Why How We Learn Matters

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Overview

- BIG Picture
- Game Theory Basics
- Nash Equilibrium
  - Equilibrium something about the other.
  - Stability
  - Basins
- Learning Rules
- Why Learning Matters

Methodological Question

Do we construct simple, illustrative, insight generating models (PD, Sandpile, El Farol) or do we construct high fidelity, realistic models?

My Answer: BOTH!

Both types of models are useful in their own right. In addition, each tells us something about the other.

My Answer: BOTH!

Knowledge of simple models helps us construct better high fidelity models.

Large models show if insights from simple models still apply.

Today's Exercise

How does how agents learn influence outcomes?

Examples

Best respond to current state

Better respond

Mimic best

Mimic better

Include portions of best or better

Random with death of the unfit

Equilibrium Science

We can start by looking at the role that learning rules play in equilibrium systems.   This will give us some insight into whether they'll matter in complex systems.

Game Theory

- Players
- Actions
- Payoffs

“Equilibrium” Based Science

Step 1: Set up game

Step 2: Solve for equilibrium

Step 3: Show how equilibrium depends on parameters of model

Step 4: Provide empirical support

Is Equilibrium Enough?

Existence: Equilibrium exists

Stability: Equilibrium is stable

Attainable: Equilibrium is attained by a learning rule.

Stability

Stability can only be defined relative to a learning dynamic. In dynamical systems, we often take that dynamic to be a best response function, but with human actors we need not assume people best respond.

Existence Theorem

Theorem: Finite number of players, finite set of actions, then there exists a Nash Equilibrium.

Pf: Show best response functions are upper hemi continuous and then apply Kakutani's fixed point theorem

Battle of Sexes Game

	EF	CG
EF	3,1	0,0
CG	0,0	1,3

Three Equilibria

	1/4	3/4
3/4	3,1 ★	0,0
1/4	0,0	1,3 ★

Unstable Mixed?

	1/4 +e	3/4 - e	
EF ★	3,1	0,0	3/4 + 3e
CG	0,0	1,3	3/4 - 3e

Note the Implicit Assumption

Our stability analysis assumed that Player 1 would best respond to Player 2’s tremble. However, the learning rule could be **go to the mixed strategy equilibrium**. If so, Player 1 would sit tight and Player 2 would return to the mixed strategy equilibrium.

Empirical Foundations

We need to have some understanding of how people learn and adapt to say anything about **stability**.

Classes of Learning Rules

Belief Based Learning Rules: People best respond given their beliefs about how other people play.

Replicator Learning Rules: People replicate successful actions of others.

Examples

Belief Based Learning Rules:

Best response functions

Replicator Learning Rules:

Replicator dynamics

Stability Results

An extensive literature provides conditions (fairly week) under which the two learning rules have identical stability property.

Synopsis: *Learning rules do not matter*

Basins Question

Do games exist in which best response dynamics and replicator dynamics produce very different basins of attraction?

Question: *Does learning matter?*

Best Response Dynamics

x = mixed strategy of Player 1

y = mixed strategy of Player 2

$$dx/dt = BR(y) - x$$

$$dy/dt = BR(x) - y$$

Replicator Dynamics

$dx_i/dt = x_i(\ \pi_i - \pi^{ave})$

$dy_i/dt = y_i(\ \pi_i - \pi^{ave})$

Symmetric Matrix Game

	A	B	C
A	60	60	30
B	30	70	20
C	70	25	35

Conceptualization

Imagine a large population of agents playing this game. Each chooses an action. The population distribution over actions creates a mixed strategy. We can then study the dynamics of that population given different learning rules.

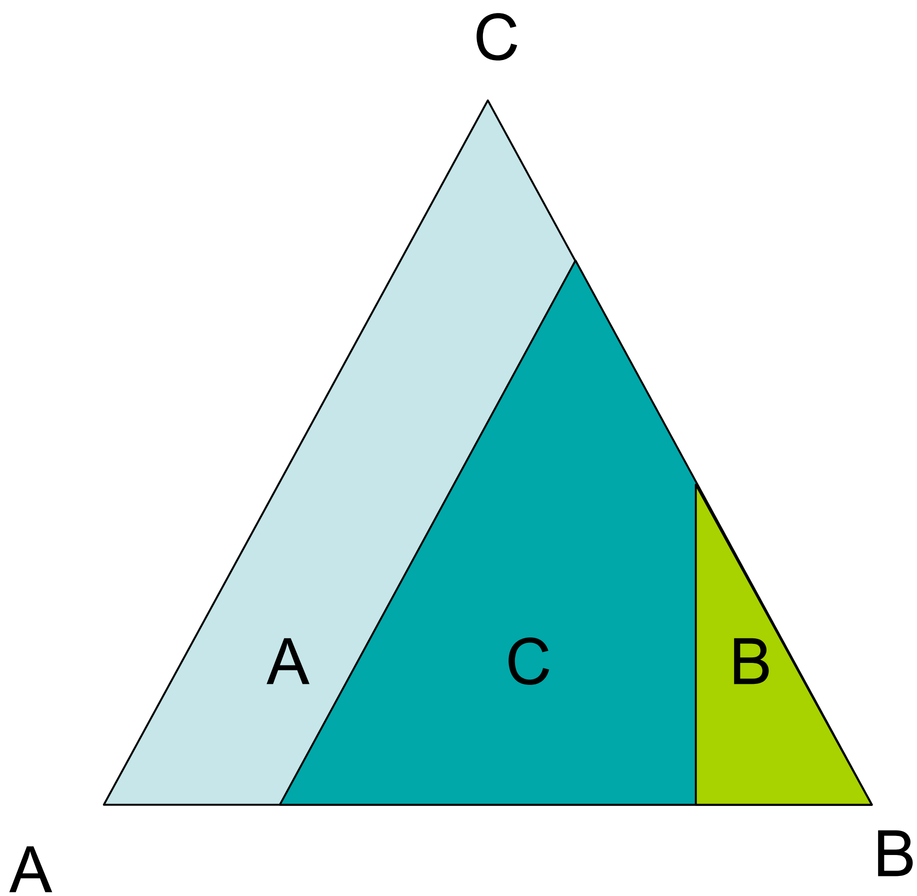
Best Response Basins

	A	B	C
A	60	60	30
B	30	70	20
C	70	25	35

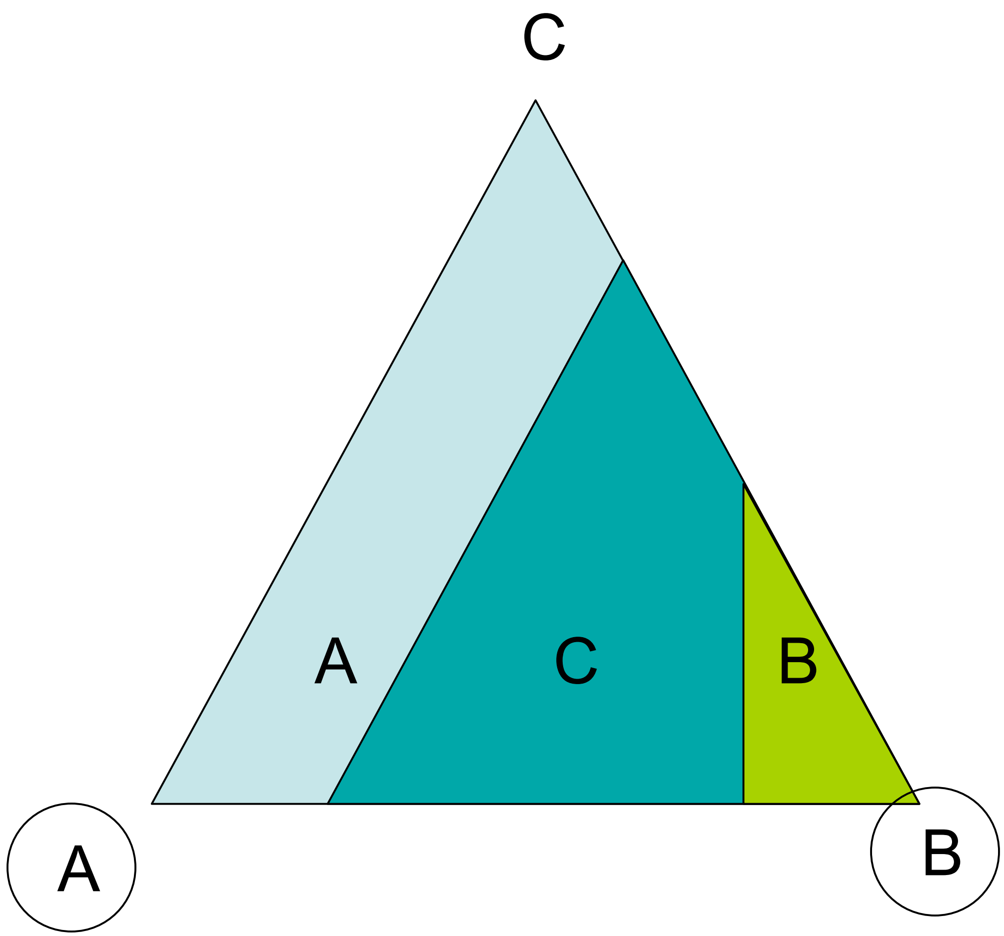
A > B iff $60p_A + 60p_B + 30p_C > 30p_A + 70p_B + 20p_C$

A > C iff $60p_A + 60p_B + 30p_C > 70p_A + 25p_B + 35p_C$

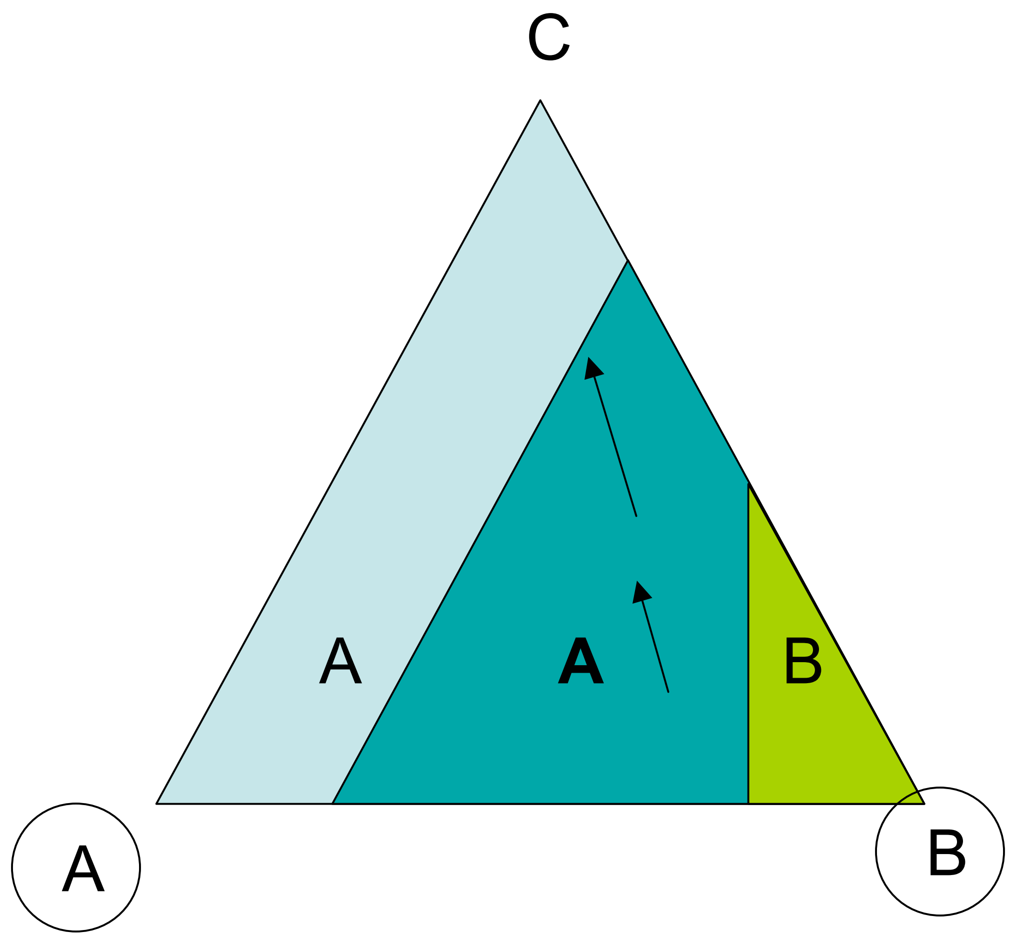
Best Response Basins



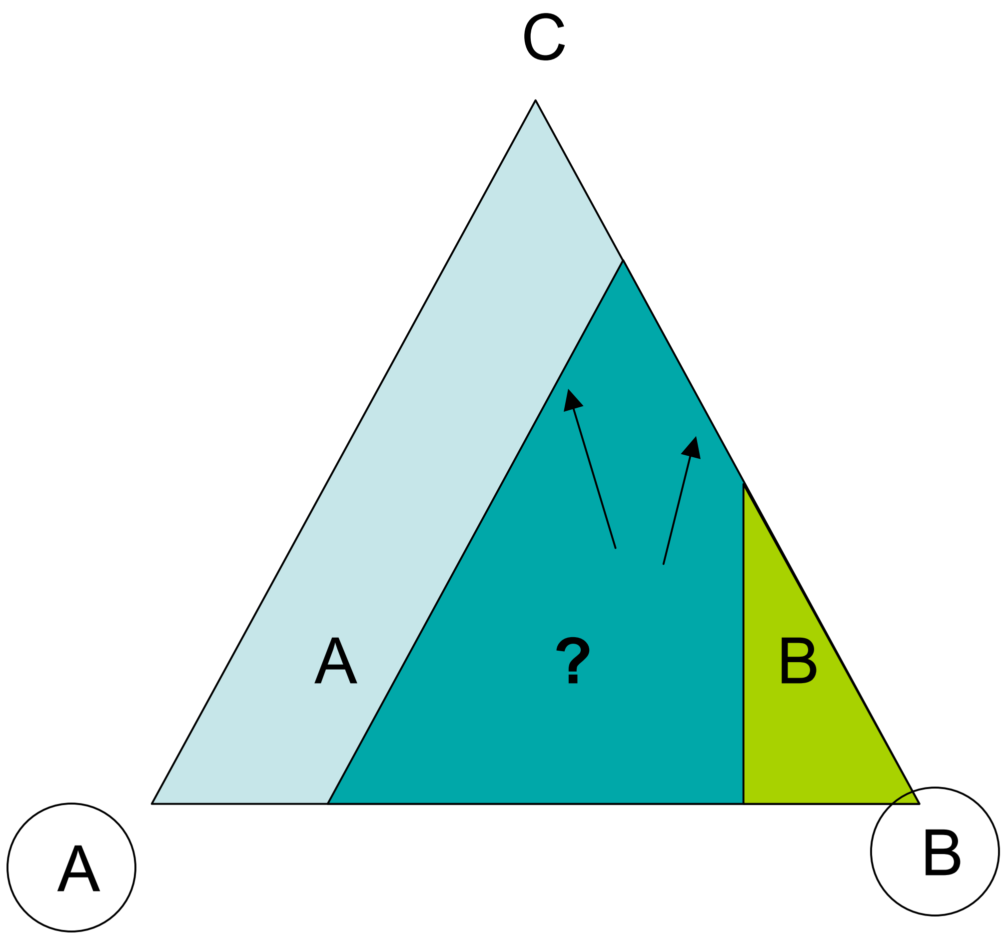
Stable Equilibria



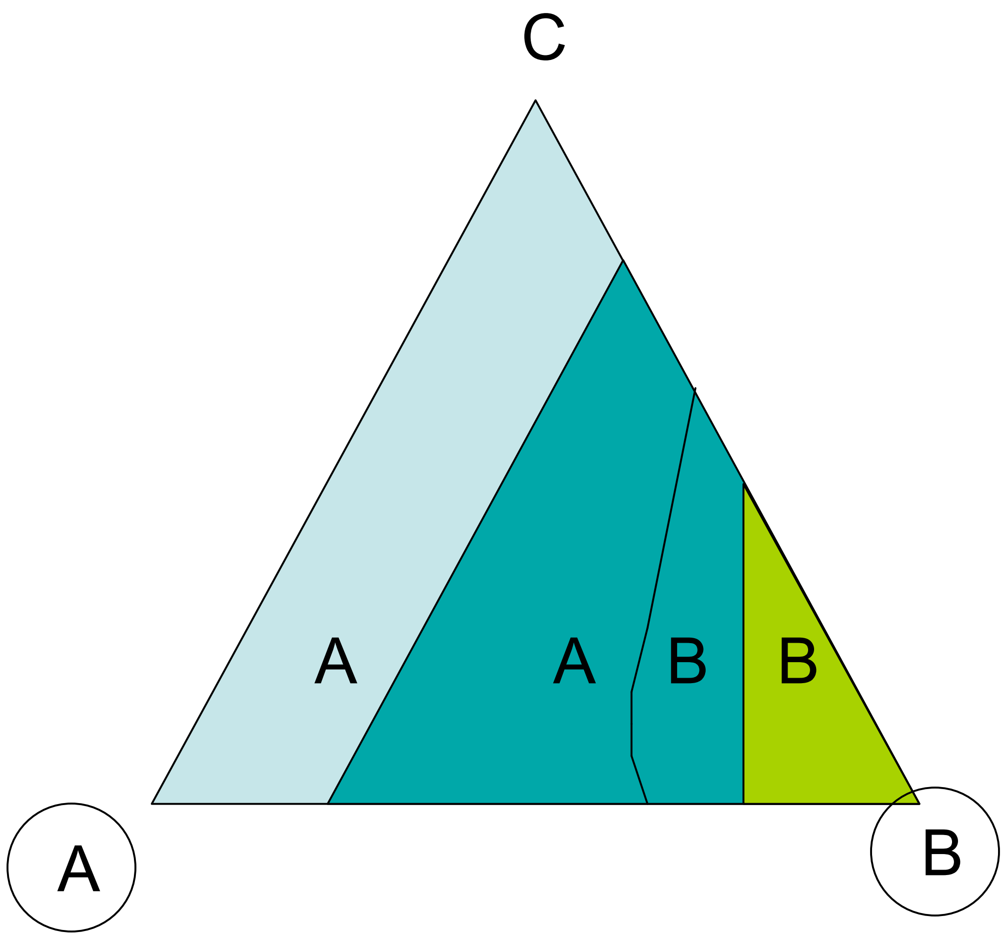
Best Response Basins



Replicator Dynamics



Replicator Dynamics Basins



Recall: Basins Question

Do games exist in which best response dynamics and replicator dynamics produce very different basins of attraction?

Question: *Does learning matter?*

Conjecture

For any $\varepsilon > 0$, There exists a symmetric matrix game such that the basins of attraction for distinct equilibria under continuous time best response dynamics and replicator dynamics overlap by less than ε

Results

Thm 1 (SP): Can be done if number of actions goes to infinity

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Thm 2 (RG): Can be done if number of actions scales with $1/\varepsilon$

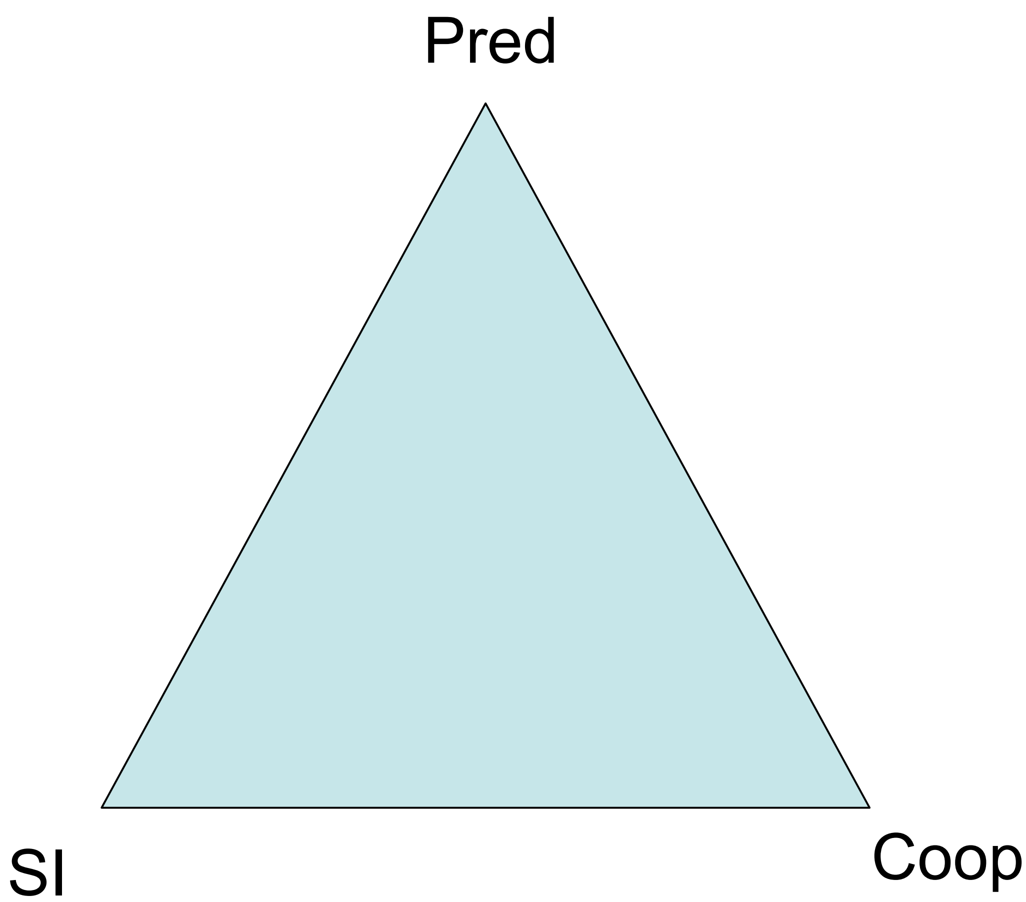
Thm 3 (RG): Cannot be done with two actions.

Thm 4 (SP): Can be done with four!

Collective Action Game

	SI	Coop	Pred	Naive
SI	2	2	2	2
Coop	1	N+1	1	1
Pred	0	0	0	N^2
Naive	0	0	$-N^2$	0

Intuition: Naïve Goes Away



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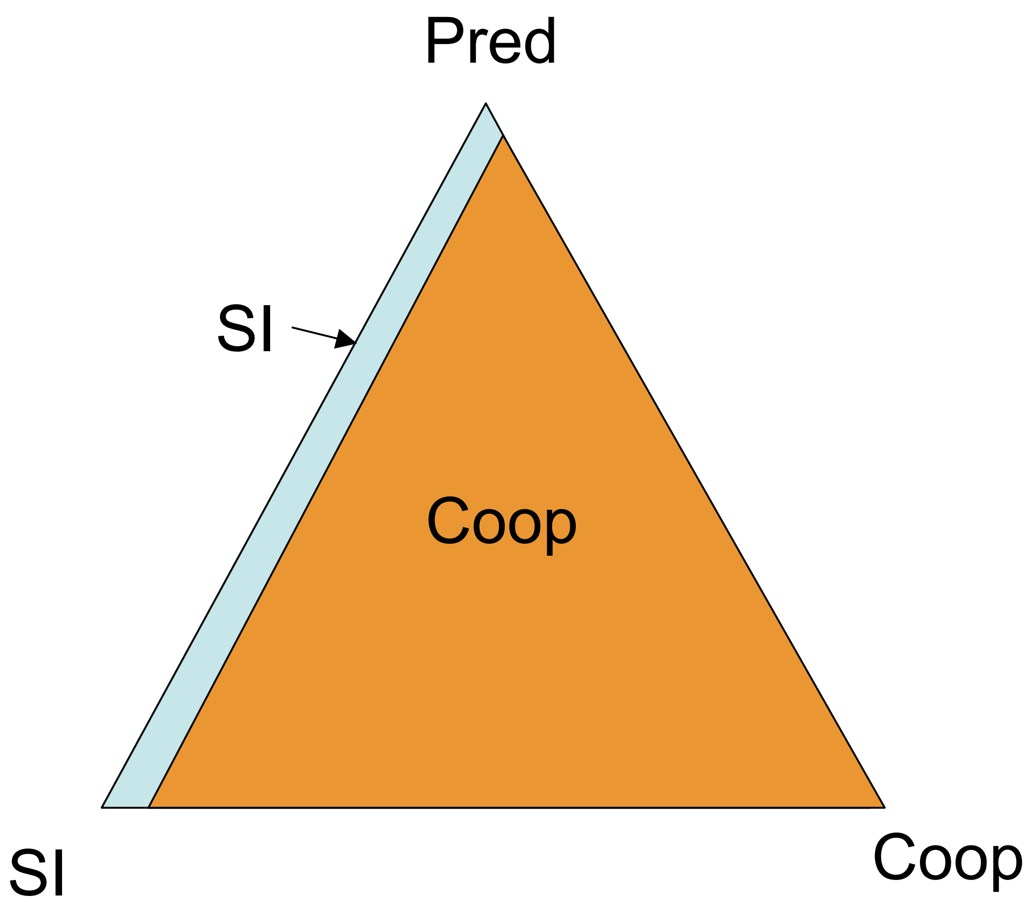
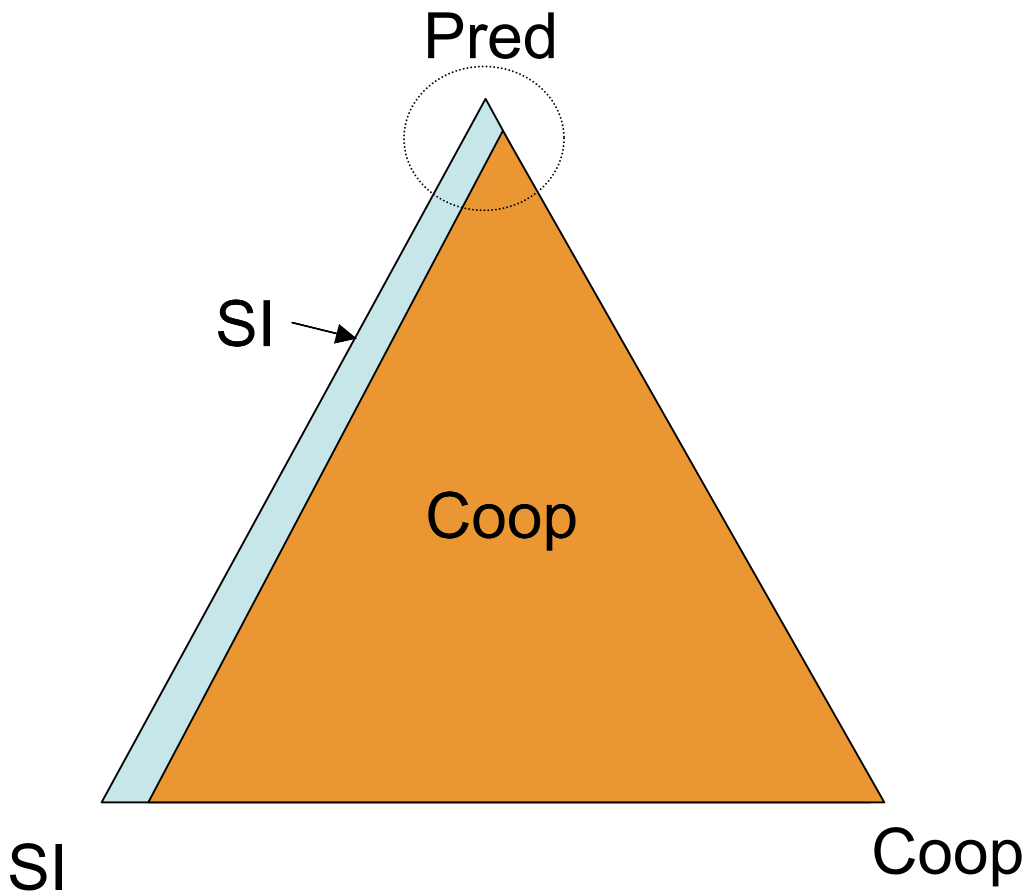
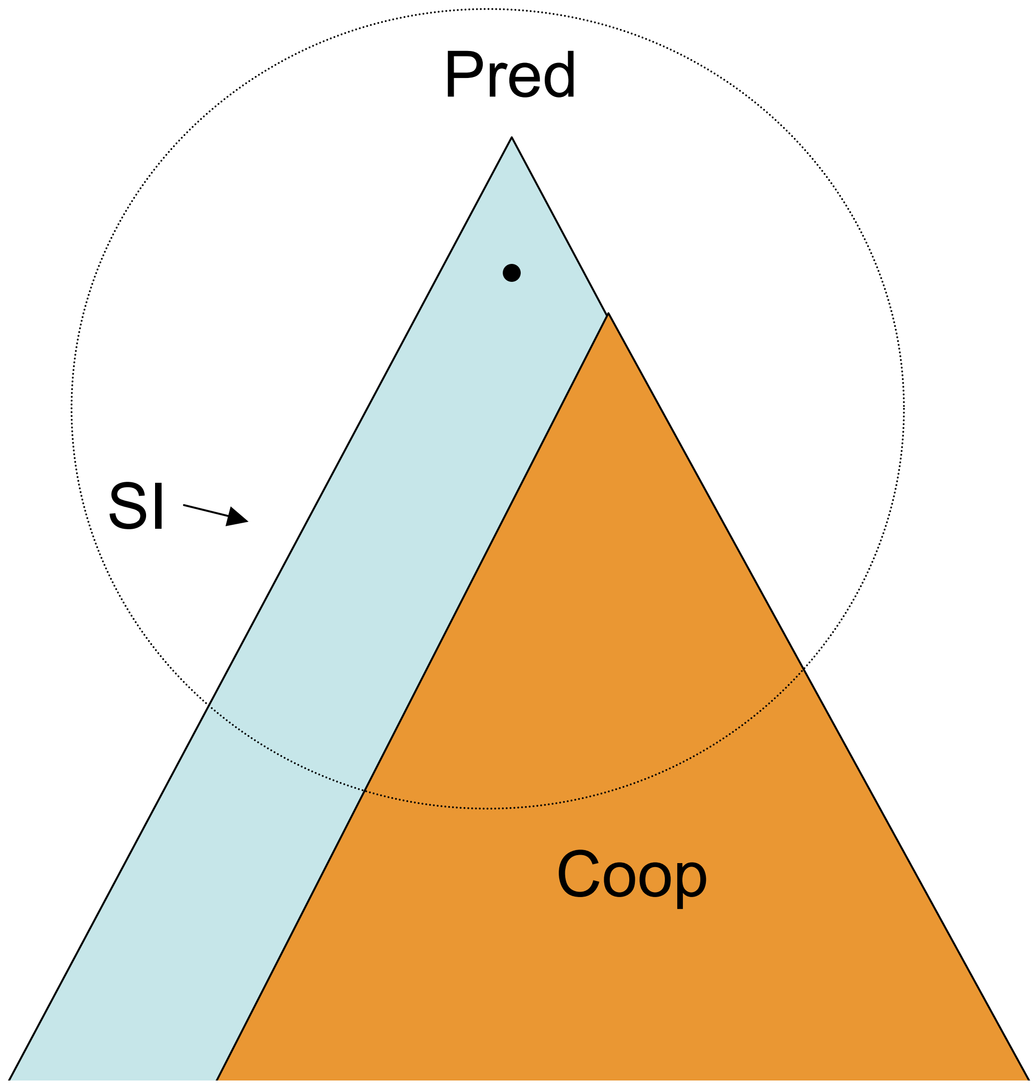


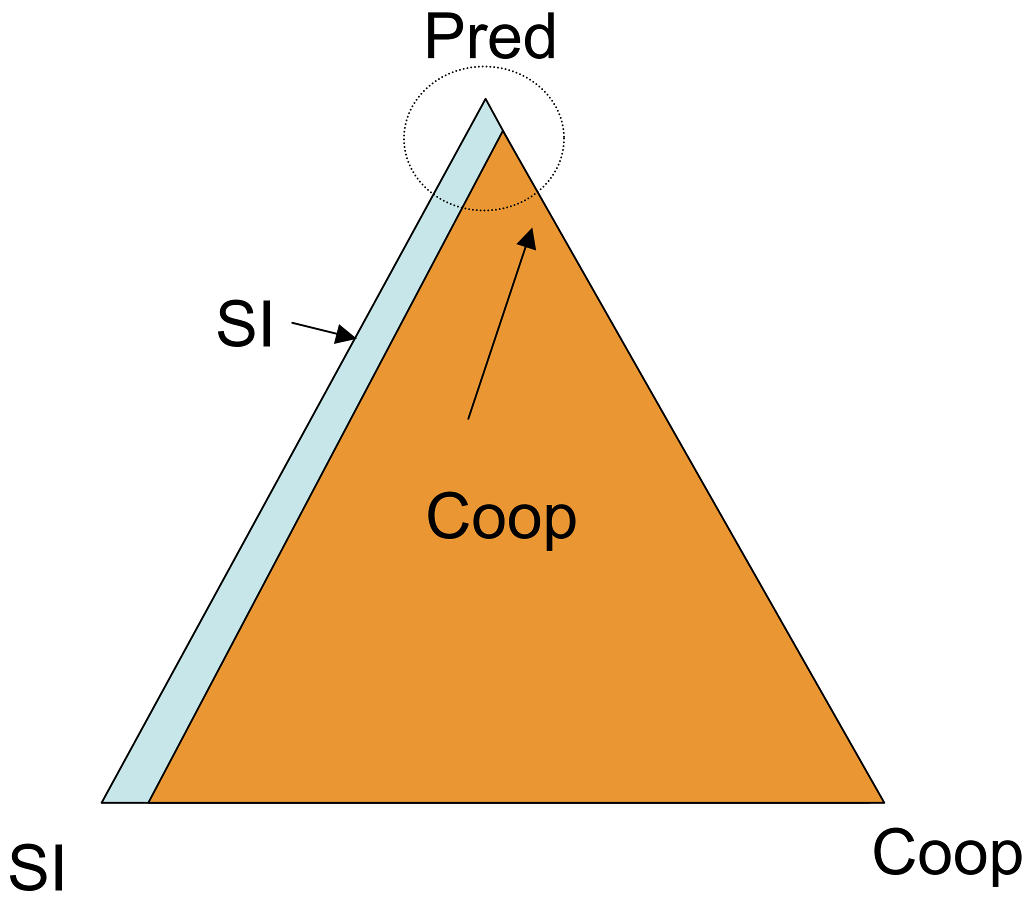
Best Response



Best Response



Replicator



The Math

$$dx_i/dt = x_i(\ \pi_i - \pi^{ave})$$

$$\pi^{ave} = 2x_S + x_C\,(1+Nx_C)$$

$$dx_c/dt = x_c[(1+ Nx_{C-} - 2x_S - x_C\,(1+Nx_C)]$$

$$dx_c/dt = x_c[(1+ Nx_C)(1- x_C) - 2x_S]$$

Choose $N > 1/\varepsilon$

Assume $x_c > \varepsilon$

$dx_c/dt = x_c[(1+ Nx_C)(1- x_C) - 2x_S]$

$dx_c/dt > \varepsilon\ [2(1- \varepsilon) - 2(1- \varepsilon)] = 0$

Therefore, x_c always increases.

Aside: Why Care?

Replicator dynamics often thought of as being *cultural learning*. Best response learning thought of as self interested learning. Societies differ by degree of individualism. These results show that how society is structure could affect the ability to solve collective action problems.

Results (Cont'd)

Conjecture (SP): There does not exist a game with three actions such that the basins have vanishing overlap.

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Conjecture (SP): There does not exist a game with three actions such that the basins have vanishing overlap.

Thm 5 (RG): There **does exist** a game with three actions such that the basins have vanishing overlap

Convex Combinations

What happens if the population learns using the following rule

a (Best Response) + (1-a) Replicator

Convex Combinations

What happens if the population learns using the following rule

p (Best Response) + (1-p) Replicator

Conjecture: Anything!

Claim: *There exists a class of games in which Best Response, Replicator Dynamics, and any fixed convex combination select distinct equilibria with probability one.*

BR -> A

Rep - > B

p Br + (1-p) Rep -> C

Genericity of Results

Example

Proof for a functional form

Proof for a class of functions

General Result

What do we have?

Examples: 3 and 4 dimensions

General Result: 3 dimensions is minimal.

Another General Result

Recall that in the (very cool) four dimensional example, that initially predatory behavior was a best response with probability one. Moreover, it was not an equilibrium.

Turns out, this is always true!!

Theorem: In any symmetric matrix game for which best response and replicator dynamics attain different equilibria with probability one, there exists an action A that is both an initial best response with probability one and is **not** an equilibrium.

From Science to Art

Insight: If I’m constructing a large scale ABM and some actions will win for a short time and then die off, then I had better experiment with lots of learning rules.