1. Preliminaries. Physicists often start their investigations by doing a qualitative analysis of the basic features of the problem, typically involving simple calculations. The purpose of these calculations is to give the researcher a sense for the lay of the land, to reveal the basic features of the terrain, before he or she embarks upon a more complicated analysis. Sometimes this kind of qualitative analysis goes by the name of “dimensional analysis”. My goal in this paper is to do the same for beliefs, i.e., to lay out the basic dimensions involved in modeling beliefs and then show how these principles can be implemented in simple back of the envelope type calculations that can tell us quite a bit about the underlying structure of beliefs. We could take as a provisional hypothesis, the following statement:

**Hypothesis:** Beliefs evolve in the interaction of the individual, community and society at large.

Therefore, the natural question is: What are the forces that influence the dynamics of belief change over the multiple scales of individual, community and society? The case of research in language is instructive. For example, we don’t want to believe that beliefs are purely learnt. Without general psychological mechanisms, it is hard to see how beliefs could be acquired at all by individuals (Fodor, 1975). On the other hand, we don’t think beliefs are entirely in our brains, otherwise how would different cultures have different beliefs (Lakoff, 1987, Varela et. al., 1991)? So the million dollar questions remain:

(a) What cognitive mechanisms underlie our ability to form, manipulate and evaluate beliefs?

(b) How do these mechanisms interact with community and cultural factors and give rise to the belief systems that characterize particular social groups?

This paper is an attempt to lay out a methodology to approach these two questions. For the theorist, the domain of beliefs presents an embarrassment of riches. One could say that the road to heaven is paved with various theories of beliefs. We have quantitative rational models of beliefs we *have* (as studied in economics, political science...
and sociology), we have normative accounts of beliefs that we *should be having* (the province of philosophy), and then, there are the *descriptive* approaches to modeling beliefs as seen in the various psychological, sociological, anthropological and historical accounts of the *origins* of beliefs. My goal is to combine the advantages of the two approaches in one package, namely, the quantitative power of the rational models and the etiological insight of the psychological and historical models and to ask “can we develop a formal framework to study the origins and dynamics of beliefs?” The main contribution of this paper is to answer the above question in the affirmative.

Normally, when people study beliefs quantitatively, they are interesting either in evaluating the rationality of the beliefs relative to the actions in the world they represent, or study how beliefs influence the choices that people make. Similarly, belief change, i.e., how people shift from believing one thing to another is studied in terms of a rational model of beliefs, whether it be Bayesian or logical. While these are valid topics for study, my goal is not to adopt a rational framework for modeling beliefs. Instead, I will be concentrating on a structural approach, which focuses on the underlying patterns that allow us to have beliefs in the first place.

Suppose I ask you “Have you seen the novel I was reading yesterday?” and you reply “In the living room”, which is a typical “daily-life” exchange involving our beliefs about the nature of the world. How do I evaluate your statement? When you say it, I have no way of knowing it to be true. Of course, I could check the truth of the statement by going to the living room, but even in order to make that effort, I have to evaluate the plausibility of your statement. What principles do I use for that purpose? As one might imagine, answering these questions requires an analysis of the underlying structure of beliefs.

From the structural perspective, the first step is to delineate some of the regularities underlying beliefs. The next step is to develop a formal scheme in which these regularities can be stated and modeled. The third step is to extract some generative principles that explain those regularities. All ready, at this stage, we should see some interesting explanations of the original phenomena, which cannot be derived in a direct way from the phenomena themselves. Only after this is done do we go to the last step, which is to put numbers on the models, in order to extract detailed quantitative
predictions. In this paper, I will only perform the first three steps, leaving the fourth step for the future.

**A Methodological Note:** While one can study beliefs using various kinds of data, I have chosen to concentrate on belief formation and the principles behind them based on what one would call typical, daily life interactions – the conversations we have, the stories we tell and the usual contexts (work, family etc) in which we live.

So, what are the phenomena that one should focus on, while trying to model beliefs in the structural mode? While I cannot claim to have a complete analysis, here are four that I find crucial:

(1) Beliefs are asymmetric. There is a difference between articulating a belief and accepting it, as there is between uttering a sentence and understanding it. The way beliefs are articulated is fundamentally underdetermined. Suppose a friend told me that New Orleans is suffering from severe flooding. Even if I was an evangelical Christian, I could reply in several ways: that it was an act of god, and we should accept it for what it is, or say it was an act of god and we should stop leading sinful lives. Certainly, there are people who believe one or the other. It is hard, if not impossible to tell how people will string their beliefs together. Even having the same starting point doesn’t ensure that we will end up at the same point.

(2) On the other hand, beliefs are fundamentally social. Beliefs are meant to be shared – which is to say that a person holding a belief has a reasonable expectation that when he communicates the belief to another person, that person can comprehend the belief and evaluate its content within the cultural norms of the community that they belong to. A conjecture in advanced mathematics is a belief only if the recipient of the conjecture can understand the statement. In other words, there are implicit constraints on what constitutes a belief in a given social context. So beliefs have to be coherent and acceptable.

(3) Beliefs are judged by norms of *acceptability* (Bach & Harnish, 1979) that are more general than the usual constraints like predictive power or explanatory adequacy often used in rational models of beliefs. For example, in a Christian community, the recent flooding in New Orleans can be acceptably explained as an act of god, which a scientist may view as being un-predictive. In other words, acceptability is relative to
the social context in which the belief is being stated\(^1\). Note that the same Christian who explains the floods as an act of god might still prefer to go to a hospital when ill rather than to a faith healer.

(4) Beliefs are nested within several contexts, each with its own domain of application, though some contexts may trump others. For example, if someone asks me, “why is this laptop in your living room”, I might say, “because I like writing on the couch”, which in turn is predicated on my having bought a laptop from Dell, the existence of computer technology and so on. A given belief may well be articulated and expanded in several contexts simultaneously as one often does, say as a “life story”\(^2\).

Any theory of beliefs has to take these four constraints in mind. Furthermore, one may need a domain in which all four aspects of beliefs and their interaction with each other can be studied in detail. Is there such a domain? I believe that stories are a good domain for studying the structure of beliefs, for two reasons: first, beliefs are usually communicated in narrative form and secondly, there has been a lot of work by people such as Joseph Campbell (Campbell, 1972) and others in trying to encapsulate the structure of stories. In the next few sections I will argue for using stories as models for beliefs, introduce the prototypical structure of a story informally, model its structure formally and outline a few formal operations underlying the narrative flow.

2. Beliefs and Stories. Why is a belief like a story? Before we go any further, let me first say what I mean by the term “belief”.

**Definition 1.** A Belief is a *stable, communicable* attitude about the nature of the world that is *acceptable* to the community with whom the belief is shared.

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\(^1\) The acceptability based norms are related to the earlier methodological remarks. Of course, a scientific hypothesis is evaluated by the scientific community in ways that are different from our daily beliefs, but for my purposes, the scientific community is a high specialized community with a process of belief formation and evaluation that are markedly different. Of course, we could say that informal conversations and meetings play a much larger role in belief formation within the scientific community as well, they are just not part of the public record of science.

\(^2\) For example, upon being asked, “why is this laptop in the living room?”, I may say “Because I like writing on the couch. I find that it makes me think more informally than if I was writing on a table. In grade school and middle school I was always forced to write on a chair and desk and I cant stand that anymore.”
Of course, not all members of a community have to share the same attitude. A famous example of a lack of shared beliefs is the old story of the blind men and the elephant, where each man, after touching one of the elephant’s parts believes that the elephant is a snake or a tree or some other object, not realizing that they are basing their inference on partial exposure to the facts. That said, until the blind men talk to each other and arrive at a common opinion of the nature of the beast in front of them, they do not have a shared belief. One could say that the blind men are having a “conversation”, each one with his point of view, but they are not telling a common story. Beliefs are like stories in that they are special “conversations”.

A typical conversation involves alternating points of view. For example, suppose I am home and I ask my wife “Have you seen the book I was reading yesterday?” and she replies “which one?” The two of us don’t share the same information (I know what book it was while she does not) and until we can deliberate and come to a position of shared knowledge, we can have a conversation but we cannot share our beliefs about the location of the book. As one can see, this situation is exactly the same as the blind men and the elephant. In a story on the other hand, there is a uniform point of view, namely, the narrators perspective. Indeed, in the story of the blind men and the elephant, the reader is in on the joke because he, unlike the blind men, can see the whole elephant because the narrator has provided a consistent point of view. More formally, I could say the following:

**Definition 2.** A *conversation* is a sequence of communications, \( C_i \), where each communication is a pair \( (S_i, P_i) \) where \( S_i \) is the semantic content of the communication and \( P_i \) is the point of view of the communication.

In general, a conversation can have multiple points of view, one point of view for each speaker (or more if a speaker changes his point of view during the conversation). A story or a belief -in the shared sense of the definition 1 above- is a conversation where everybody shares the *same* point of view.

**Definition 3.** A *story* or a *belief* is a conversation \( C_i \) such that \( P_i = P_j \) for all \( i, j \).
A belief or a story is a special conversation that admits an invariant point of view, or one could say that a story or a belief is a highly non-accidental conversation. A person listening to a conversation will quickly conclude when the conversation is articulating a common belief. Given that beliefs are like stories, one should expect that the structure of stories illuminates the structure of beliefs.

3. Stories. One of the oldest maxims of storytelling, one that can be found in every book on writing fiction or screenplays (McKee, 1997) is the following:

   Conflict drives stories

In order to understand this maxim, let’s start with a simple narrative. Everyday you walk to your office at 8:00 AM in the morning. You take the elevator to the fifth floor, walk to your office door, take your keys out, open the door, boot up your computer and you are ready to go. Now think of a slightly alternative situation:

Suppose you are walking to your office in the morning with the intention of doing some work. You take the elevator to the fifth floor, walk to your office door, fish in your pockets for your keys out, when you realize that you left them on your mantelpiece at home.

The first one of the two narratives is reminiscent of “boy meets girl, they fall in love, get married and live happily ever after”, while the second one is more like “boy meets girl, they fall in love but their families forbid the match”. It is also clear that the first narrative is complete while the second one is not in that the second one invites a further articulation of events until the door is opened.

What does this mean for a theory of stories? Here are three observations:

(a) Both narratives start with a non-accidental event, i.e., Adam meets Eve and not some other possible mate.
(b) Both narratives take two agents (me and the door, Adam and Eve) and incorporate them into a new frame or context.
(c) However, the first narrative stops there, i.e., no other events occur in the new frame that pertains to the original non-accidental event. The second narrative on the other hand involves a new force (family, lack of keys) that “impacts” the stability of the original non-accidental configuration.
Given these intuitions, one may hazard the following hypothesis:

**Hypothesis.** A story is a sequence of stable events such that:

(i) Each event is a non-accidental mapping of two contexts.
(ii) Each event “forces” the previous one.

Of course, stories are not just about stringing non-accidental events together. As we all know, there are millions of stories, since each culture has its own myths, fables, heroic sagas and what not (Leeming, 2001). However, some scholars have pointed out how stories the world over share similar themes and forms, even going to the extent, as Christopher Booker (Booker, 2005) has in his recent book, to say that there are only seven basic plots. For our purpose, we need not even get to the magic number seven, for even the most general fact about stories is enough to stimulate the theorist of beliefs. As we saw in the case of opening a door (or failing to do so), a story at its most basic has the following structure:

Story begins → Life going on as usual → Conflict intervenes → Conflict resolved → End

For example, considers the typical “hero’s journey” which can be caricatured as “The hero is sitting comfortably in his village smoking a cigar, when a demon comes threatening to kill everybody. The hero takes up arms, kills the demon and comes home to a heroic welcome. End.” Or you have the “romantic tragedy” where “Boy meets girl, they fall in love, their families object, they commit suicide”. From a structural perspective we can model the story logic as follows:

Story begins → Protagonist 1 in Context 1, Protagonist 2 in Context 2 → Conflict between the two protagonists forces Context 3 → Conflict resolved in Context 3, story descends back to Contexts 1 and 2 (possibly to a different state) → End

In other words, stories are about moving in and out of nested contexts. Furthermore, each new context has a force that destabilizes the event in the previous context. The theorist needs to model these nested contexts in a manner that takes into account the acceptability and the social character of beliefs? I believe that certain tools from cognitive linguistics (in particular, Talmy’s analysis of force dynamics) can be a
useful part of a formal approach to modeling the dynamics of narrative. Here are the four components of the formal model in my system:

(a) A Superstructure of causality, i.e., a catalogue of the types of forces that are possible. An old but still useful classification, usually attributed to Aristotle, classifies causes into four types: Material (what the object is made of), Formal (what it is, say a cup), Efficient (who made it) and Final (why it was made).

(b) A representation of how causal forces interact in the belief world, which would be something like Talmy’s force dynamics, where agents are classified as having a positive or negative force valuation with respect to each other.

(c) A theory of how new contexts are formed from old ones.

(d) A theory of acceptability of beliefs that shows how beliefs are evaluated by shared societal norms.

Now, let’s see how to introduce these formal elements into the structural description of stories. Given these four elements of a narrative, what is the structure of a story? For example, Romeo and Juliet goes as follows: Boy meets girl, they fall in love, families intervene, tragedy ensues and then the families come together as a result. Structurally, one can identify several elements of this story schema:

(a) Story starts with the principal agents separated – they do not exist within a single frame or context.

(b) The agents come together. Their coming together indexes a new frame which subsumes the earlier two. Furthermore, each agent assigns a positive or negative value to the other (friend or foe for example). A new frame/context is formed. Now, one can think of two sub-possibilities:

(b1) The valuations of the two agents are respected, the story comes to a natural conclusion. For example if the two characters come together, fall in love and get married. End of story. Or, bad man enters town, sheriff gets into a gunfight and shoots him down. End of story.

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3 We can see how society by changing the valuation of one agent for another can strongly influence the structure of beliefs.
(b2) The new context enables forces that change the natural valuation. For example, in Romeo and Juliet, the intervention of the two families causes the tragedy. Here the outcome is different from what one would expect as the default.\(^4\)

(c) In case of (b2), the story keeps moving until no new forces are introduced and the situation is resolved. What is interesting in both the a→b and b→c transitions is that an unexpected event moves the story along. This will be crucial for us later – that the dynamics of stories involve non-accidental events.

To summarize, we can see three structural principles at play – one principle that indexes into a new frame/context when an unexpected event happens, another principle that forms the new context out of two (or more) old ones and the third that indexes an unexpected force in the new context. Let us see how to model these three aspects formally.

4. Partial Order of Contexts. The simplest model of a story, as we have seen, is a linear sequence that moves forward to a higher context by mapping two contexts (that are related non-accidentally) together. We can model the linear story as follows:

Definition 4. A partial ordering is a relation, Γ, that is reflexive, anti-symmetric and transitive.

Definition 5. Let Γ and Δ be two partial orders. Then the mapped partial order Γ●Δ is the smallest partial order that dominates the two.

Note that if Γ and Δ are two partial orders, there are natural inclusion maps Γ→Γ●Δ and Δ→Γ●Δ as well as natural projection maps Γ●Δ→Γ and Γ●Δ→Δ

Definition 6. Let Γ₁ and Γ₂ be two partial orderings of closed directed line segments. A Story is a sequence of inclusions and projections Γ₁,Γ₂→Γ₁●Γ₂→Γ₁,Γ₂.

Note that there is no unique partial order that contains the two partial orders that we start with. The minimal partial order is uniquely determined only if the context is fully defined, which it rarely is. In general there are multiple minimal mappings, which is not a

\(^4\) As mentioned earlier, the asymmetry of beliefs/stories means that they are fundamentally underdetermined. Therefore, the new context is not fully determined by the contexts that feed into it, which leaves open the possibility of unexpected forces.
problem really, since as we saw earlier, beliefs are asymmetric – we want the formal framework to be underdetermined since the underlying beliefs are as well.

The partial orderings show how the contexts are arranged relative to each other, but they don’t tell us how the story moves within a context. For this we need a theory of force dynamics. To start with, one can get away with a relatively simple model of agent-agent interactions.

Suppose we have two agents, x and y, each with its own positive or negative valuation of the other. Let us call these valuations the source valuation. After their interaction (which may invoke other agents, like in Romeo and Juliet) the new context ends up with a target valuation of the two agents with respect to each other. This target valuation has four possible qualitative outcomes, assuming that the source valuation has two choices (positive, negative) to start with, some of which are default and the others are non-accidental. For example, if the two agents have a negative valuation of each other, but the interaction leads to them having a positive valuation of the other, it is non-accidental. Lets us see how this plays out in full, formally.

**Definition 7.** An agent-agent interaction is a map $F: R^2 \rightarrow (+, -)$.

Here is the matrix of expected/default outcomes.

- $A(+) \leftrightarrow B(+) \rightarrow AB$ (they come together)
- $A(-) \leftrightarrow B(-) \rightarrow A$ if $\text{val}(A) > \text{val}(B)$; $\rightarrow B$ if $\text{val}(B) > \text{val}(A)$ (one beats the other)
- $A(+) \leftrightarrow B(-)$ or $A(-) \leftrightarrow B(+)$, indeterminate. (there is no natural solution)

What does this mean though? It means that the story ends in that context if one of the following happens:

(a) If the two agents value each other positively, they come together to form one unit.
(b) If the two agents value each other negatively, they fight it out and the stronger one of the two wins.
(c) The agents are of mixed opinions and they quit, i.e., they decide not to play the game.
Given this default valuation rule, we can also state the non-accidental interaction in which context shift happens as follows:

**Principle of context change**: A story moves from one context to another when the target valuation does not have the expected sign.

The story moves on to the next frame if one of the following happens:

(a) The two agents value each other positively but they do not form a unit.
(b) The two agents value each other negatively and the weaker one wins.
(c) The two agents have mixed opinions but they get to have the same valuation, positive or negative.

A story (or a belief), as it develops, invokes new characters (agents) and new forces, but it can also loop back and change the mutual valuation of old characters (agents) and contexts. Enemies can become friends and vice versa. The above scheme is capable of generating infinitely long stories that in principle can loop back\(^5\). But, the fact that the story or belief can go on for ever does not mean it will. Its unfolding depends on the acceptability of new context shifts each time that happens. In the next section, I try to address this issue.

### 5. Acceptability and Such

Now, what we want to encode in our framework is that people, for the most part, do not find beliefs that are too far away or inconsistent with their own beliefs acceptable. How to model this? Similarity metrics that measure how close a belief is to another are hard to come by. Furthermore, how to compare beliefs that are not that similar to each other? For example, if I believe in a vengeful God, I might believe that Hurricane Katrina was a result of the sins of the people of New Orleans. How is the second belief like the first? Indeed the second belief is more like an inference from the first than anything else. So what we need is a theory of inference built around notions

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\(^5\) A fascinating question is whether stories are finite structures that lead to an infinite inferential leap. For example, the series of negative and positive valuations in a given story form a tree structure. As the story progresses, and loops come into play, the shape of the tree approximates a fractal structure. A natural question is whether the limiting fractal tree structure is inferred by the listener? In other words, is a listener, upon hearing a story, capable of inferring an infinite recursive structure?
of acceptability. For that task, partial orders are more than adequate. Partial orderings give a precise mathematical framework in which to study notions like inference and acceptability. Here is how:

**Definition 8.** Let $\Gamma$ be a partial ordering and let $d(a_{\text{max}}, a_{\text{min}})$ be the number of steps from a maximal element, $a_{\text{max}}$, to a minimal element, $a_{\text{min}}$. The depth of the partial ordering is $\text{Maximum}(d(a_{\text{max}}, a_{\text{min}}), \text{over all pairs } (a_{\text{max}}, a_{\text{min}}))$.

We can define the relative depth of a story as follows:

**Definition 9.** Let $\Gamma_1$ and $\Gamma_2$ be the two protagonists of the story with their own individual partial orders. Then the relative depth of the story is the $\text{Depth}(\Gamma_1 \bullet \Gamma_2) - \text{min}(\text{Depth}(\Gamma_1), \text{Depth}(\Gamma_2))$.

In simple terms, the relative depth measures the number of additional contexts it takes to enfold both the protagonists of the story. This gives an easy measure of acceptability, which is that people would like to keep the number of new contexts down, i.e., if you have to invoke a large number of intermediate contexts to wrap up the story, then it is not believable. In more abstract terms, we can state the restriction on the number of new contexts in terms of *locality*, where by locality, I mean the following: a new belief should be a small perturbation of old beliefs. In the case of partial orders, the locality condition can be stated as follows:

**Locality Condition.** Given two partial orders, the mapped partial order is acceptable only if the depth of the new partial order is a small perturbation of the original two.

So the locality condition gives us a principle by which we can study the dynamics of beliefs. In the extreme situation, we have the following condition:

**The depth 1 condition:** A story is acceptable only if the context jump is of depth 1.

Suppose we go back to the earlier example. I am working on my laptop in the living room. You come in and ask me “why is the computer here” to which I reply “I like working on the laptop in the living room”. So far, there is nothing new. A default context has been set up, of answering the question in the proximate spatio-temporal context. But if you say “but why the PC?” there is a conflict, which cannot be resolved in the default
context. What do I do? I can say: “Because my Mac is being repaired” or “I have switched to PC’s” or “It is my brother’s laptop, I am loading some music in it for him”. There is no correct answer, but I cannot say: “Because computers have been invented” except as a wisecrack, since it is not a minimal answer. The question arises, whether the locality principle applies to other examples of belief change, more real world kind of examples, so to speak. I believe so, but the full analysis will have to wait for another day.

6. Summary. In this paper, I outlined a few steps towards formulating a structural theory of beliefs, organized around the idea that beliefs are like stories. The fundamental lesson for belief change is the following: new beliefs come from an acceptable mixture of old beliefs, which maps neatly on to the old idea that stories have to a conflict as a driving force. From a structural perspective, we could say belief change occurs when a new context is induced by two old contexts. Finally, one can combine the contextuality and acceptability conditions into one locality condition for partial orders. This locality condition predicts when new beliefs are acceptable.

References


