Conformity, Consistency and Cultural Heterogeneity

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Overview

Culture:

Behavior, Beliefs, Roles, Categories, Values, Artifacts, Networks

GameS Theory

Agents evolve automata to play multiple games

The automata exhibit something like culture

- consistency within an individual (nice)
- consistency across people within group
- behavioral stickiness
- contextual effects
- suboptimal behavior

Simplest Model

N agents

M attributes.

A values per attributes

$(a_1, a_2, ...a_M)$ agent

Rules

External Coordination Rule (Voter Model, Axelrod's Culture Model: *The first paired agent randomly chooses an attribute and sets the value of that attribute equal to the value that the other agent assigns to that attribute.*

(0,0,0,0,1) meets (1,1,0,0,1) and becomes (1,0,0,0,1)

Internal Consistency Rule: *The agent randomly chooses two random distinct attributes and changes the value of the first attribute to match the value of the second.*

(0,0,0,1,1) looks internally and becomes (0,0,0,0,1)

Consistent Coordination Rule CC(p): *An agent is chosen at random and with probability* p *the internal consistency rule is chosen and with probability* $(1-p)$ *the external coordination rule is chosen.*

Claim 1 *Consider a population of* N *agents with* M *binary attributes, and an agent whose first* x *attributes take value e 0. Let* S_i *equal the number of other agents in the population who have value 0 on attribute* i*, the probability that* x *increases by one equals*

$$p\frac{x(M-x)}{M(M-1)} + (1-p)\frac{1}{M}\sum_{i=x+1}^{M}\frac{S_i}{N-1}$$

and the probability that x *decreases by one equals*

$$p\frac{x(M-x)}{M(M-1)} + (1-p)\frac{1}{M}\sum_{i=1}^{x}\frac{N-1-S_i}{N-1}$$

States of System

State	*Agents*	*Prob*
Coordinated & Consistent (C&C)	(a,a) (a,a)	$\frac{1}{8}$
Consistent Not Coordinated (CON)	(a,a) (b,b)	$\frac{1}{8}$
Coordinated Not Consistent (CRD)	(a,b) (a,b)	$\frac{1}{8}$
Off By One (OBO)	(a,b) (a,a)	$\frac{1}{2}$
Not Coordinated Not Consistent (NOT)	(a,b) (b,a)	$\frac{1}{8}$

Consistency Model

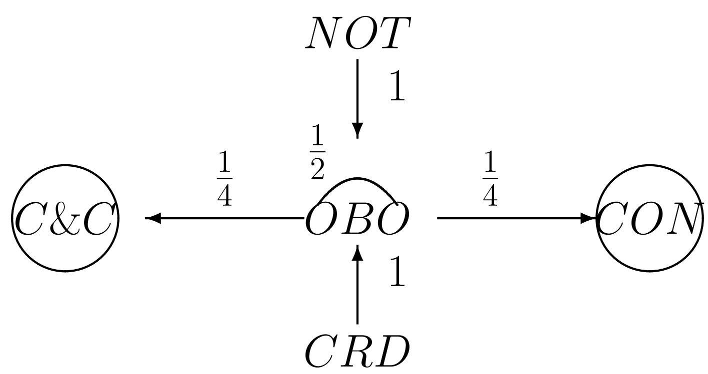

Figure 1: The Dynamics of the Internal Consistency Rule

Claim 2 *The expected time to equilibrium for the Internal Consistency Rule equals* $1\frac{3}{4}$.

Coordination Model

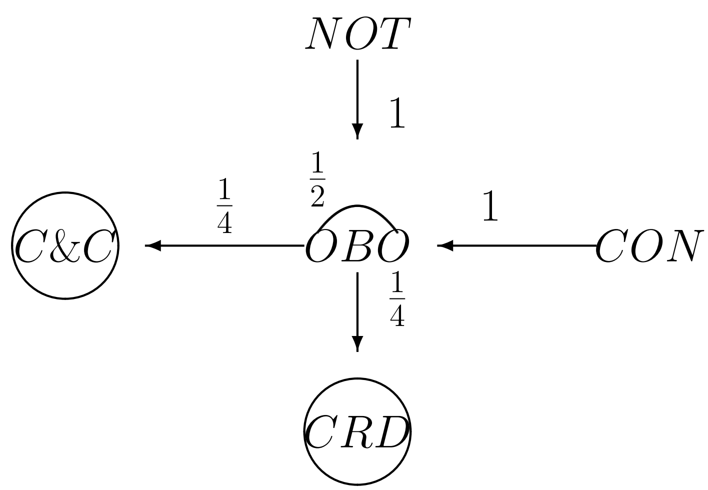

Figure 2: The Dynamics of the External Coordination Rule

Claim 3 *The expected time to equilibrium for the External Coordination Rule equals* $1\frac{3}{4}$.

CC(p) Model

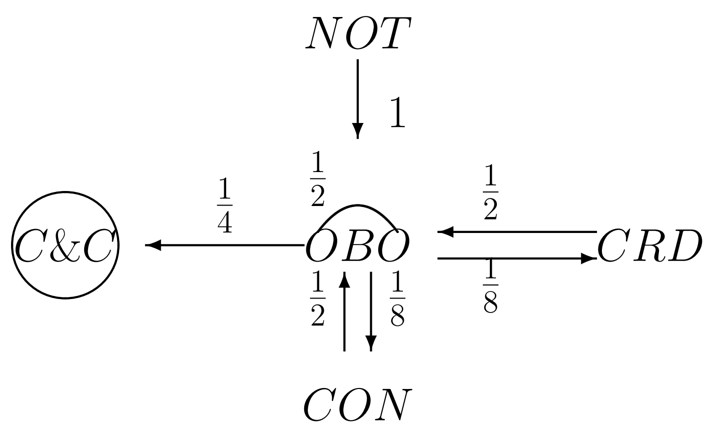

Figure 3: The Dynamics CC($\frac{1}{2}$)

Claim 4 *The expected time to equilibrium for the CC(p) Rule equals* $1\frac{7}{8} + \frac{1}{p(1-p)}$

Note 1: $p = \frac{1}{2}$ this gives $5\frac{7}{8}$

Note 2: time to convergence minimized at $\frac{1}{2}$

1+1 = 3: Equilibrium Distributions with Noise

ϵ probability of random flip

Consistency Model

$T/T+1$	CNS	OBO	NCN
CNS	$1-\epsilon$	ϵ	0
OBO	$\frac{1}{2}$	$\frac{1-\epsilon}{2}$	$\frac{\epsilon}{2}$
NCN	0	1	0

$P_{CNS} = \frac{1}{1+2\epsilon+\epsilon^2}$

$P_{OBO} = \frac{2\epsilon}{1+2\epsilon+\epsilon^2}$

$P_{NCN} = \frac{\epsilon^2}{1+2\epsilon+\epsilon^2}$

Coordination Model

$T/T+1$	CDC	OBO	NCD
CDC	$1-\epsilon$	ϵ	0
OBO	$\frac{1}{2}$	$\frac{1-\epsilon}{2}$	$\frac{\epsilon}{2}$
NCD	0	1	0

$P_{CDC} = \frac{1}{1+2\epsilon+\epsilon^2}$

$P_{OBO} = \frac{2\epsilon}{1+2\epsilon+\epsilon^2}$

$P_{NCD} = \frac{\epsilon^2}{1+2\epsilon+\epsilon^2}$

CC(p) Model

T /T+1	$C\&C$	OBO	CRD	CON	NOT
$C\&C$	$1-\epsilon$	ϵ	0	0	0
OBO	$\frac{1}{4}$	$\frac{(1-\epsilon)}{2}$	$\frac{p+\epsilon-\epsilon p}{4}$	$\frac{1-p-\epsilon p}{4}$	$\frac{\epsilon}{4}$
CRD	0	$1-p+\epsilon p$	$p-\epsilon p$	0	0
CON	0	$p+\epsilon+\epsilon p$	0	$1-p-\epsilon+\epsilon p$	0
NOT	0	1	0	0	0

$P_{C\&C} = \frac{1}{1+4\epsilon+\epsilon^2+\alpha\epsilon+\alpha^{-1}\epsilon}$

$P_{OBO} = \frac{4\epsilon}{1+4\epsilon+\epsilon^2+\alpha\epsilon+\alpha^{-1}\epsilon}$

$P_{CRD} = \frac{\alpha\epsilon}{1+4\epsilon+\epsilon^2+\alpha\epsilon+\alpha^{-1}\epsilon}$

$P_{CON} = \frac{\alpha^{-1}\epsilon}{1+4\epsilon+\epsilon^2+\alpha\epsilon+\alpha^{-1}\epsilon}$

$P_{NOT} = \frac{\epsilon^2}{1+4\epsilon+\epsilon^2+\alpha\epsilon+\alpha^{-1}\epsilon}$

$\alpha = \frac{(p+\epsilon-\epsilon p)}{(1-p+\epsilon p)}$

Numerical Experiments

External Coordination Rule: Time to Convergence

Number of Dimensions	*Number of Attribute Values* 2	3	4	5
4	35.10 (1.20)	61.34 (2.08)	127.60 (9.02)	237.88 (14.88)
5	34.60 (0.99)	68.86 (3.49)	132.76 (8.40)	223.32 (15.02)

Internal Consistency Rule: Time to Convergence

Number of Dimensions	*Number of Attribute Values* 2	3	4	5
4	7.52 (0.11)	16.92 (0.26)	31.04 (0.54)	49.68 (0.77)
5	8.32 (0.17)	18.96 (0.33)	34.56 (0.59)	53.72 (0.74)

$CC(\frac{1}{2})$ Rule: Time to Convergence

Number of Dimensions	*Number of Attribute Values* 2	3	4	5
4	70.78 (3.26)	168.80 (9.72)	394.78 (27.66)	869.56 (54.75)
5	72.62 (2.88)	195.18 (10.55)	426.82 (30.00)	851.72 (57.31)

Observations

Heterogeneity consistent with forces for convergence

If 1+1 = 3, does 1+1+1 = 4,6, or 9?

Heterogeneity