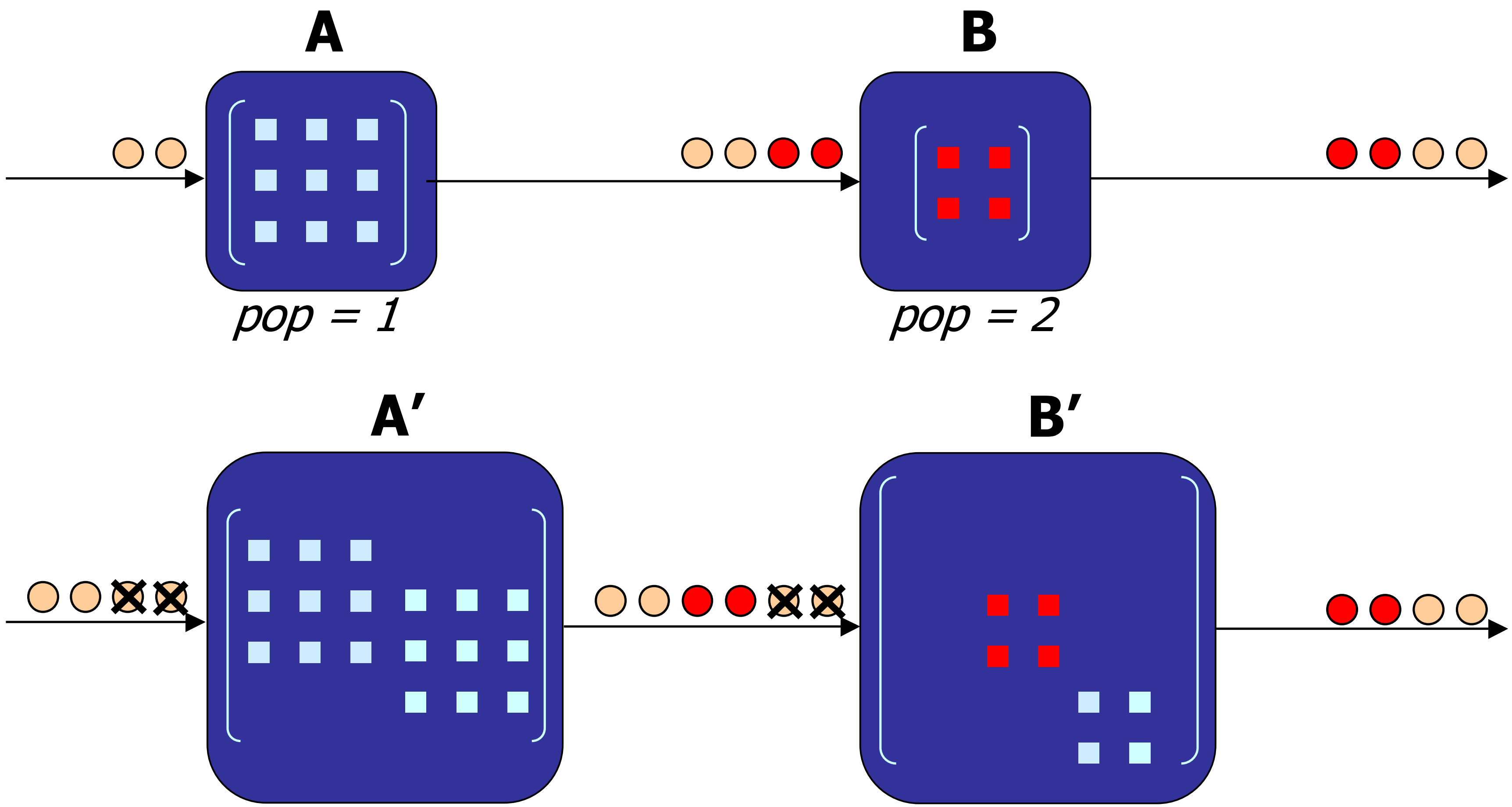

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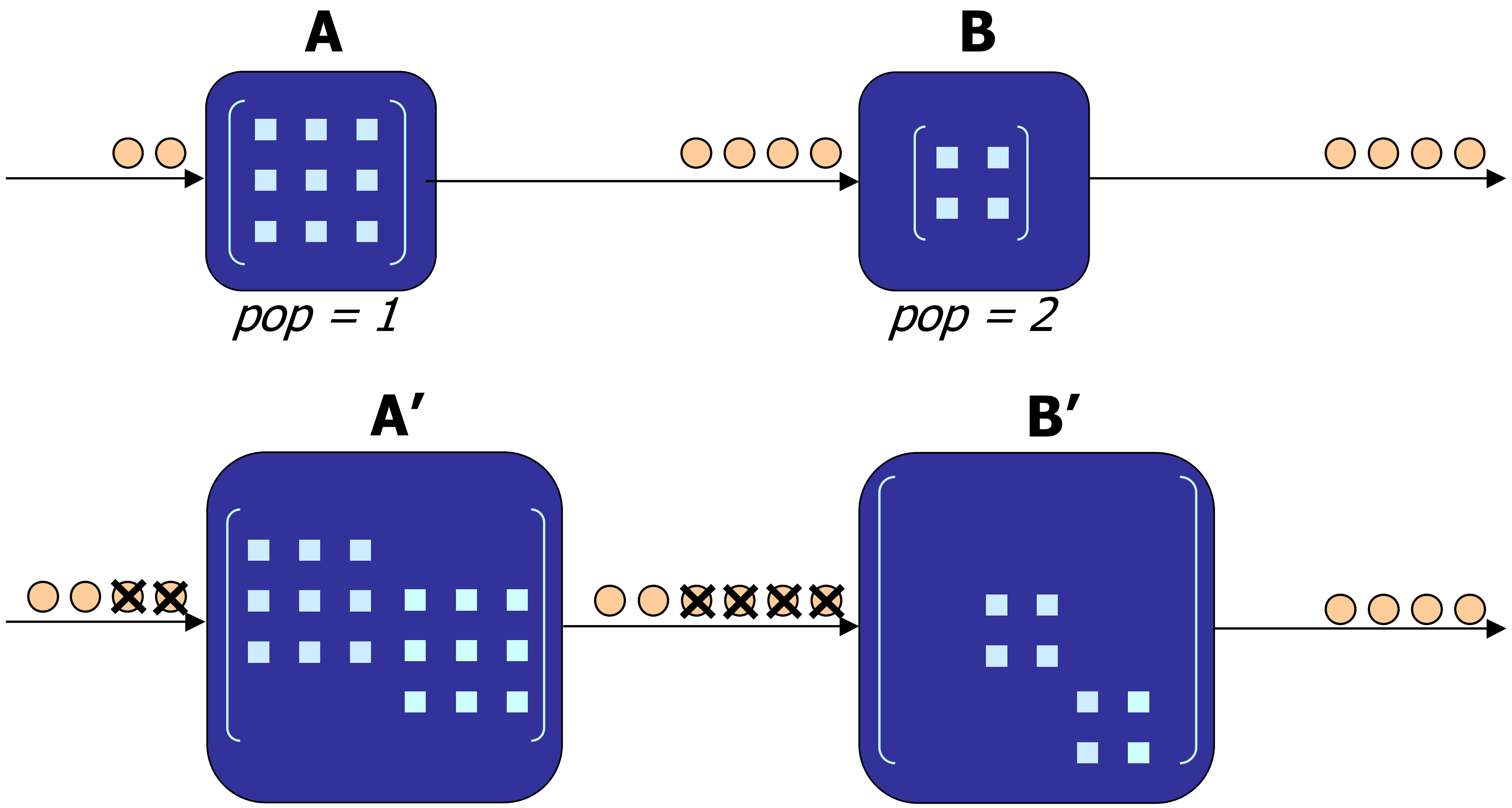

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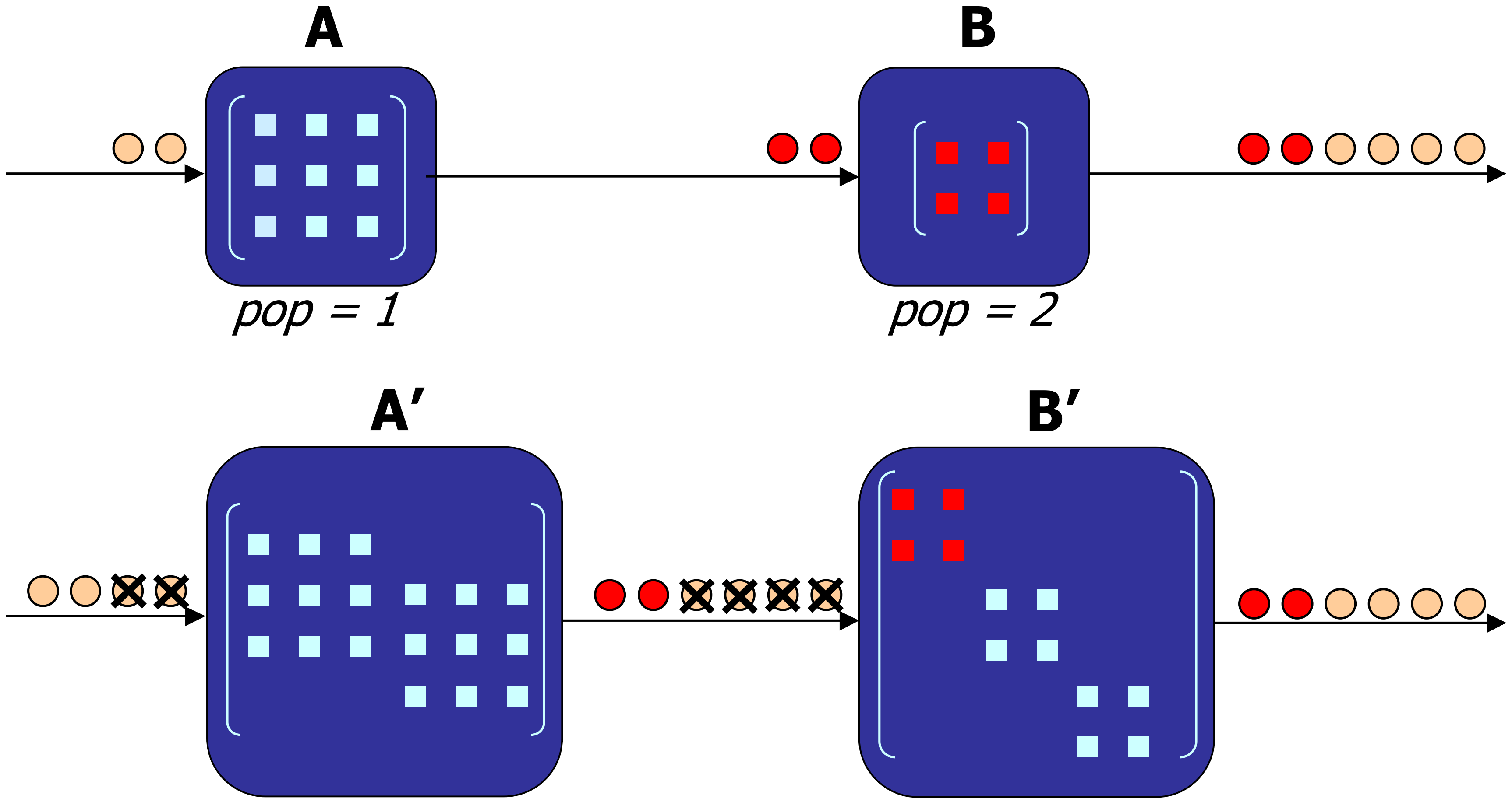

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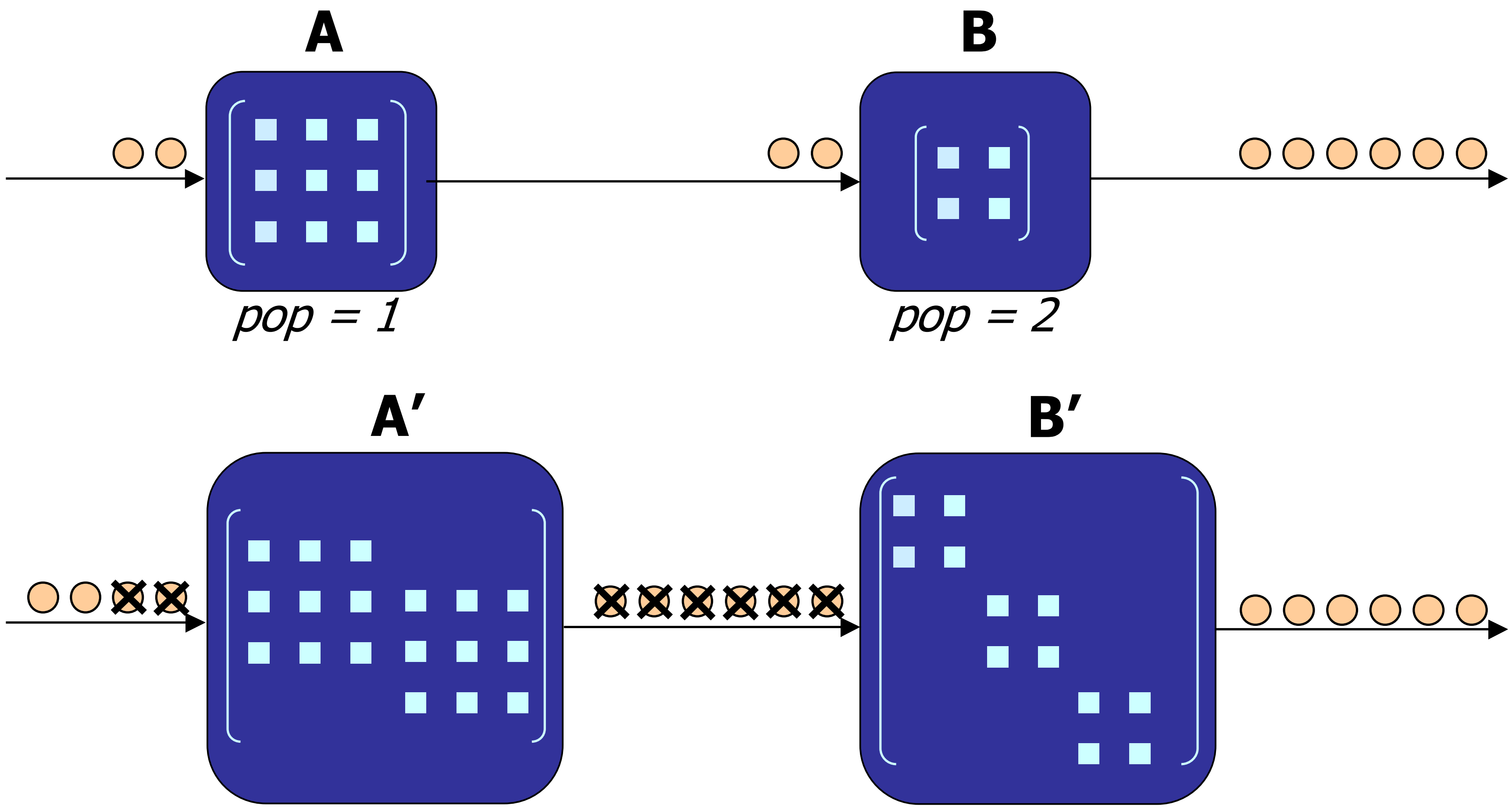

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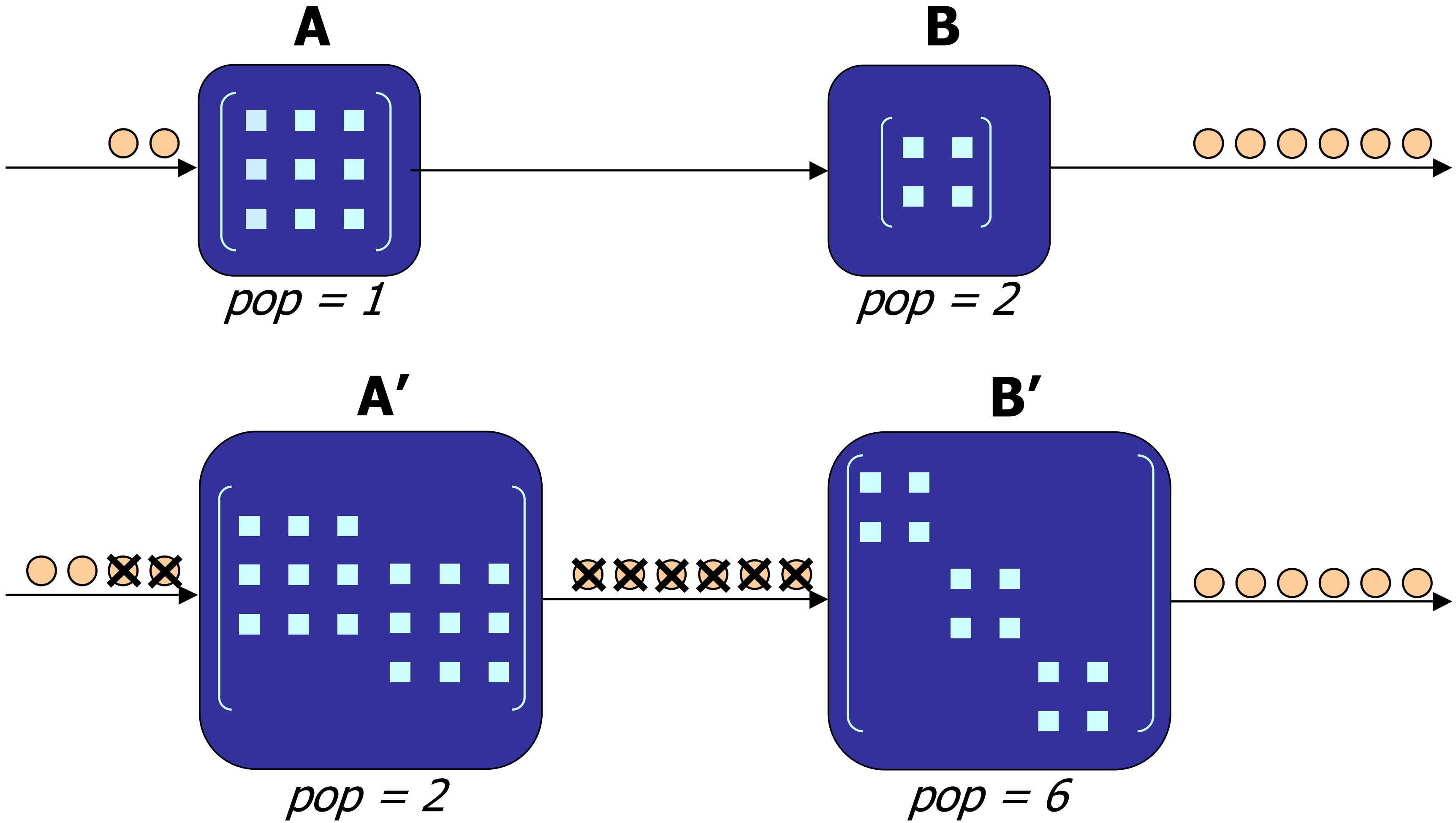

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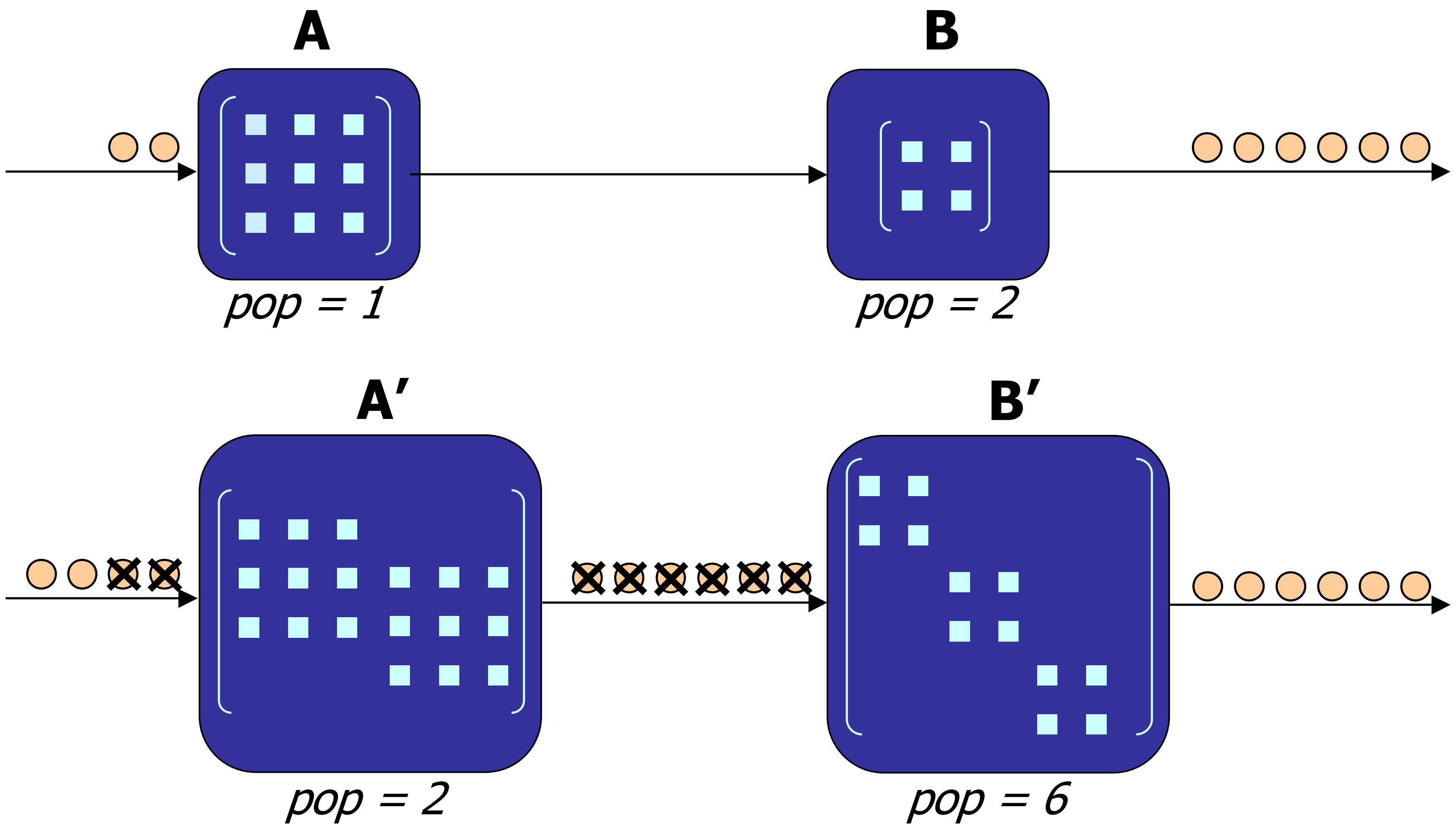
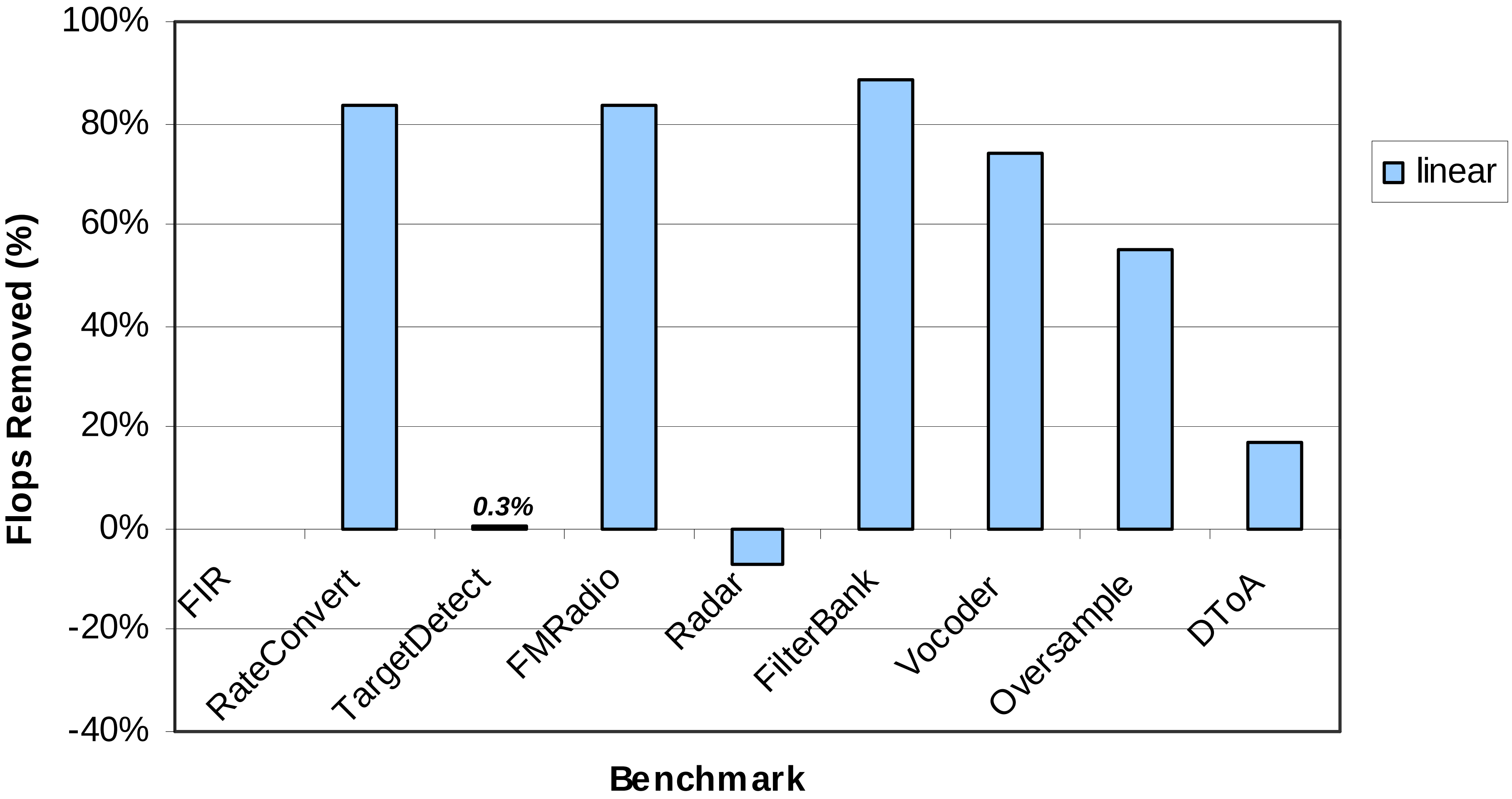
Expanded dimensions match

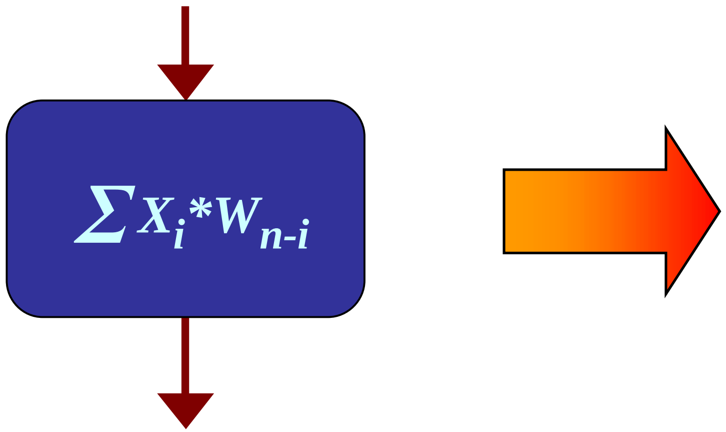


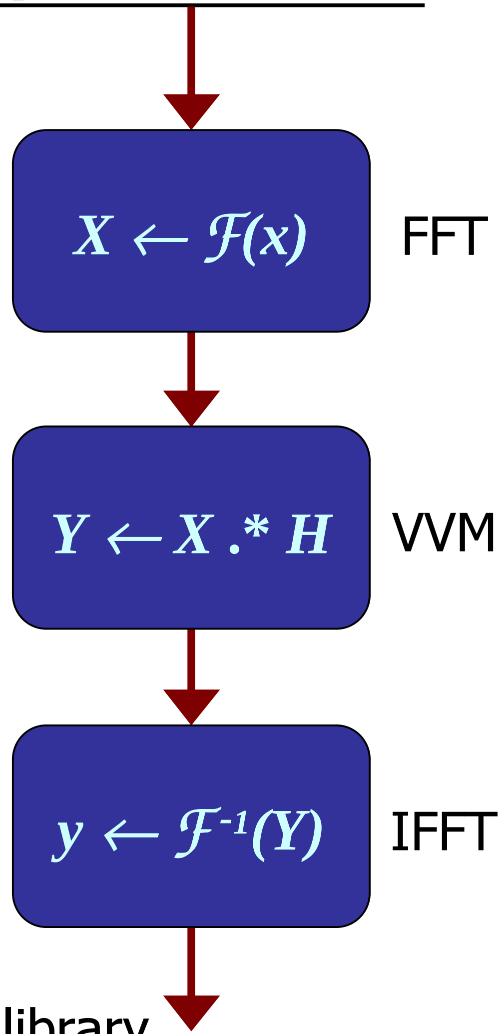
Floating-Point Operations Reduction



2) From Time to Frequency Domain

- Convolutions can be done cheaply in the Frequency Domain





- Painful to do by hand
  - Blocking
  - Coefficient calculations
  - Startup
  - Multiple outputs
  - Interfacing with FFT library

Generic Freq. Implementation

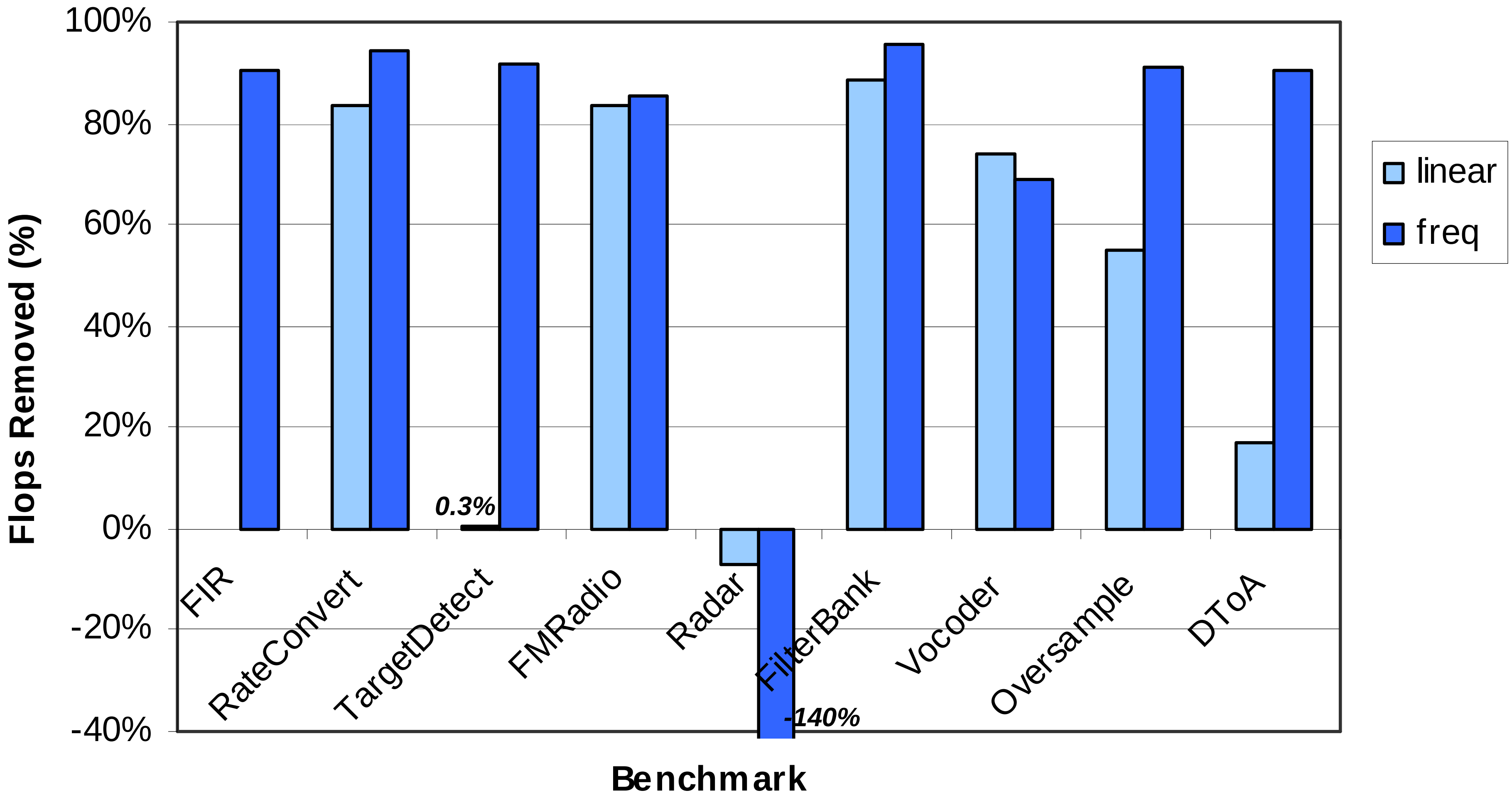
```
float → float pipeline optimizedFreq (A, b⃗, e, o, u) {
  add float → float filter {
    N ← 2^⌈lg(2e)⌉
    m ← N − 2e + 1
    partials⃗ ← new array[0 ... e − 2, 0 ... u − 1]
    r ← m + e − 1

    init {
      for j = 0 to u − 1
        H⃗[*, j] ← FFT(N, A[*, u − 1 − j])
    }

    prework peek r pop r push u * m {
      x⃗ ← pop(0 ... m + e − 2)
      X⃗ ← FFT(N, x⃗)
      for j = 0 to u − 1 {
        Y⃗[*, j] ← X⃗ .* H⃗[*, j]
        y⃗[*, j] ← IFFT(N, Y⃗[*, j])
        partials⃗[*, j] ← y⃗[m + e − 1 ... m + 2e − 3, j]
      }
      for i = 0 to m − 1
        for j = 0 to u − 1
          push(y⃗[i + e − 1, j] + b⃗[j])
    }

    work peek r pop r push u * r {
      x⃗ ← pop(0 ... m + e − 2)
      X⃗ ← FFT(N, x⃗)
      for j = 0 to u − 1 {
        Y⃗[*, j] ← X⃗ .* H⃗[*, j]
        y⃗[*, j] ← IFFT(N, Y⃗[*, j])
      }
      for i = 0 to e − 1
        for j = 0 to u − 1 {
          push(y⃗[i, j] + partials⃗[i, j])
          partials⃗[i, j] ← y⃗[m + e − 1 + i, j]
        }
      for i = 0 to m − 1
        for j = 0 to u − 1
          push(y⃗[i + e − 1, j] + b⃗[j])
    }
  }
  add Decimator(o, u)
}
```

Floating-Point Operations Reduction



3) Transformation Selection

- When to apply what transformations?
  - Linear filter combination can increase the computation cost
  - Shifting to the Frequency domain is expensive for filters with pop > 1
    - Compute all outputs, then decimate by pop rate
  - Some expensive transformations may later enable other transformations, reducing the overall cost

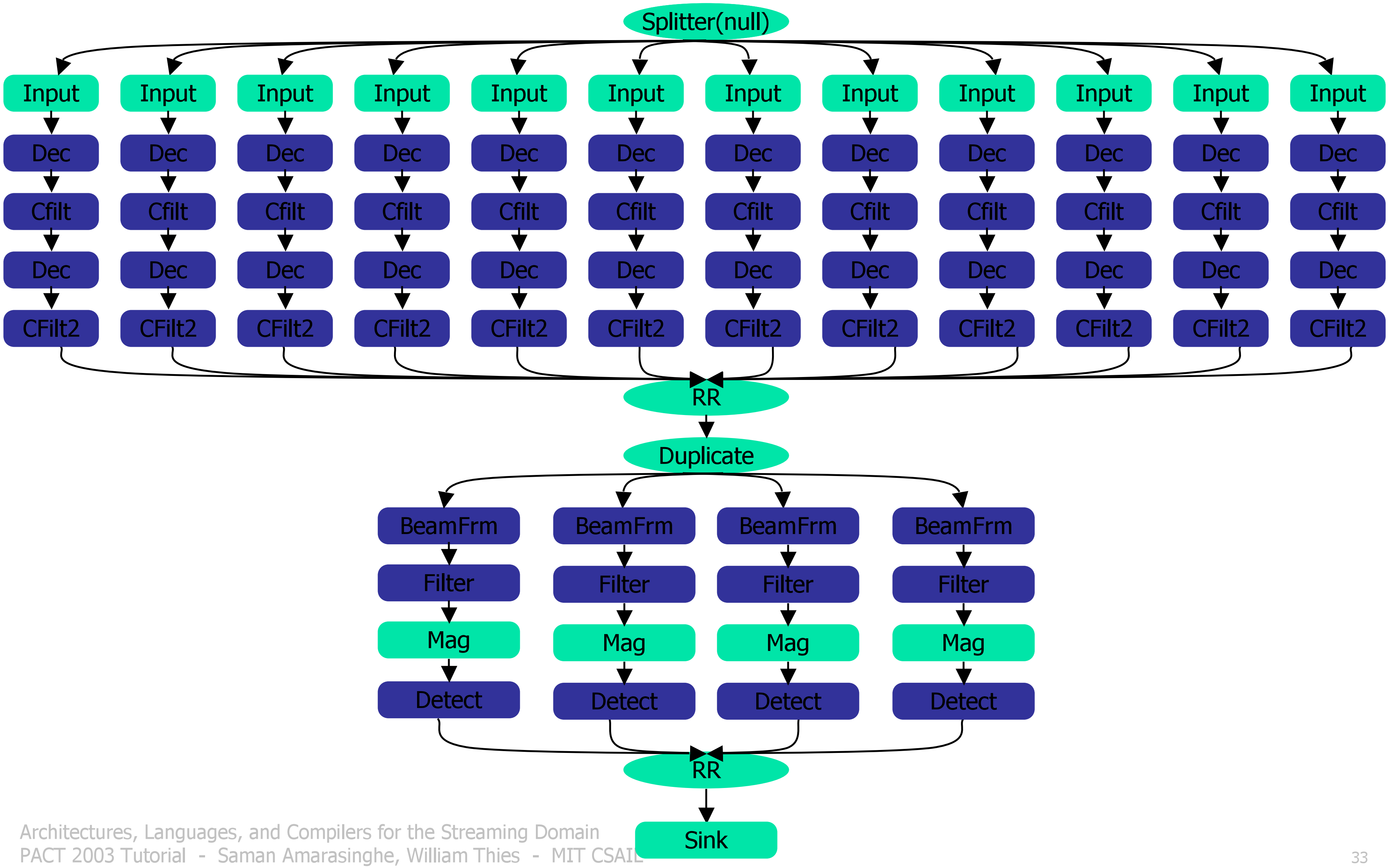
Selection Algorithm

- Estimate minimal cost for each structure:
  - Linear combination
  - Frequency translation
  - No transformation
    - If hierarchical, consider all possible groupings of children

Cost function based on profiler feedback

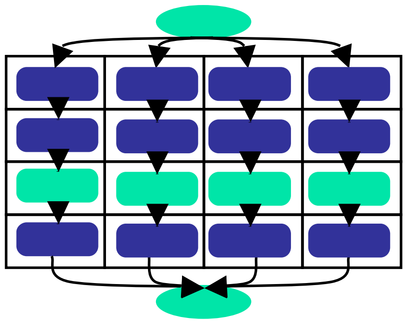
- Overlapping sub-problems allows efficient dynamic programming search

Radar (Transformation Selection)



Radar (Transformation Selection)

First compute cost of individual filters:



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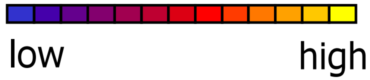

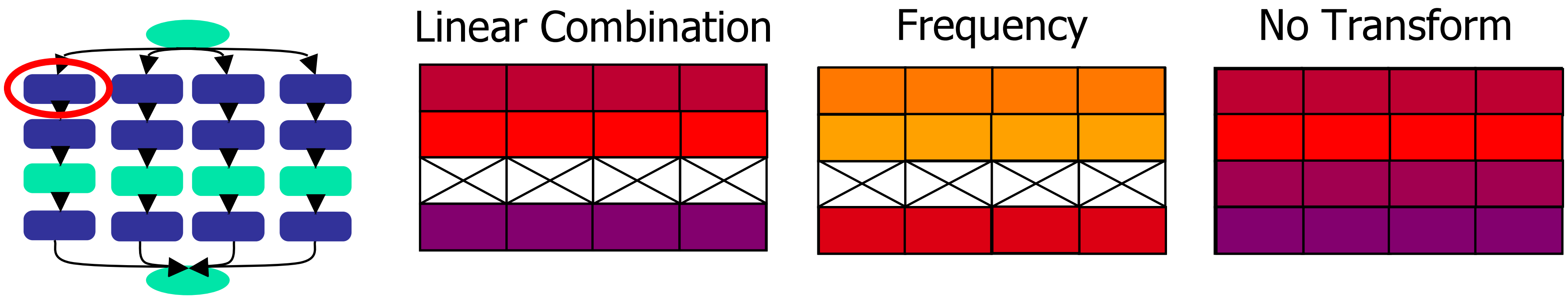

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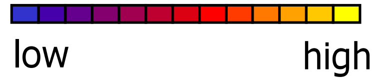
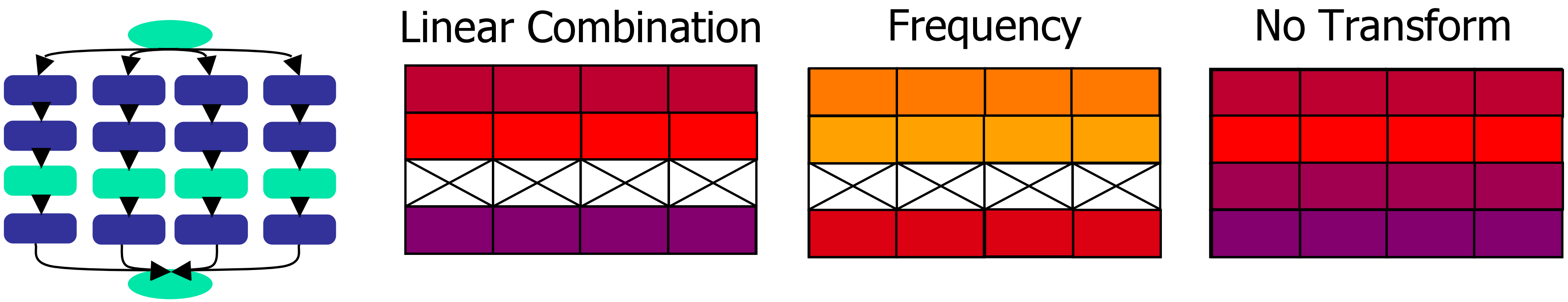
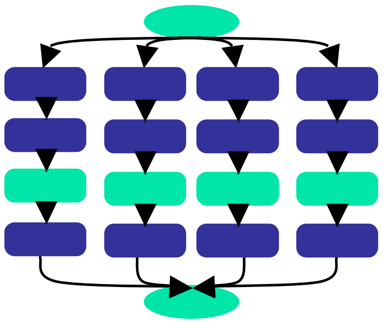




1x1

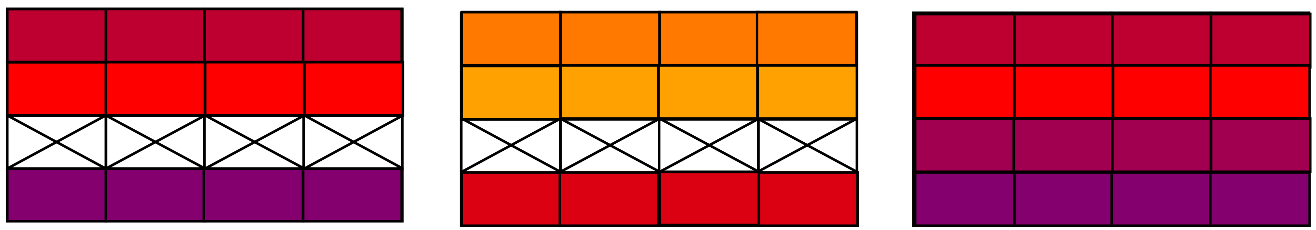
Radar (Transformation Selection)

Then, compute cost of 1x2 nodes:



Linear Combination Frequency No Transform

1x1



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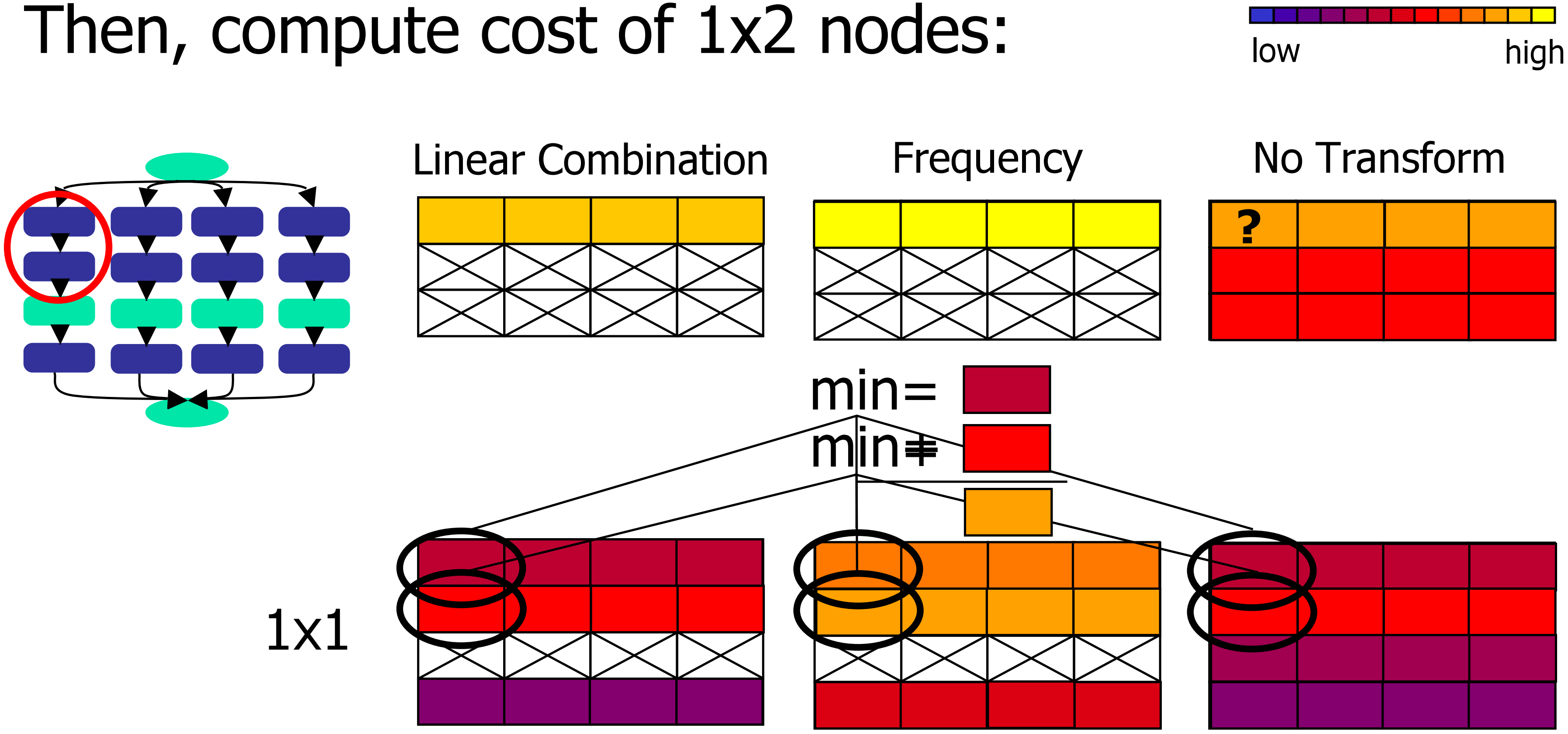

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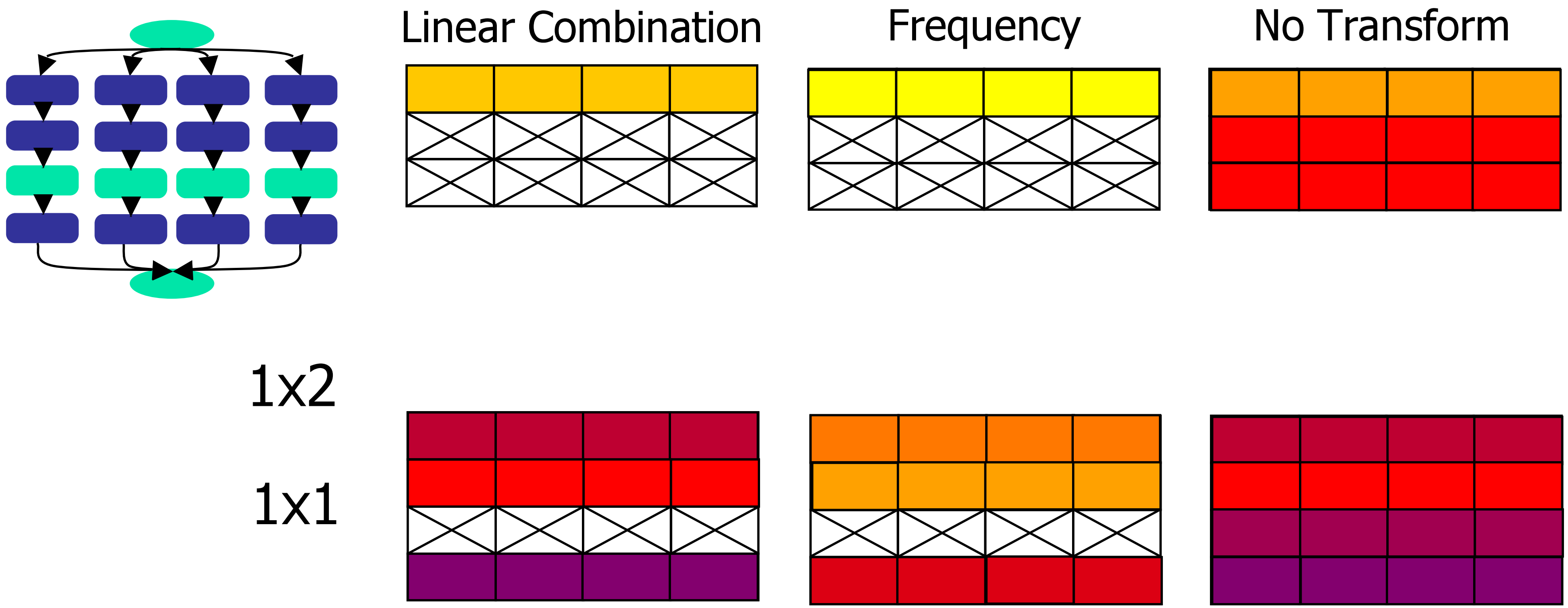
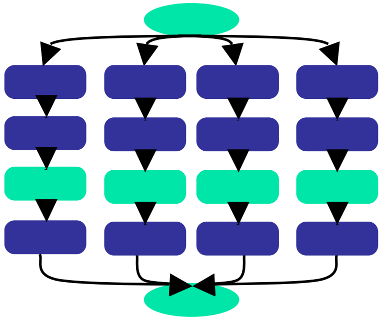


Radar (Transformation Selection)

Continue with	1x3	2x1	3x1	4x1
	1x4	2x2	3x2	4x2
		2x3	3x3	4x3
		2x4	3x4	4x4 ⇨ Overall solution



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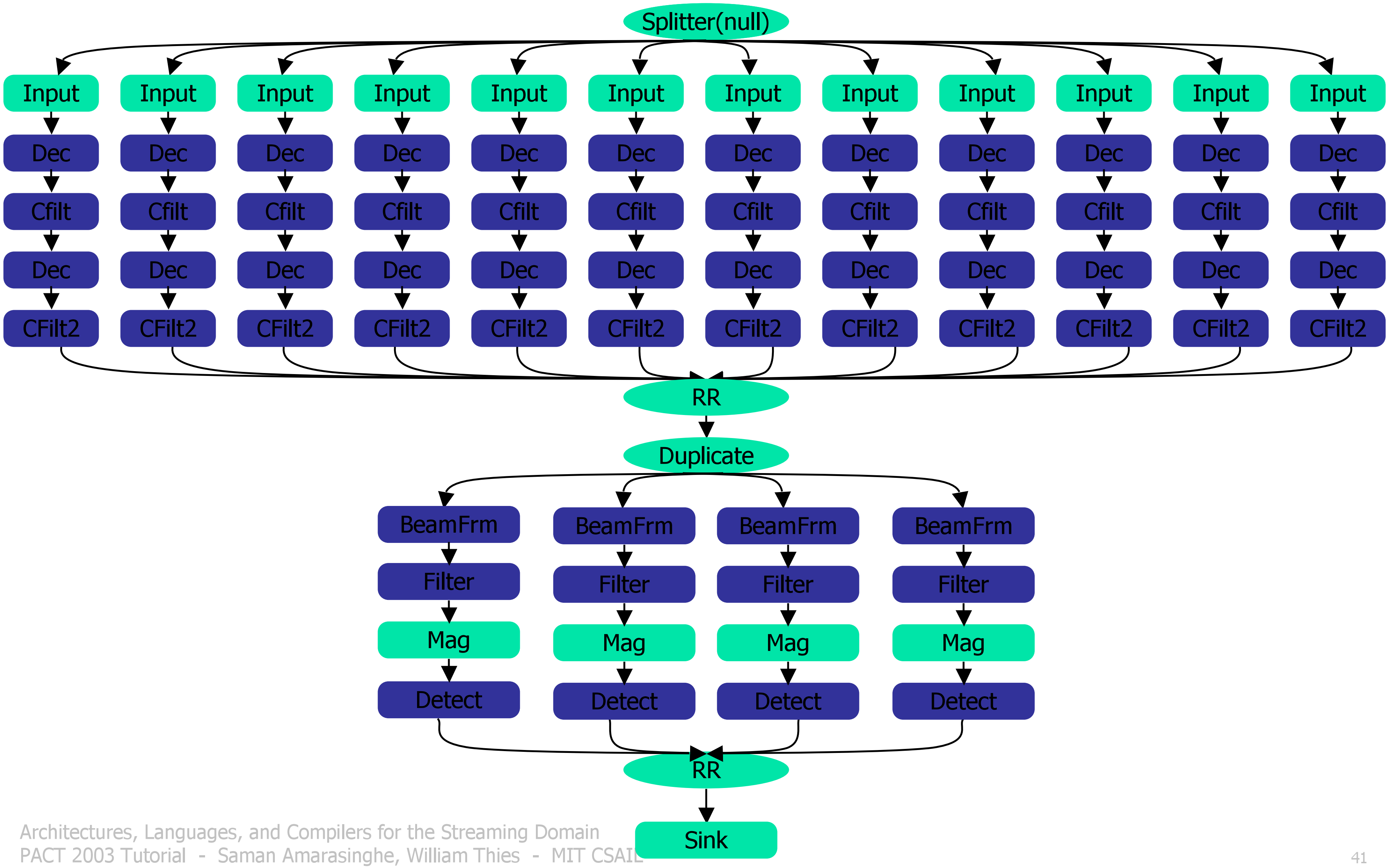

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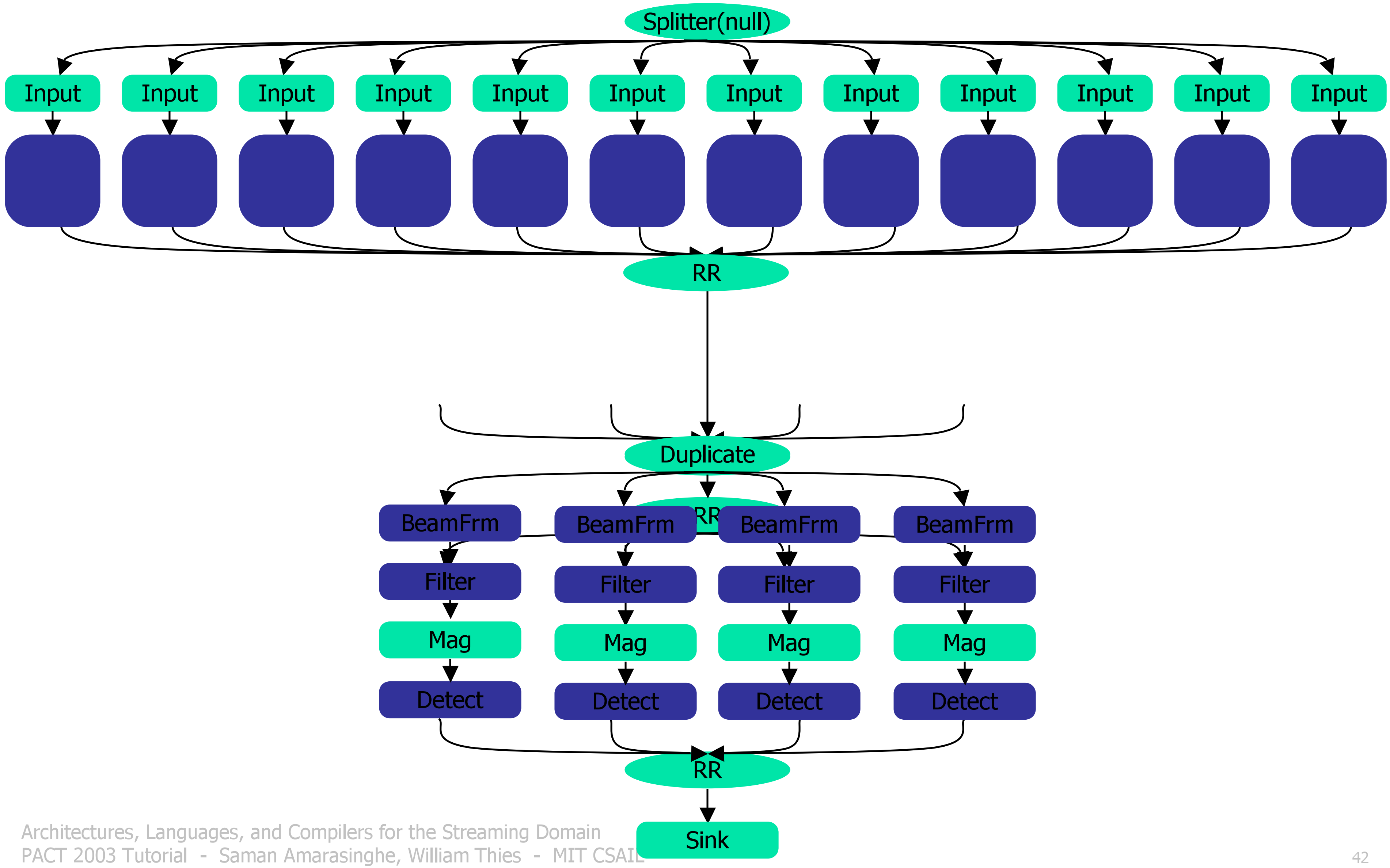

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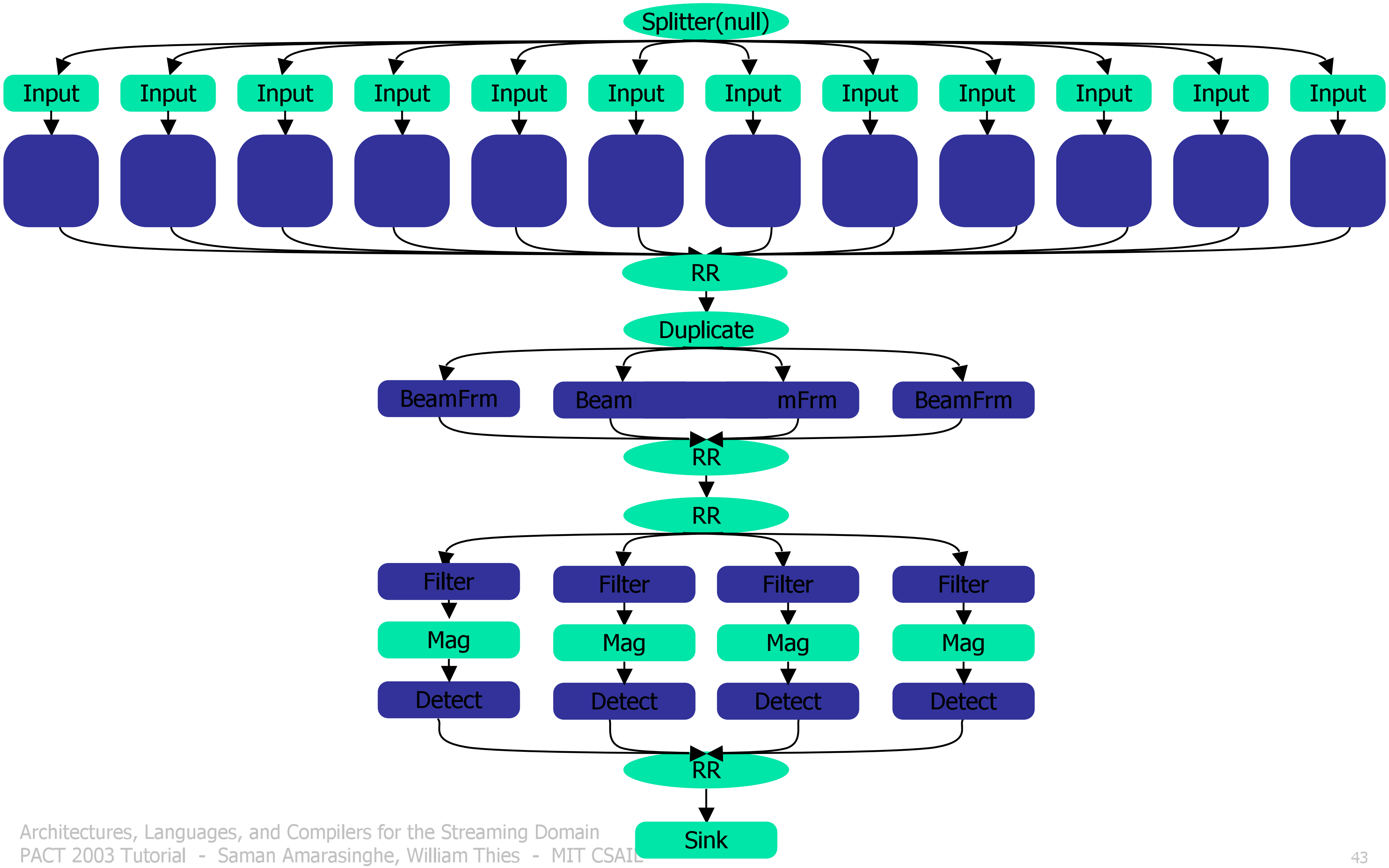
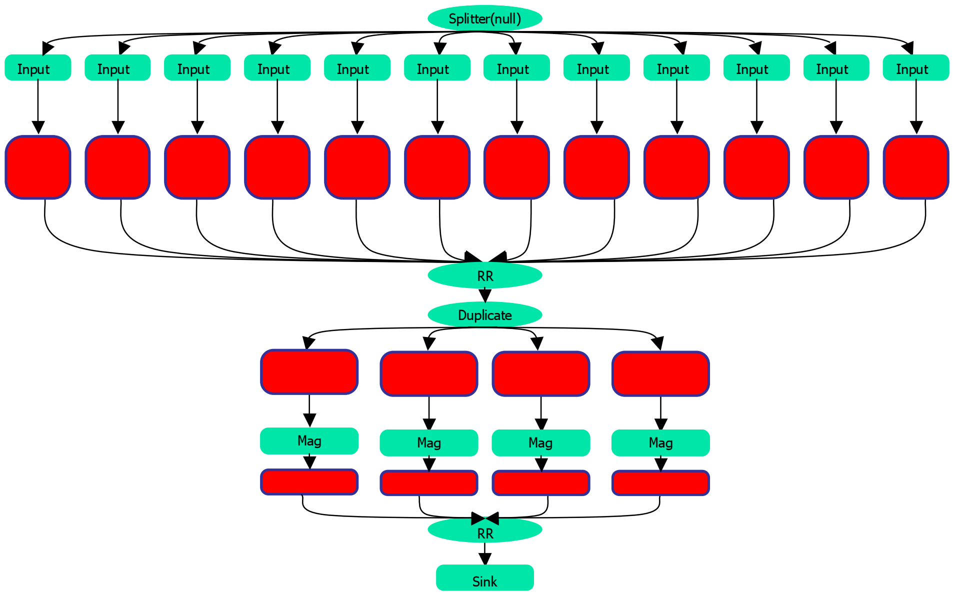
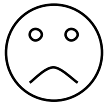


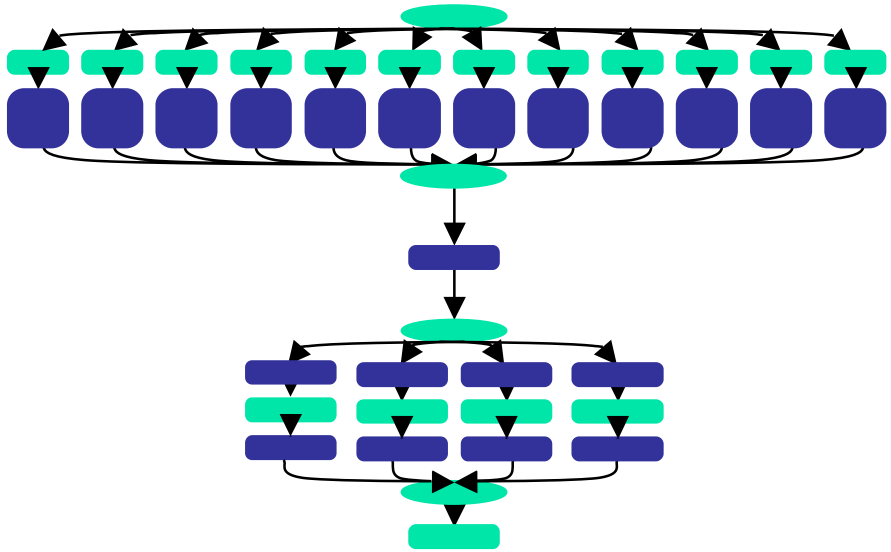
Radar



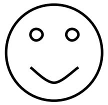
Maximal Combination and Shifting to Frequency Domain



2.4 times as many FLOPS

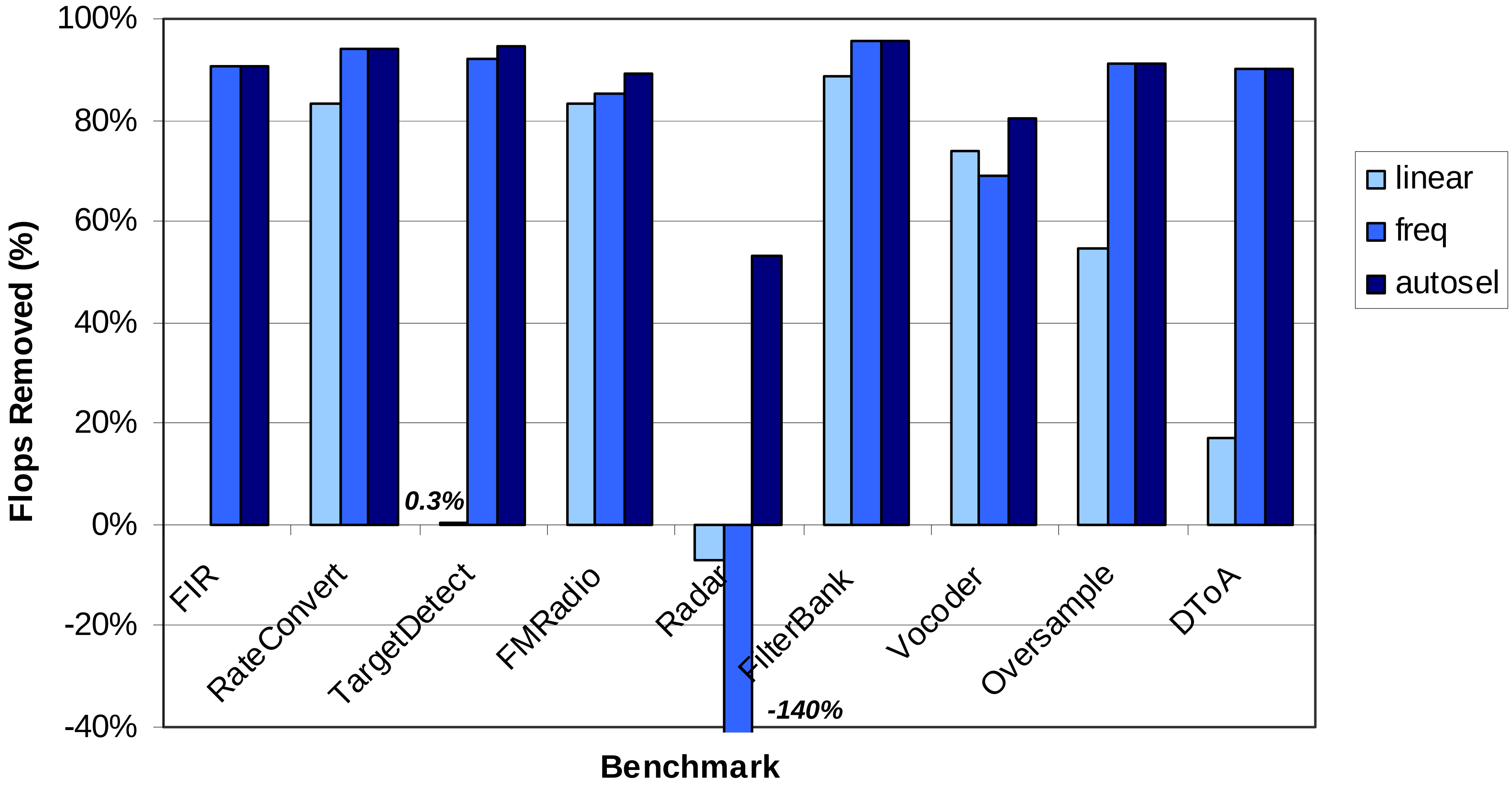


Using Transformation Selection



half as many FLOPS

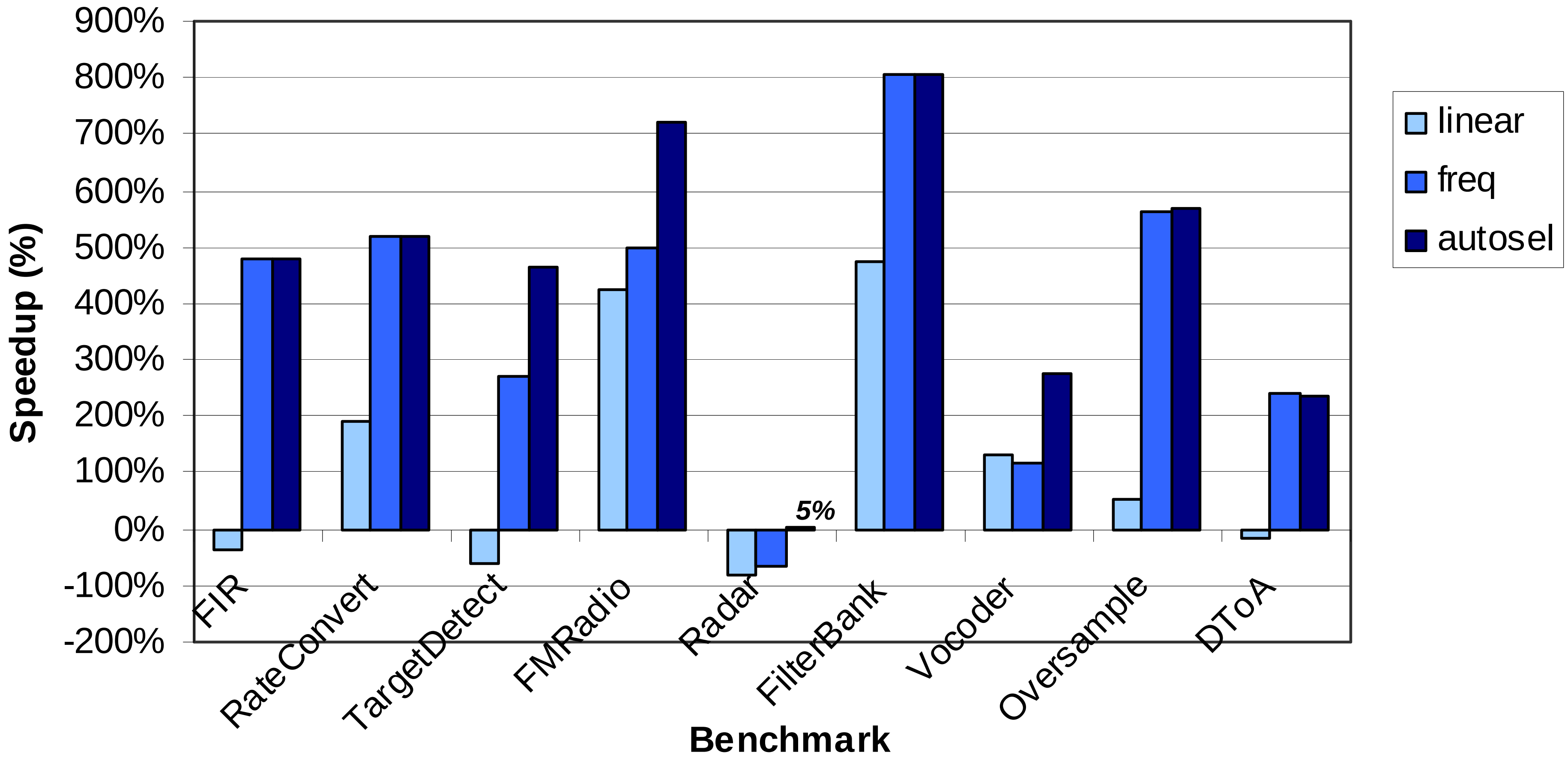
Floating-Point Operations Reduction



Experimental Results

- Fully automatic implementation
  - StreamIt compiler
- StreamIt to C compilation
  - FFTW for shifting to the frequency domain
- Benchmarks all written in StreamIt
- Measurements
  - Dynamic floating-point instruction counting
  - Speedups on a general purpose processor

Execution Speedup



On a Pentium IV

Related Work

- SPIRAL/SPL (Püschel et. al)
  - Automatic derivation of DSP transforms
- FFTW (Friego et. al)
  - Wicked fast FFT
- ADE (Covell, MIT PhD Thesis, 1989)
- Affine Analysis (Karr, Acta Informatica, 1976)
  - *Affine relationships among variables of a program*
- Linear Analysis (Cousot, Halbwatchs, POPL, 1978)
  - *Automatic discovery of linear restraints among variables of a program*

Conclusions

- A DSP Program Representation: *Linear Filters*
  - A dataflow analysis that recognizes linear filters

- Three Optimizations using Linear Information
  - Adjacent Linear Structure Combination
  - Time Domain to Frequency Domain Transformation
  - Automatic Transformation Selection

- First Step in Replacing the DSP Engineer from the Design Flow
  - On the average 90% of the FLOPs eliminated
  - Average performance speedup of 450%

- StreamIt: A Unified High-level Abstraction for DSP Programming
  - Increased abstraction does not have to sacrifice performance

http://cag.lcs.mit.edu/linear/